CHAPTER TWO

Solutions for Section 2.1

EXERCISES

1. No. Reordering the expression on the left gives us $3x \cdot x$ or $3x^2$ but not $4x$.

5. Yes. The expression $(3x)(4y)(2x)$ can be reordered as $3 \cdot 4 \cdot 2 \cdot x \cdot x \cdot y$ which is equivalent to $24x^2y$.

9. Correct. Think of h^2 as a thing. If you have 3 things and 2 of the same thing, you have 5 things altogether.

13. We have $(x+1)+(x+2)+(x+3) = (x+x+x)+(1+2+3) = 3x+6$.

17. We have $(2x)(3y)+4x+5y+(6x)(3y) = 6xy+4x+5y+18xy = (6xy+18xy)+4x+5y = 24xy+4x+5y$.

PROBLEMS

21. We regroup in the expression $(3x)(2y)(5z)$ so that we can use the information that $xyz = 100$. We have

$$\begin{aligned}(3x)(2y)(5z) &= (xyz)(3 \cdot 2 \cdot 5)\\ &= 30(xyz)\\ &= 30(100)\\ &= 3000.\end{aligned}$$

25. The area of a rectangle is length times width. If the original length is l and the original width is w, the area is $l \cdot w$, so we have $l \cdot w = 50$. Since the length is increased by 25%, the new length is $l + 0.25l = 1.25l$. Since the width is increased by 10%, the new width is $w + 0.10w = 1.10w$. We have

$$\begin{aligned}\text{Area} &= \text{Length} \times \text{Width}\\ &= (1.25l)(1.10w)\\ &= (1.25 \cdot 1.10) \cdot (lw)\\ &= (1.375) \cdot (50)\\ &= 68.75.\end{aligned}$$

The area of the new rectangle is 68.75 square meters.

Solutions for Section 2.2

EXERCISES

1. Distributing the 2, we have

$$2(x+3y) = 2 \cdot x + 2 \cdot 3y = 2x + 6y.$$

5. Distributing the $2x$,we have

$$2x(x^2 - 3x + 4) = 2x \cdot x^2 + 2x \cdot (-3x) + 2x \cdot 4 = 2x^3 - 6x^2 + 8x.$$

9. Taking out the common factor of x, we have

$$2ax - 3bx = x(2a - 3b).$$

13. Taking out the common factors of -1 and m and n, we have

$$-m^2n - 3mn^2 = -mn(m + 3n).$$

17. The two terms each have a common factor of $(b + 3)$.
Factoring out the $(b + 3)$ gives $(b + 3)(b - 6)$.

21. No. Taking out the common factor 3 in the expression on the left, we have $3(x^2 + 2x + 1)$.

25. No. You cannot distribute a factor through a product. The expression on the left is equivalent to $24xy$ while the expression on the right is equivalent to $48xy$.

PROBLEMS

29. We have

$$\begin{aligned} 2(a+1) - (b+3) + (2c - b) &= 2a + 2 - b - 3 + 2c - b \\ &= 2a - 2b + 2c - 1 \\ &= 2(a - b + c) - 1 = 2 \cdot 17 - 1 = 33. \end{aligned}$$

33. To write the expression in the form $k(x + A)$, we factor out the coefficient of x. Factoring out 2, we have:

$$2x + 50 = 2(x + 25).$$

We see that $k = 2$ and $A = 25$.

37. To write the expression in the form $k(x + A)$, we factor out the coefficient of x. Factoring out 0.2, we have:

$$0.2x - 60 = 0.2\left(x - \frac{60}{0.2}\right) = 0.2(x - 300) = 0.2(x + (-300)).$$

We see that $k = 0.2$ and $A = -300$.

41.

a	-2	-1	0	1	2
$-(1/2)(a+1)+1$	$3/2$	1	$1/2$	0	$-1/2$
$-(1/2)a+(1/2)$	$3/2$	1	$1/2$	0	$-1/2$

Since the values are the same, the two expressions could be equivalent. Rewriting the first expression using the distributive law and collecting like terms we get

$$-(1/2)(a+1) + 1 = (-1/2)a - (1/2) + 1 = (-1/2)a + (1/2),$$

so it is equivalent to the second expression.

Solutions for Section 2.3

EXERCISES

1. First multiply x by $(x + 5)$, then multiply 2 by $(x + 5)$. This gives $x(x + 5) = x^2 + 5x$ and $2(x + 5) = 2x + 10$. Thus,

$$(x + 5)(x + 2) = x(x + 5) + 2(x + 5) = x^2 + 5x + 2x + 10 = x^2 + 7x + 10.$$

5. First multiply b by $(3b + c)$, then multiply $2c$ by $(3b + c)$. This gives us

$$(3b + c)(b + 2c) = b(3b + c) + 2c(3b + c) = 3b^2 + bc + 6bc + 2c^2 = 3b^2 + 7bc + 2c^2.$$

9. We begin by noting that $(x-8)^2 = (x-8)(x-8)$. We then distribute the first $x-8$ through:

$$(x-8)(x-8) = (x-8)x - (x-8)8.$$

Rearranging and distributing further, we have

$$x(x-8) - 8(x-8) = x \cdot x - x8 - 8x - 8(-8).$$

We complete the multiplications and combine like terms:

$$x(x-8) - 8(x-8) = x \cdot x - x8 - 8x - 8(-8) = x^2 - 8x - 8x + 64 = x^2 - 16x + 64.$$

13. The first and last terms of $(x+y)^2$ are the squares of the first and last terms of $(x+y)$. The middle term is twice the product of the first and last term.

Therefore, $(x+y)^2 = x^2 + 2xy + y^2$.

17. Rewrite $(2a-3b)^3$ as $(2a-3b)^2(2a-3b)$. We know that $(2a-3b)^2 = 4a^2 - 12ab + 9b^2$. Use the distributive law and combine like terms:

$$\begin{aligned}(2a-3b)^2(2a-3b) &= (4a^2 - 12ab + 9b^2)(2a-3b)\\ &= 2a(4a^2 - 12ab + 9b^2) - 3b(4a^2 - 12ab + 9b^2)\\ &= 8a^3 - 24a^2b + 18ab^2 - 12a^2b + 36ab^2 - 27b^3\\ &= 8a^3 - 36a^2b + 54ab^2 - 27b^3.\end{aligned}$$

21. We distribute the first parentheses through:

$$(x-11)(3x+2) = (x-11)3x + (x-11)2.$$

Rearranging and distributing further, we have

$$3x(x-11) + 2(x-11) = 3x \cdot x + 3x(-11) + 2x + 2(-11).$$

We complete the multiplications and combine like terms:

$$3x \cdot x + 3x(-11) + 2x + 2(-11) = 3x^2 - 33x + 2x - 22 = 3x^2 - 31x - 22.$$

25. Multiplying and collecting like terms gives

$$\begin{aligned}(s-3)(s+5) + s(s-2) &= s^2 + 2s - 15 + s^2 - 2s\\ &= 2s^2 - 15.\end{aligned}$$

29. This could be equivalent to an expression of the form $(g+r)(g+s)$. If so, then $r+s$ must equal -12 and rs must equal 20. The numbers $r=-10$ and $s=-2$ satisfy both conditions.

Therefore, $g^2 - 12g + 20 = (g-10)(g-2)$.

33. This could be equivalent to an expression of the form $(x+ry)(x+sy)$. If so, then $r+s$ must equal 11 and rs must equal 24. The numbers $r=3$ and $s=8$ satisfy both conditions.

Therefore, $x^2 + 11xy + 24y^2 = (x+3y)(x+8y)$.

37. First, factor out the common factor of 3.

$$3w^2 + 12w - 36 = 3(w^2 + 4w - 12).$$

The remaining factor is a quadratic expression. This could be equivalent to an expression of the form $(w+r)(w+s)$. If so, then $r+s$ must equal 4 and rs must equal -12. The numbers $r=6$ and $s=-2$ satisfy both conditions.

Therefore, $3w^2 + 12w - 36 = 3(w^2 + 4w - 12) = 3(w+6)(w-2)$.

41. The first and last terms are both squares. If $s^2 - 12st + 36t^2$ is a square of a sum of two terms, the two terms could be $\pm s$ and $\pm 6t$. Twice the product of s and $-6t$ is $-12st$, which is the middle term of $s^2 - 12st + 36t^2$.

Thus,
$$s^2 - 12st + 36t^2 = (s - 6t)^2.$$

45. This could be equivalent to an expression of the form $(x + r)(x + s)$. If so, then $r + s$ must equal 10 and rs must equal 25. The numbers $r = 5$ and $s = 5$ satisfy both conditions. Therefore, $x^2 + 10x + 25 = (x + 5)(x + 5)$.

49. We multiply the coefficient of the x^2 term by the constant term: $5 \cdot (-72) = -360$. Now we try to write $-37x$ as a sum of two terms whose coefficients multiply to -360. Writing $-37x = -45x + 8x$ works, since $-45 \cdot 8 = -360$. So we write $5x^2 - 37x - 72$ as $5x^2 - 45x + 8x - 72$, and factor by grouping.
$$\begin{aligned}5x^2 - 37x - 72 &= 5x^2 - 45x + 8x - 72\\ &= (5x^2 - 45x) + (8x - 72)\\ &= 5x(x - 9) + 8(x - 9)\\ &= (5x + 8)(x - 9).\end{aligned}$$

53. We first take out a common factor of x, giving
$$x^3 - 16x^2 + 64x = x(x^2 - 16x + 64).$$
This could be equivalent to an expression of the form $x(x + r)(x + s)$. If so, then $r + s$ must equal -16 and rs must equal 64. The numbers $r = -8$ and $s = -8$ satisfy both conditions. Therefore, $x^3 - 16x^2 + 64x = x(x - 8)(x - 8)$.

57. Factor out the common factor $2x$.
$$18x^7 + 48x^4z^2 + 32xz^4 = 2x(9x^6 + 24x^3z^2 + 16z^4).$$
The first and last terms of $9x^6 + 24x^3z^2 + 16z^4$ are both squares. If $9x^6 + 24x^3z^2 + 16z^4$ is a square of a sum of two terms, the two terms could be $\pm 3x^3$ and $4z^2$. Twice the product of $3x^3$ and $4z^2$ is $24x^3z^2$, which is the middle term of $9x^6 + 24x^3z^2 + 16z^4$. Thus,
$$18x^7 + 48x^4z^2 + 32xz^4 = 2x(3x^3 + 4z^2)^2.$$

PROBLEMS

61. First we factor out the common 3 to give $3(4-9(t+1)^2)$. Now factor the difference of squares to give $3(2-3(t+1))(2+3(t+1))$. Combining like terms inside the parentheses and factoring out a -1 gives $12-27(t+1)^2 = -3(3t+1)(3t+5)$.

65. First factor out a common $2w$. This gives $2w(w^2 - 8w + 16)$. Since $(w^2 - 8w + 16)$ is a perfect square, we obtain $2w(w - 4)^2$.

69.
$$\begin{aligned}((x + h) + 1)((x + h) - 1) &= (x + h)^2 - 1\\ &= x^2 + 2xh + h^2 - 1.\end{aligned}$$

Solutions for Section 2.4

EXERCISES

1. We have
$$\frac{m}{2} + \frac{m}{3} = \frac{3m}{6} + \frac{2m}{6} = \frac{5m}{6}.$$

5. We have
$$\frac{-1}{x} - \frac{1}{-x} + \frac{-1}{-x} = -\frac{1}{x} + \frac{1}{x} + \frac{1}{x} = \frac{1}{x}.$$

9. We have
$$\frac{1}{a-b} + \frac{1}{a+b} = \frac{a+b}{(a-b)(a+b)} + \frac{a-b}{(a-b)(a+b)} = \frac{a+b+a-b}{(a-b)(a+b)} = \frac{2a}{(a-b)(a+b)}.$$

13. Multiply the two numerators and the two denominators and cancel like terms:

$$\frac{5p}{6q^2}\cdot\frac{3pq}{5p}=\frac{15p^2q}{30pq^2}=\frac{\cancel{15}p^{\cancel{2}\,1}\cancel{q}}{\cancel{30}^{2}\cancel{p}q^{\cancel{2}\,1}}=\frac{p}{2q}.$$

17. Multiply the two numerators and the two denominators:

$$\frac{2r+3s}{4s}\cdot\frac{6r}{6r+9s}=\frac{6r(2r+3s)}{4s(6r+9s)}.$$

Factor the 3 out in the denominator, multiply 4 by 3 and cancel like terms:

$$\frac{6r(2r+3s)}{4s(6r+9s)}=\frac{6r(2r+3s)}{4s\cdot 3(2r+3s)}=\frac{\cancel{6}r\cancel{(2r+3s)}}{\cancel{12}^{2}s\cancel{(2r+3s)}}=\frac{r}{2s}.$$

21. Multiply the two numerators and the two denominators:

$$\frac{w^2r+4wr}{2r^2w+2wr}\cdot\frac{r+r^2}{4w+16}=\frac{(w^2r+4wr)(r+r^2)}{(2r^2w+2wr)(4w+16)}.$$

Factor the numerator and denominator, and cancel like terms:

$$\frac{(w^2r+4wr)(r+r^2)}{(2r^2w+2wr)(4w+16)}=\frac{wr(w+4)\cdot r(1+r)}{2wr(r+1)\cdot 4(w+4)}=\frac{wr^2(w+4)(1+4)r}{8wr(r+1)(w+4)}=\frac{\cancel{wr}^{\cancel{2}\,1}\cancel{(w+4)(1+r)}}{8\cancel{wr(r+1)(w+4)}}=\frac{r}{8}.$$

25. $\dfrac{z+1}{2}=\dfrac{z}{2}+\dfrac{1}{2}.$

29. $\dfrac{6p-3}{6}=\dfrac{6p}{6}-\dfrac{3}{6}=p-\dfrac{1}{2}$, which can be thought of as a difference of fractions by writing it as $\dfrac{p}{1}-\dfrac{1}{2}$.

33. $\dfrac{2xh+h^2}{h}=\dfrac{2xh}{h}+\dfrac{h^2}{h}=2x+h$, which can be thought of as a sum of two fractions by writing it as $\dfrac{2x}{1}+\dfrac{h}{1}$.

37. $\dfrac{3}{t(r+s)}=\dfrac{3}{t}\cdot\dfrac{1}{r+s}.$

41. $\dfrac{(x+1)^2-y}{xy}=\dfrac{(x+1)^2}{xy}-\dfrac{y}{xy}=\dfrac{(x+1)^2}{xy}-\dfrac{1}{x}.$

PROBLEMS

45. We have a common denominator, so we can write

$$\frac{1}{x}+\frac{1}{x}=\frac{1+1}{x}=\frac{2}{x}.$$

This matches expression (c).

49. This expression cannot be rewritten to match any of the expressions (a)–(f).

53. By first writing the denominator of the original fraction as a single fraction whose denominator is 36, we obtain $\dfrac{p+q}{\dfrac{p}{12}+\dfrac{q}{18}}=\dfrac{p+q}{\dfrac{3p+2q}{36}}$. Inverting the denominator and multiplying, gives $\dfrac{36(p+q)}{3p+2q}$.

57. Writing the numerator and denominator as single fractions gives

$$\frac{\dfrac{1}{25}-\dfrac{1}{x^2}}{\dfrac{1}{x}-\dfrac{1}{5}}=\frac{\dfrac{x^2-25}{25x^2}}{\dfrac{5-x}{5x}}.$$

Inverting the denominator and multiplying fractions gives

$$\frac{\frac{x^2-25}{25x^2}}{\frac{5-x}{5x}} = \frac{x^2-25}{25x^2}\cdot\frac{5x}{5-x}.$$

Since the numerator $x^2-25=(x+5)(x-5)$ and the denominator $(5-x)=-(x-5)$, we have

$$\frac{x^2-25}{25x^2}\cdot\frac{5x}{5-x} = \frac{(x-5)(x+5)}{25x^2}\cdot\frac{5x}{-(x-5)}.$$

Canceling out common factors of 5, x, and $(x-5)$ gives

$$\frac{(x-5)(x+5)}{25x^2}\cdot\frac{5x}{-(x-5)} = -\frac{x+5}{5x}.$$

61. Writing the numerator a single fraction, we have

$$\frac{\frac{1}{m-1}+\frac{2}{m+2}}{\frac{3}{m+2}} = \frac{\frac{(m+2)+2(m-1)}{(m-1)(m+2)}}{\frac{3}{m+2}}.$$

We invert the denominator and multiply the fractions producing $\frac{(m+2)+2(m-1)}{(m-1)(m+2)}\cdot\frac{m+2}{3}$. Since the numerator $(m+2)+2(m-1)$ is equal to $3m$, and since there are a common factors of $m+2$ and 3, we have

$$\frac{(m+2)+2(m-1)}{(m-1)(m+2)}\cdot\frac{m+2}{3} = \frac{3m}{(m-1)(m+2)}\cdot\frac{m+2}{3} = \frac{m}{m-1}.$$

Solutions for Chapter 2 Review

EXERCISES

1. Yes. The expression $x(5x)$ can be reordered to $5x\cdot x$, which is equivalent to $5x^2$.

5. We have

$$\begin{aligned}3(z+r)^2+6rz+\frac{r-z-2}{r+z} &= 3(2-5)^2+6(-5)\cdot 2+\frac{-5-2-2}{-5+2}\\ &= 3(-3)^2-60+\frac{-9}{-3}\\ &= 3\cdot 9-60+3\\ &= -30.\end{aligned}$$

9. We combine all three terms to find $(5/12)A$.

13. We have

$$\begin{aligned}2(x+5)+3(x-4) &= 2x+10+3x-12\\ &= 5x-2.\end{aligned}$$

17. We have

$$\begin{aligned} 2x(3x+4)+3(x^2-5x+6) &= 6x^2+8x+3x^2-15x+18 \\ &= 9x^2-7x+18. \end{aligned}$$

21. We have

$$\begin{aligned} mn(m+2n)+3mn(2m+n)+5m^2n &= m^2n+2mn^2+6m^2n+3mn^2+5m^2n \\ &= 12m^2n+5mn^2. \end{aligned}$$

25. This could be equivalent to an expression of the form $(v+r)(v+s)$. If so, then $r+s$ must equal -4 and rs must equal -32. The numbers $r=-8$ and $s=4$ satisfy both conditions.

Therefore, $v^2-4v-32=(v-8)(v+4)$.

29. There is no common factor. We multiply the coefficient of the x^2 term by the constant term: $6\cdot(-6)=-36$.

Now we try to write $5x$ as a sum of two terms whose coefficients multiply to -36. Writing $5x=9x-4x$ works since $9\cdot(-4)=-36$.

So, we write $6x^2+5x-6$ as $6x^2+9x-4x-6$ and factor by grouping.

$$\begin{aligned} 6x^2+5x-6 &= 6x^2+9x-4x-6 \\ &= (6x^2+9x)+(-4x-6) \\ &= 3x(2x+3)-2(2x+3) \\ &= (2x+3)(3x-2). \end{aligned}$$

33. Factor out the common factor of 2.

$$12w^2-10w-8=2(6w^2-5w-4).$$

We multiply the coefficient of the w^2 term by the constant term: $6\cdot(-4)=-24$.

Now we try to write $-5w$ as a sum of two terms whose coefficients multiply to -24. Writing $-8w+3w$ works since $(-8)\cdot 3=-24$.

So, we write $6w^2-5w-4$ as $6w^2-8w+3w-4$ and factor by grouping.

$$\begin{aligned} 12w^2-10w-54 &= 2(6w^2-5w-4) \\ &= 2(6w^2-8w+3w-4) \\ &= 2(2w(3w-4)+(3w-4)) \\ &= 2(3w-4)(2w+1). \end{aligned}$$

37. We note a common factor of x in the two terms, so we factor it out:

$$x^3+4x=x(x^2+4).$$

41. This could be equivalent to an expression of the form $(x+r)(x+s)$. If so, then $r+s$ must equal -22 and rs must equal 121. The numbers $r=-11$ and $s=-11$ satisfy both conditions. Therefore, $x^2-22x+121=(x-11)(x-11)$.

45. We can group the first two terms and the second two terms as $(ax+bx)+(-ay-by)$. Factoring out an x from the first group and a $-y$ from the second group gives us

$$ax+bx-ay-by=(ax+bx)+(-ay-by)=x(a+b)-y(a+b)=(a+b)(x-y).$$

49. First, factor out the common factor y.

$$y^3+7y^2-18y=y(y^2+7y-18).$$

The remaining factor is a quadratic expression. This could be equivalent to an expression of the form $(y+r)(y+s)$. If so, then $r+s$ must equal 7 and rs must equal -18. The numbers $r=9$ and $s=-2$ satisfy both conditions.

Therefore,

$$y^3+7y^2-18y=y(y^2+7y-18)=y(y+9)(y-2).$$

53. $\dfrac{4a-8}{16} = \dfrac{4(a-2)}{16} = \dfrac{a-2}{4}$.

57. $\dfrac{3t^3+12t}{4t^2+16} = \dfrac{3t(t^2+4)}{4(t^2+4)} = \dfrac{3t}{4}$.

61. $\dfrac{p^2q-pq^2}{(p-q)^2} = \dfrac{pq(p-q)}{(p-q)(p-q)} = \dfrac{pq}{p-q}$.

65. We have

$$\begin{aligned}\frac{r^4-1}{r^3p-rp} &= \frac{(r^2-1)(r^2+1)}{rp(r^2-1)}\\ &= \frac{r^2+1}{rp}.\end{aligned}$$

PROBLEMS

69. We have

$$\frac{3a}{2b} = \frac{3}{2}\cdot\frac{a}{b} = \frac{3}{2}\cdot\frac{2}{3} = 1.$$

73. We have

$$\begin{aligned}6x^2+12 &= 2\underbrace{(3x^2+6)}_{r}\\ &= 2r,\end{aligned}$$

so $r = 3x^2+6$.

77. We have

$$\begin{aligned}\frac{3(x+5)-7x}{x+5} &= \frac{3(x+5)-7x}{x+5}\\ &= \frac{3(x+5)}{x+5} - \frac{7x}{x+5}\\ &= 3 - \frac{7x}{x+5}\\ &= 3 - \frac{7x}{x-(-5)},\end{aligned}$$

so $k = 3, m = 7, n = -5$.

81. We have

$$\begin{aligned}\left(x^2+x+6\right)\left(3x^3+6x^2\right) &= \left(x^2+x+6\right)\left(3x^2\cdot x+3x^2\cdot 2\right) && \text{regroup}\\ &= \left(x^2+x+6\right)\left(\underbrace{3x^2}_{a}\cdot\underbrace{x}_{b}+\underbrace{3x^2}_{a}\cdot\underbrace{2}_{c}\right) && \text{identify } a,b,c\\ &= \left(x^2+x+6\right)\cdot\underbrace{3x^2}_{a}\cdot\left(\underbrace{x}_{b}+\underbrace{2}_{c}\right) && \text{factor (distribute law)}\\ &= 3x^2\left(x^2+x+6\right)(x+2), && \text{reorder as required}\end{aligned}$$

$$\text{so}\quad \begin{aligned} a &= 3x^2\\ b &= x\\ c &= 2.\end{aligned}$$

Note that here, we used the distributive law to factor part of this expression, writing $3x^3+6x^2$ as $3x^2(x+2)$. The other part of the expression, x^2+x+6, was not involved.