# CHAPTER TWO

## Solutions for Section 2.1

### EXERCISES

1. No. Reordering the expression on the left gives us $3x \cdot x$ or $3x^2$ but not $4x$.
2. No. The two expressions are not equivalent.
3. Yes. The expression $5 - x$ is equivalent to $5 + (-x)$ which can be reordered to $-x + 5$.
4. No. The expression $(2x)(2y)$ is equivalent to $4xy$ which is not equivalent to $2xy$.
5. Yes. The expression $(3x)(4y)(2x)$ can be reordered as $3 \cdot 4 \cdot 2 \cdot x \cdot x \cdot y$ which is equivalent to $24x^2y$.
6. Yes. Since $4 + x$ is equivalent to $x + 4$, the two expressions are equivalent.
7. Incorrect; for $x = 2$, we have $32 \neq 160$.
8. Incorrect; for $A = 2, B = 1$, we have $16 \neq 40$.
9. Correct. Think of $h^2$ as a thing. If you have 3 things and 2 of the same thing, you have 5 things altogether.
10. Incorrect. Replace $b$ by 2, for example. $14 \neq 40$.
11. We have $(2x + 1) + (5x + 8) = (2x + 5x) + (1 + 8) = 7x + 9$.
12. We have $(4 - 2x) + (5x - 9) = (-2x + 5x) + (4 - 9) = 3x - 5$.
13. We have $(x + 1) + (x + 2) + (x + 3) = (x + x + x) + (1 + 2 + 3) = 3x + 6$.
14. We have $(7x + 1) + (5 - 3x) + (2x - 4) = (7x - 3x + 2x) + (1 + 5 - 4) = 6x + 2$.
15. We have $5x^2 + 5x + 3x^2 = (5x^2 + 3x^2) + 5x = 8x^2 + 5x$.
16. There are no like terms to combine.
17. We have $(2x)(3y) + 4x + 5y + (6x)(3y) = 6xy + 4x + 5y + 18xy = (6xy + 18xy) + 4x + 5y = 24xy + 4x + 5y$.
18. We have $(2x)(5x)+(3x)(2x)+5(3x)+x(3x) = 10x^2+6x^2+15x+3x^2 = (10x^2+6x^2+3x^2)+15x = 19x^2+15x$.

### PROBLEMS

19. We regroup in the expression $(a+5)+(b-3)+(c+8)$ so that we can use the information that $a+b+c = 12$. We have

$$\begin{aligned}(a + 5) + (b - 3) + (c + 8) &= a + 5 + b + (-3) + c + 8 \\ &= a + b + c + 5 + (-3) + 8 \\ &= (a + b + c) + (5 - 3 + 8) \\ &= (a + b + c) + 10 \\ &= 12 + 10 \\ &= 22.\end{aligned}$$

20. We regroup in the expression $(y - 10) + (z + 8) + (x - 5)$ so that we can use the information that $x + y + z = 25$. We have

$$\begin{aligned}(y - 10) + (z + 8) + (x - 5) &= y + (-10) + z + 8 + x + (-5) \\ &= x + y + z + (-10) + 8 + (-5) \\ &= (x + y + z) + (-10 + 8 - 5)\end{aligned}$$

$$\begin{aligned} &= (x+y+z) - 7 \\ &= 25 - 7 \\ &= 18. \end{aligned}$$

**21.** We regroup in the expression $(3x)(2y)(5z)$ so that we can use the information that $xyz = 100$. We have

$$\begin{aligned} (3x)(2y)(5z) &= (xyz)(3 \cdot 2 \cdot 5) \\ &= 30(xyz) \\ &= 30(100) \\ &= 3000. \end{aligned}$$

**22.** We regroup in the expression $(2z)(\frac{x}{4})(6y)$ so that we can use the information that $xyz = 20$. We have

$$\begin{aligned} (2z)(\frac{x}{4})(6y) &= 2 \cdot z \cdot \frac{1}{4} \cdot x \cdot 6 \cdot y \\ &= (xyz) \cdot (2 \cdot \frac{1}{4} \cdot 6) \\ &= (xyz) \cdot \frac{2 \cdot 6}{4} \\ &= 3(xyz) \\ &= 3(20) \\ &= 60. \end{aligned}$$

**23.** We have

$$\begin{aligned} a + 2(b-a) - 3(c+b) &= a + \underbrace{(b-a) + (b-a)}_{\text{2 groups}} - \underbrace{(c+b) - (c+b) - (c+b)}_{\text{3 groups}} \\ &= a - a - a + b + b - b - b - b - c - c - c && \text{regrouping terms} \\ &= -a - b - 3c && \text{collecting terms.} \end{aligned}$$

**24.** When the speed is $r$ and the time is $t$, the distance traveled is $r \cdot t$. We know the car travels 200 miles, so $r \cdot t = 200$.

If the car travels half as fast, its new rate is $(1/2)r$, and if it travels three times as long, its new time is $3t$. We have

$$\begin{aligned} \text{Distance} &= \text{Rate} \times \text{Time} \\ &= \left(\frac{r}{2}\right) \cdot (3t) \\ &= \left(\frac{3}{2}\right) \cdot (rt) \\ &= \left(\frac{3}{2}\right) \cdot (200) \\ &= 300. \end{aligned}$$

The car travels 300 miles.

**25.** The area of a rectangle is length times width. If the original length is $l$ and the original width is $w$, the area is $l \cdot w$, so we have $l \cdot w = 50$. Since the length is increased by 25%, the new length is $l + 0.25l = 1.25l$. Since the width is increased by 10%, the new width is $w + 0.10w = 1.10w$. We have

$$\text{Area} = \text{Length} \times \text{Width}$$

$$\begin{aligned} &= (1.25l)(1.10w) \\ &= (1.25 \cdot 1.10) \cdot (lw) \\ &= (1.375) \cdot (50) \\ &= 68.75. \end{aligned}$$

The area of the new rectangle is $68.75$ square meters.

**26.** **(a)** The amount at the end of the month is the starting amount plus the flow in minus the flow out. We have

$$\begin{aligned} \text{Amount at end of January} &= \text{Amount at start of January} + \text{Flow in} - \text{Flow out} \\ &= 412 + A - B. \end{aligned}$$

The amount at the end of January is $412 + A - B$ billion gallons.

**(b)** The amount at the end of February is the amount at the end of January plus the flow in minus the flow out. The flow in in February is $A - 20$ and the flow out in February is $B + 12$. We have

$$\begin{aligned} \text{Amount at end of February} &= \text{Amount at start of February} + \text{Flow in} - \text{Flow out} \\ &= (412 + A - B) + (A - 20) - (B + 12) \end{aligned}$$

The amount at the end of February is $(412 + A - B) + (A - 20) - (B + 12)$ billion gallons.

**(c)** Combining like terms, we have

$$\begin{aligned} (412 + A - B) + (A - 20) - (B + 12) &= 412 + A - B + A - 20 - B - 12 \\ &= 412 - 20 - 12 + A + A - B - B \\ &= 380 + 2A - 2B. \end{aligned}$$

## Solutions for Section 2.2

### EXERCISES

**1.** Distributing the 2, we have

$$2(x + 3y) = 2 \cdot x + 2 \cdot 3y = 2x + 6y.$$

**2.** Distributing the $3x$,we have

$$3x(x + 4) = 3x \cdot x + 3x \cdot 4 = 3x^2 + 12x.$$

**3.** Distributing the $-5$,we have

$$-5(2x - 3) = -5 \cdot 2x + (-5) \cdot (-3) = -10x + 15.$$

Notice that the 15 is positive since it is the product of two negative numbers.

**4.** Distributing the $3ab$,we have

$$3ab(2a - 5b) = 3ab \cdot 2a + 3ab \cdot (-5b) = 6a^2b - 15ab^2.$$

**5.** Distributing the $2x$,we have

$$2x(x^2 - 3x + 4) = 2x \cdot x^2 + 2x \cdot (-3x) + 2x \cdot 4 = 2x^3 - 6x^2 + 8x.$$

**6.** Distributing the $-3x$,we have

$$-3x(5-3x-2x^2) = -3x \cdot 5 + (-3x)\cdot(-3x) + (-3x)\cdot(-2x^2) = -15x + 9x^2 + 6x^3.$$

**7.** Distributing the 2 through the expression $5x-3y$,we have

$$2(5x-3y)+5 = 2\cdot 5x + 2\cdot(-3y) + 5 = 10x - 6y + 5.$$

**8.** We distribute the 3 through the expression $C-D$, and we distribute the $D$ through the expression $C-D$. We have

$$3(C-D)D = (3C-3D)D = 3CD - 3D^2.$$

**9.** Taking out the common factor of $x$, we have

$$2ax - 3bx = x(2a-3b).$$

**10.** Since $100 = 5\cdot 20$, we take out the common factor of 5. We have

$$5x + 100 = 5\cdot x + 5\cdot 20 = 5(x+20).$$

**11.** Taking out the common factor of $1/5$, we have

$$\frac{x}{5} + \frac{y}{5} = \frac{1}{5}(x+y).$$

**12.** Taking out the common factors of 2 and $x$, we have

$$2x^2 - 6x = 2x(x-3).$$

**13.** Taking out the common factors of $-1$ and $m$ and $n$, we have

$$-m^2n - 3mn^2 = -mn(m+3n).$$

**14.** Taking out the common factor of 3, we have

$$9x^2 + 18x + 3 = 3(3x^2 + 6x + 1).$$

**15.** Taking out the common factors of $-2$ and $a$ and $b$, we have

$$-4a^2b - 6ab^2 - 2ab = -2ab(2a + 3b + 1).$$

**16.** Taking out the common factor of $(x+1)$, we have

$$5x(x+1) + 7(x+1) = (x+1)(5x+7).$$

**17.** The two terms each have a common factor of $(b+3)$.
Factoring out the $(b+3)$ gives $(b+3)(b-6)$.

**18.** The two terms each have a common factor of $6(s-2)$.
Factoring out the $6(s-2)$ gives $6(s-2)(r-2)$.

**19.** The two terms each have a common factor of $2x(x+4)$. Factoring out the $2x(x+4)$ gives $2x(x+4)(2a-1)$.

**20.** No. Distributing the $-2$ through $x-4$, we have $-2x+8$.

**21.** No. Taking out the common factor 3 in the expression on the left, we have $3(x^2+2x+1)$.

**22.** Yes. Distributing the $ab$ through gives us the expression on the right.

**23.** No. Factoring out the 5 in the expression on the left gives $5(x+20)$.

**24.** Yes. In the expression on the left, we distribute the $x-3$ through $x+2$ to get the expression on the right.

**25.** No. You cannot distribute a factor through a product. The expression on the left is equivalent to $24xy$ while the expression on the right is equivalent to $48xy$.

**26.** No. You cannot distribute an exponent.

**27.** No. You can see that the two expressions are not equivalent by substituting values for $a$, $b$, and $c$.

## PROBLEMS

**28.** Rewriting this expression gives

$$\begin{aligned}p(2qr+3r)+3r(pq-p) &= 2pqr+3pr+3pqr-3pr\\ &= 5pqr\\ &= 5\cdot 17\\ &= 85.\end{aligned}$$

**29.** We have

$$\begin{aligned}2(a+1)-(b+3)+(2c-b) &= 2a+2-b-3+2c-b\\ &= 2a-2b+2c-1\\ &= 2(a-b+c)-1 = 2\cdot 17-1 = 33.\end{aligned}$$

**30.** We have

$$3(x^2+2)-3x(1-x) = 3x^2+6-3x+3x^2 = 6x^2-3x+6,$$

which is (ii).

**31.** We know that:

- There are $n$ items costing \$$p$, which amounts to a cost of $n\cdot p$.
- There are twice as many items, amounting to $2n$ items, each costing \$1 more, or $p+1$. The total cost for these items is $2n\cdot(p+1)$.

We conclude that

$$\begin{aligned}\text{Total cost} &= np+2n(p+1)\\ &= np+2n\cdot p+2n\cdot 1\\ &= 3np+2n.\end{aligned}$$

**32.** **(a)** The total output in January is $q+r+s$, so

$$\text{Output in February } = 2(q+r+s).$$

**(b)** The output in February from the first factory is $2q$, from the second factory is $2r$, and from the third factory is $2s$. Adding them up, we have

$$\text{Output in February } = 2q + 2r + 2s.$$

**(c)** Yes. Both expressions double the total output from January. Using the distributive law, we see

$$2(q + r + s) = 2q + 2r + 2s.$$

**33.** To write the expression in the form $k(x + A)$, we factor out the coefficient of $x$. Factoring out 2, we have:

$$2x + 50 = 2(x + 25).$$

We see that $k = 2$ and $A = 25$.

**34.** To write the expression in the form $k(x + A)$, we factor out the coefficient of $x$. Factoring out 3, we have:

$$3x - 18 = 3(x - 6) = 3(x + (-6)).$$

We see that $k = 3$ and $A = -6$.

**35.** To write the expression in the form $k(x + A)$, we factor out the coefficient of $x$. Factoring out $-5$, we have:

$$15 - 5x = -5(-3 + x) = -5(x + (-3)).$$

We see that $k = -5$ and $A = -3$.

**36.** To write the expression in the form $k(x + A)$, we factor out the coefficient of $x$. Factoring out $0.05$, we have:

$$0.05x + 100 = 0.05\left(x + \frac{100}{0.05}\right) = 0.05(x + 2000).$$

We see that $k = 0.05$ and $A = 2000$.

**37.** To write the expression in the form $k(x + A)$, we factor out the coefficient of $x$. Factoring out $0.2$, we have:

$$0.2x - 60 = 0.2\left(x - \frac{60}{0.2}\right) = 0.2(x - 300) = 0.2(x + (-300)).$$

We see that $k = 0.2$ and $A = -300$.

**38.** To write the expression in the form $k(x + A)$, we factor out the coefficient of $x$. Factoring out $-0.1$, we have:

$$50 - 0.1x = -0.1\left(\frac{50}{-0.1} + x\right) = -0.1(-500 + x) = -0.1(x + (-500)).$$

We see that $k = -0.1$ and $A = -500$.

**39.** To answer this question, we separate the fraction $(x+3)/x$ into two separate fractions, each with a denominator of $x$. We then simplify:

$$\frac{x+3}{x} = \frac{x}{x} + \frac{3}{x} = 1 + \frac{3}{x}.$$

We can see that the two expressions are equivalent.

**40.** We cannot simplify the fraction

$$\frac{x}{x+3}.$$

Thus, the two expressions do not appear to be equivalent.

An alternate approach is to write $1 + x/3$ as a single fraction. We do this in the following manner:

$$1 + \frac{x}{3} = \frac{1}{1} + \frac{x}{3} = \frac{1 \cdot 3 + x \cdot 1}{3} = \frac{3 + x}{3}.$$

Again the two expressions do not appear to be equivalent.

In order to show that they are not equivalent, we try to find a value of $x$ for which the two expressions have different values. We try $x = 0$, giving

$$\frac{x}{x+3} = \frac{0}{0+3} = 0$$

and

$$1 + \frac{x}{3} = 1 + \frac{0}{3} = 1.$$

Since the two expression have different values when $x = 0$, they are not equivalent.

**41.**

| $a$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $-(1/2)(a+1)+1$ | $3/2$ | $1$ | $1/2$ | $0$ | $-1/2$ |
| $-(1/2)a+(1/2)$ | $3/2$ | $1$ | $1/2$ | $0$ | $-1/2$ |

Since the values are the same, the two expressions could be equivalent. Rewriting the first expression using the distributive law and collecting like terms we get

$$-(1/2)(a+1)+1 = (-1/2)a-(1/2)+1 = (-1/2)a+(1/2),$$

so it is equivalent to the second expression.

**42.** Writing the left-hand side as

$$\begin{aligned}(2x+3)^3 &= \underbrace{(2x+3)^2}_{a}(\underbrace{2x}_{b}+\underbrace{3}_{c})\\ &= \underbrace{(2x+3)^2\cdot 2x}_{ab}+\underbrace{(2x+3)^2\cdot 3}_{ac},\end{aligned}$$

we see that $a=(2x+3)^2, b=2x, c=3$.

**43.** Writing the left-hand side as

$$\begin{aligned}x^2(x+r+3) &= \underbrace{x^2}_{a}(\underbrace{(x+r)}_{b}+\underbrace{3}_{c})\\ &= \underbrace{x^2(x+r)}_{ab}+\underbrace{3x^2}_{ac},\end{aligned}$$

we see that $a=x^2, b=x+r, c=3$.

**44.** **(a)** We have

$$\begin{aligned}\text{Total cost} &= (\text{Cost at site 1})+(\text{Cost at site 2})+(\text{Cost at site 3})\\ &= 12c+2p+4e\\ &\quad +14c+5p+3e\\ &\quad +17c+p+5e\\ &= 43c+8p+12e.\end{aligned}$$

**(b)** We have

$$\begin{aligned}\text{Difference} &= (\text{Cost at site 1})-(\text{Cost at site 3})\\ &= 12c+2p+4e-(17c+p+5e)\\ &= -5c+p-e.\end{aligned}$$

**(c)** We have

$$\begin{aligned}\begin{matrix}\text{Amount}\\ \text{remaining}\end{matrix} &= (\text{Original budget})-(\text{Total cost})\\ &= 50c+10p+20e-(43c+8p+12e)\\ &= 7c+2p+8e.\end{aligned}$$

## Solutions for Section 2.3

### EXERCISES

**1.** First multiply $x$ by $(x+5)$, then multiply 2 by $(x+5)$. This gives $x(x+5) = x^2+5x$ and $2(x+5) = 2x+10$. Thus,

$$(x+5)(x+2) = x(x+5) + 2(x+5) = x^2 + 5x + 2x + 10 = x^2 + 7x + 10.$$

**2.** First multiply $y$ by $(y+3)$, then multiply $-1$ by $(y+3)$. This gives $y(y+3) = y^2+3y$ and $-1(y+3) = -y-3$. Thus,

$$(y+3)(y-1) = y(y+3) - 1(y+3) = y^2 + 3y - y - 3 = y^2 + 2y - 3.$$

**3.** First multiply $z$ by $(z-5)$, then multiply $-6$ by $(z-5)$. This gives us

$$(z-5)(z-6) = z(z-5) - 6(z-5) = z^2 - 5z - 6z + 30 = z^2 - 11z + 30.$$

**4.** First multiply $3a$ by $(2a+3)$, then multiply $-2$ by $(2a+3)$. This gives us

$$(2a+3)(3a-2) = 3a(2a+3) - 2(2a+3) = 6a^2 + 9a - 4a - 6 = 6a^2 + 5a - 6.$$

**5.** First multiply $b$ by $(3b+c)$, then multiply $2c$ by $(3b+c)$. This gives us

$$(3b+c)(b+2c) = b(3b+c) + 2c(3b+c) = 3b^2 + bc + 6bc + 2c^2 = 3b^2 + 7bc + 2c^2.$$

**6.** First multiply $a$ by $(a+b+c)$, then multiply $-b$ by $(a+b+c)$, and then multiply $-c$ by $(a+b+c)$. This gives us

$$\begin{aligned}(a+b+c)(a-b-c) &= a(a+b+c) - b(a+b+c) - c(a+b+c)\\ &= a^2 + ab + ac - ab - b^2 - bc - ac - bc - c^2\\ &= a^2 - b^2 - 2bc - c^2.\end{aligned}$$

**7.** We have

$$3(x-4)^2 + 8x - 48 = 3(x^2 - 8x + 16) + 8x - 48 = 3x^2 - 24x + 48 + 8x - 48 = 3x^2 - 16x.$$

**8.** We begin by noting that $(x+6)^2 = (x+6)(x+6)$. We then distribute the first $x+6$ through:

$$(x+6)(x+6) = (x+6)x + (x+6)6.$$

Rearranging and distributing further, we have

$$x(x+6) + 6(x+6) = x \cdot x + x6 + 6x + 6 \cdot 6.$$

We complete the multiplications and combine like terms:

$$x \cdot x + x6 + 6x + 6 \cdot 6 = x^2 + 6x + 6x + 36 = x^2 + 12x + 36.$$

**9.** We begin by noting that $(x-8)^2 = (x-8)(x-8)$. We then distribute the first $x-8$ through:

$$(x-8)(x-8) = (x-8)x - (x-8)8.$$

Rearranging and distributing further, we have

$$x(x-8) - 8(x-8) = x \cdot x - x8 - 8x - 8(-8).$$

We complete the multiplications and combine like terms:

$$x(x-8) - 8(x-8) = x \cdot x - x8 - 8x - 8(-8) = x^2 - 8x - 8x + 64 = x^2 - 16x + 64.$$

**10.** We begin by noting that $(x+11)^2 = (x+11)(x+11)$. We then distribute the first $x+11$ through:

$$(x+11)(x+11) = (x+11)x + (x+11)11.$$

Rearranging and distributing further, we have

$$x(x+11) + 11(x+11) = x \cdot x + x11 + 11x + 11 \cdot 11.$$

We complete the multiplications and combine like terms:

$$x \cdot x + x11 + 11x + 11 \cdot 11 = x^2 + 11x + 11x + 121 = x^2 + 22x + 121.$$

**11.** We begin by noting that $(x-13)^2 = (x-13)(x-13)$. We then distribute the first $x-13$ through:

$$(x-13)(x-13) = (x-13)x - (x-13)13.$$

Rearranging and distributing further, we have

$$x(x-13) - 13(x-13) = x \cdot x + x(-13) - 13x - 13(-13).$$

We complete the multiplications and combine like terms:

$$x \cdot x + x(-13) - 13x - 13(-13) = x^2 - 13x - 13x + 169 = x^2 - 26x + 169.$$

**12.** We distribute the first parentheses through:

$$(x+7)(x-7) = (x+7)x - (x+7)7.$$

Rearranging and distributing further, we have

$$x(x+7) - 7(x+7) = x \cdot x + x7 - 7x - 7 \cdot 7.$$

We complete the multiplications and combine like terms:

$$x \cdot x + x7 - 7x - 7 \cdot 7 = x^2 + 7x - 7x - 49 = x^2 - 49.$$

**13.** The first and last terms of $(x+y)^2$ are the squares of the first and last terms of $(x+y)$. The middle term is twice the product of the first and last term.
Therefore, $(x+y)^2 = x^2 + 2xy + y^2$.

**14.** The first and last terms of $(2a+3b)^2$ are the squares of the first and last terms of $(2a+3b)$. The middle term is twice the product of the first and last term.
Therefore, $(2a+3b)^2 = 4a^2 + 12ab + 9b^2$.

**15.** The first and last terms of $(5p^2-q)^2$ are the squares of the first and last terms of $(5p^2-q)$. The middle term is twice the product of the first and last term.
Therefore, $(5p^2-q)^2 = 25p^4 - 10p^2q + q^2$.

**16.** Rewrite $(x-y)^3$ as $(x-y)^2(x-y)$. We know that $(x-y)^2 = x^2 - 2xy + y^2$. Use the distributive law and combine like terms:

$$\begin{aligned}(x-y)^2(x-y) &= (x^2-2xy+y^2)(x-y)\\ &= x(x^2-2xy+y^2) - y(x^2-2xy+y^2)\\ &= x^3 - 2x^2y + xy^2 - yx^2 + 2xy^2 - y^3\\ &= x^3 - 3x^2y + 3xy^2 - y^3.\end{aligned}$$

**17.** Rewrite $(2a-3b)^3$ as $(2a-3b)^2(2a-3b)$. We know that $(2a-3b)^2 = 4a^2 - 12ab + 9b^2$. Use the distributive law and combine like terms:

$$\begin{aligned}(2a-3b)^2(2a-3b) &= (4a^2 - 12ab + 9b^2)(2a-3b)\\ &= 2a(4a^2 - 12ab + 9b^2) - 3b(4a^2 - 12ab + 9b^2)\\ &= 8a^3 - 24a^2b + 18ab^2 - 12a^2b + 36ab^2 - 27b^3\\ &= 8a^3 - 36a^2b + 54ab^2 - 27b^3.\end{aligned}$$

**18.** We distribute the first parentheses through:

$$(x+9)(x-9) = (x+9)x - (x+9)9.$$

Rearranging and distributing further, we have

$$x(x+9) - 9(x+9) = x\cdot x + x9 - 9x - 9\cdot 9.$$

We complete the multiplications and combine like terms:

$$x\cdot x + x9 - 9x - 9\cdot 9 = x^2 + 9x - 9x - 81 = x^2 - 81.$$

**19.** We distribute the first parentheses through:

$$(x-8)(x+8) = (x-8)x + (x-8)8.$$

Rearranging and distributing further, we have

$$x(x-8) + 8(x-8) = x\cdot x + x(-8) + 8x + 8(-8).$$

We complete the multiplications and combine like terms:

$$x\cdot x + x(-8) + 8x + 8(-8) = x^2 - 8x + 8x - 64 = x^2 - 64.$$

**20.** We distribute the first parentheses through:

$$(x-12)(x+12) = (x-12)x + (x-12)12.$$

Rearranging and distributing further, we have

$$x(x-12) + 12(x-12) = x\cdot x + x(-12) + 12x + 12(-12).$$

We complete the multiplications and combine like terms:

$$x\cdot x + x(-12) + 12x + 12(-12) = x^2 - 12x + 12x - 144 = x^2 - 144.$$

**21.** We distribute the first parentheses through:

$$(x-11)(3x+2) = (x-11)3x + (x-11)2.$$

Rearranging and distributing further, we have

$$3x(x-11) + 2(x-11) = 3x\cdot x + 3x(-11) + 2x + 2(-11).$$

We complete the multiplications and combine like terms:

$$3x\cdot x + 3x(-11) + 2x + 2(-11) = 3x^2 - 33x + 2x - 22 = 3x^2 - 31x - 22.$$

**22.** We begin by distributing the $x$ through the $4x - 7$, giving

$$x(4x-7)(2x+2) = (4x \cdot x - 7x)(2x+2) = (4x^2 - 7x)(2x+2).$$

We distribute the first parentheses through:

$$(4x^2 - 7x)(2x+2) = (4x^2 - 7x)2x + (4x^2 - 7x)2.$$

Rearranging and distributing further, we have

$$2x(4x^2 - 7x) + 2(4x^2 - 7x) = 2x \cdot 4x^2 + 2x(-7x) + 2 \cdot 4x^2 + 2(-7x).$$

We complete the multiplications and combine like terms:

$$2x \cdot 4x^2 + 2x(-7x) + 2 \cdot 4x^2 + 2(-7x) = 8x^3 - 14x^2 + 8x^2 - 14x = 8x^3 - 6x^2 - 14x.$$

**23.** We begin by distributing the $x$ through the $3x + 7$, giving

$$x(3x+7)(5x-8) = (x \cdot 3x + x \cdot 7)(5x-8) = (3x^2 + 7x)(5x-8).$$

We distribute the first parentheses through:

$$(3x^2 + 7x)(5x-8) = (3x^2 + 7x)5x - (3x^2 + 7x)8.$$

Rearranging and distributing further, we have

$$5x(3x^2 + 7x) - 8(3x^2 + 7x) = 5x \cdot 3x^2 + 5x(7x) - 8 \cdot 3x^2 - 8 \cdot 7x.$$

We complete the multiplications and combine like terms:

$$5x \cdot 3x^2 + 5x(7x) - 8 \cdot 3x^2 - 8 \cdot 7x = 15x^3 + 35x^2 - 24x^2 - 56x = 15x^3 + 11x^2 - 56x.$$

**24.** We begin by distributing the $2x$ through the $5x + 8$, giving

$$2x(5x+8)(7x+2) = (2x \cdot 5x + 2x \cdot 8)(7x+2) = (10x^2 + 16x)(7x+2).$$

We distribute the first parentheses through:

$$(10x^2 + 16x)(7x+2) = (10x^2 + 16x)7x + (10x^2 + 16x)2.$$

Rearranging and distributing further, we have

$$7x(10x^2 + 16x) + 2(10x^2 + 16x) = 7x \cdot 10x^2 + 7x \cdot 16x + 2 \cdot 10x^2 + 2 \cdot 16x.$$

We complete the multiplications and combine like terms:

$$7x \cdot 10x^2 + 7x \cdot 16x + 2 \cdot 10x^2 + 2 \cdot 16x = 70x^3 + 112x^2 + 20x^2 + 32x = 70x^3 + 132x^2 + 32x.$$

**25.** Multiplying and collecting like terms gives

$$\begin{aligned}(s-3)(s+5) + s(s-2) &= s^2 + 2s - 15 + s^2 - 2s \\ &= 2s^2 - 15.\end{aligned}$$

**26.** This could be equivalent to an expression of the form $(x + r)(x + s)$. If so, then $r + s$ must equal 5 and $rs$ must equal 6. The numbers $r = 2$ and $s = 3$ satisfy both conditions.

Therefore, $x^2 + 5x + 6 = (x + 2)(x + 3)$.

**27.** This could be equivalent to an expression of the form $(y+r)(y+s)$. If so, then $r+s$ must equal $-5$ and $rs$ must equal $-6$. The numbers $r=-6$ and $s=1$ satisfy both conditions.

Therefore, $y^2-5y-6=(y-6)(y+1)$.

**28.** This could be equivalent to an expression of the form $(n+r)(n+s)$. If so, then $r+s$ must equal $-1$ and $rs$ must equal $-30$. The numbers $r=-6$ and $s=5$ satisfy both conditions.

Therefore, $n^2-n-30=(n-6)(n+5)$.

**29.** This could be equivalent to an expression of the form $(g+r)(g+s)$. If so, then $r+s$ must equal $-12$ and $rs$ must equal 20. The numbers $r=-10$ and $s=-2$ satisfy both conditions.

Therefore, $g^2-12g+20=(g-10)(g-2)$.

**30.** This could be equivalent to an expression of the form $(t+r)(t+s)$. If so, then $r+s$ must equal $-27$ and $rs$ must equal 50. The numbers $r=-2$ and $s=-25$ satisfy both conditions.

Therefore, $t^2-27t+50=(t-2)(t-25)$.

**31.** This could be equivalent to an expression of the form $(q+r)(q+s)$. If so, then $r+s$ must equal 15 and $rs$ must equal 50. The numbers $r=10$ and $s=5$ satisfy both conditions.

Therefore, $q^2+15q+50=(q+10)(q+5)$.

**32.** This could be equivalent to an expression of the form $(b+r)(b+s)$. If so, then $r+s$ must equal 2 and $rs$ must equal $-24$. The numbers $r=6$ and $s=-4$ satisfy both conditions.

Therefore, $b^2+2b-24=(b+6)(b-4)$.

**33.** This could be equivalent to an expression of the form $(x+ry)(x+sy)$. If so, then $r+s$ must equal 11 and $rs$ must equal 24. The numbers $r=3$ and $s=8$ satisfy both conditions.

Therefore, $x^2+11xy+24y^2=(x+3y)(x+8y)$.

**34.** First, factor out the common factor 2.

$$2z^2+12z-14=2(z^2+6z-7).$$

The remaining factor is a quadratic expression. This could be equivalent to an expression of the form $(z+r)(z+s)$. If so, then $r+s$ must equal 6 and $rs$ must equal $-7$. The numbers $r=7$ and $s=-1$ satisfy both conditions.

Therefore,

$$2z^2+12z-14=2(z^2+6z-7)=2(z+7)(z-1).$$

**35.** There is no common factor. We multiply the coefficient of the $z^2$ term by the constant term: $4\cdot 12=48$.

Now we try to write $19z$ as a sum of two terms whose coefficients multiply to 48. Writing $19z=16z+3z$ works since $16\cdot 3=48$.

So, we write $4z^2+19z+12$ as $4z^2+16z+3z+12$ and factor by grouping.

$$\begin{aligned}4z^2+19z+12&=4z^2+16z+3z+12\\&=(4z^2+16z)+(3z+12)\\&=4z(z+4)+3(z+4)\\&=(z+4)(4z+3).\end{aligned}$$

**36.** This could be equivalent to an expression of the form $(y+r)(y+s)$ where $r$ and $s$ are integers. If so, then $r+s$ must equal $-6$ and $rs$ must equal 7. There are no integers that satisfy both conditions.

Therefore, $y^2-6y+7$ cannot be factored this way.

**37.** First, factor out the common factor of 3.

$$3w^2+12w-36=3(w^2+4w-12).$$

The remaining factor is a quadratic expression. This could be equivalent to an expression of the form $(w+r)(w+s)$. If so, then $r+s$ must equal 4 and $rs$ must equal $-12$. The numbers $r=6$ and $s=-2$ satisfy both conditions.

Therefore, $3w^2+12w-36=3(w^2+4w-12)=3(w+6)(w-2)$.

**38.** First, factor out the common factor of 2.

$$2n^2 - 12n - 54 = 2(n^2 - 6n - 27).$$

The remaining factor is a quadratic expression. This could be equivalent to an expression of the form $(n + r)(n + s)$. If so, then $r + s$ must equal $-6$ and $rs$ must equal $-27$. The numbers $r = -9$ and $s = 3$ satisfy both conditions.

Therefore, $2n^2 - 12n - 54 = 2(n^2 - 6n - 27) = 2(n - 9)(n + 3)$.

**39.** This could be equivalent to an expression of the form $(a + r)(a + s)$ where $r$ and $s$ are integers. If so, then $r + s$ must equal $-1$ and $rs$ must equal $-16$. There are no integers that satisfy both conditions.

Therefore, $a^2 - a - 16$ cannot be factored this way.

**40.** This is the difference of two squares.

$$x^2 - 16 = (x + 4)(x - 4).$$

**41.** The first and last terms are both squares. If $s^2 - 12st + 36t^2$ is a square of a sum of two terms, the two terms could be $\pm s$ and $\pm 6t$. Twice the product of $s$ and $-6t$ is $-12st$, which is the middle term of $s^2 - 12st + 36t^2$.

Thus,

$$s^2 - 12st + 36t^2 = (s - 6t)^2.$$

**42.** We note a common factor of $x$ in the two terms, so we factor it out:

$$x^2 + 7x = x(x + 7).$$

**43.** We note that this is a difference of squares. Thus, we know that

$$x^2 - 36 = (\sqrt{x^2} + \sqrt{36})(\sqrt{x^2} - \sqrt{36}) = (x + 6)(x - 6).$$

**44.** We note that this is a difference of squares. Thus, we know that

$$x^2 - 169 = (\sqrt{x^2} + \sqrt{169})(\sqrt{x^2} - \sqrt{169}) = (x + 13)(x - 13).$$

**45.** This could be equivalent to an expression of the form $(x + r)(x + s)$. If so, then $r + s$ must equal 10 and $rs$ must equal 25. The numbers $r = 5$ and $s = 5$ satisfy both conditions. Therefore, $x^2 + 10x + 25 = (x + 5)(x + 5)$.

**46.** This could be equivalent to an expression of the form $(x + r)(x + s)$. If so, then $r + s$ must equal 26 and $rs$ must equal 169. The numbers $r = 13$ and $s = 13$ satisfy both conditions. Therefore, $x^2 + 26x + 169 = (x + 13)(x + 13)$.

**47.** This could be equivalent to an expression of the form $(x + r)(x + s)$. If so, then $r + s$ must equal 15 and $rs$ must equal 56. The numbers $r = 7$ and $s = 8$ satisfy both conditions. Therefore, $x^2 + 15x + 56 = (x + 7)(x + 8)$.

**48.** This could be equivalent to an expression of the form $(x + r)(x + s)$. If so, then $r + s$ must equal $-19$ and $rs$ must equal 90. The numbers $r = -9$ and $s = -10$ satisfy both conditions. Therefore, $x^2 - 19x + 90 = (x - 9)(x - 10)$.

**49.** We multiply the coefficient of the $x^2$ term by the constant term: $5 \cdot (-72) = -360$. Now we try to write $-37x$ as a sum of two terms whose coefficients multiply to $-360$. Writing $-37x = -45x + 8x$ works, since $-45 \cdot 8 = -360$. So we write $5x^2 - 37x - 72$ as $5x^2 - 45x + 8x - 72$, and factor by grouping.

$$\begin{aligned} 5x^2 - 37x - 72 &= 5x^2 - 45x + 8x - 72 \\ &= (5x^2 - 45x) + (8x - 72) \\ &= 5x(x - 9) + 8(x - 9) \\ &= (5x + 8)(x - 9). \end{aligned}$$

**50.** We begin by recognizing that this is a quadratic expression, because we can write $-5x + dx$ as $(-5 + d)x$. This could be equivalent to an expression of the form $(x + r)(x + s)$. If so, then $r + s$ must equal $-5 + d$ and $rs$ must equal $-5d$. The numbers $r = -5$ and $s = d$ satisfy both conditions. Therefore, $x^2 - 5x + dx - 5d = (x - 5)(x + d)$.

**51.** We can group the first two terms and the second two terms as $(8x^2 - 4xy) + (-6x + 3y)$. Factoring out a $4x$ from the first group and a $-3$ from the second group gives us

$$8x^2 - 4xy - 6x + 3y = (8x^2 - 4xy) + (-6x + 3y) = 4x(2x - y) - 3(2x - y) = (2x - y)(4x - 3).$$

**52.** We begin by recognizing that this is a quadratic expression, because we can write $pqx - 14x$ as $(pq - 14)x$. We multiply the coefficient of the $x^2$ term by the constant term: $2q \cdot (-7p) = -14qp$. Now we try to write $(pq - 14)x$ as a sum of two terms whose coefficients multiply to $-14qp$. Writing $(pq - 14)x = pqx - 14x$ works, since $pq \cdot (-14) = -14pq$. So we write $2qx^2 + pqx - 14x - 7p$ as it is and factor by grouping.

$$\begin{aligned} 2qx^2 + pqx - 14x - 7p &= (2qx^2 + pqx) - (14x + 7p) \\ &= qx(2x + p) - 7(2x + p) \\ &= (qx - 7)(2x + p). \end{aligned}$$

**53.** We first take out a common factor of $x$, giving

$$x^3 - 16x^2 + 64x = x(x^2 - 16x + 64).$$

This could be equivalent to an expression of the form $x(x + r)(x + s)$. If so, then $r + s$ must equal $-16$ and $rs$ must equal $64$. The numbers $r = -8$ and $s = -8$ satisfy both conditions. Therefore, $x^3 - 16x^2 + 64x = x(x - 8)(x - 8)$.

**54.** We think of $x^6 - 2x^3 - 63$ as $(x^3)^2 - 2(x^3) - 63$. If it is factorable, it would be equivalent to an expression of the form $(x^3 + r)(x^3 + s)$. If so, then $r + s$ must equal $-2$ and $rs$ must equal $-63$. The numbers $r = -9$ and $s = 7$ satisfy both conditions.

Therefore, $x^6 - 2x^3 - 63 = (x^3 - 9)(x^3 + 7)$.

**55.** Factor out the common factor $r$.

$$r^3 - 14r^2s^3 + 49rs^6 = r(r^2 - 14rs^3 + 49s^6).$$

The first and last terms of $r^2 - 14rs^3 + 49s^6$ are both squares. If $r^2 - 14rs^3 + 49s^6$ is a square of a sum of two terms, the two terms could be $\pm r$ and $\pm 7s^3$. Twice the product of $r$ and $-7s^3$ is $-14rs^3$, which is the middle term of $r^2 - 14rs^3 + 49s^6$. Thus,

$$r^3 - 14r^2s^3 + 49rs^6 = r(r - 7s^3)^2.$$

**56.** First, factor out the common factor $2a$.

$$8a^3 + 50ab^2 = 2a(4a^2 + 25b^2).$$

The remaining term is the sum of two squares, which cannot be factored. Thus,

$$8a^3 + 50ab^2 = 2a(4a^2 + 25b^2).$$

**57.** Factor out the common factor $2x$.

$$18x^7 + 48x^4z^2 + 32xz^4 = 2x(9x^6 + 24x^3z^2 + 16z^4).$$

The first and last terms of $9x^6 + 24x^3z^2 + 16z^4$ are both squares. If $9x^6 + 24x^3z^2 + 16z^4$ is a square of a sum of two terms, the two terms could be $\pm 3x^3$ and $4z^2$. Twice the product of $3x^3$ and $4z^2$ is $24x^3z^2$, which is the middle term of $9x^6 + 24x^3z^2 + 16z^4$. Thus,

$$18x^7 + 48x^4z^2 + 32xz^4 = 2x(3x^3 + 4z^2)^2.$$

## PROBLEMS

**58.** First factor out the common term $ay$. Therefore $ay - a^3y^3 = ay(1 - a^2y^2)$. Then, factoring the difference of squares, we have $ay - a^3y^3 = ay(1 - ay)(1 + ay)$.

**59.** First factor out $2x^2$ which produces $2x^2(9 - x^2z^6)$. Then factor the difference of perfect squares to give $2x^2(3 - xz^3)(3 + xz^3)$.

**60.** We have the difference of two perfect squares. Therefore $(a + b)^2 - 100 = ((a + b) - 10)((a + b) + 10)$. This is the same as $(a + b - 10)(a + b + 10)$.

**61.** First we factor out the common 3 to give $3(4-9(t+1)^2)$. Now factor the difference of squares to give $3(2-3(t+1))(2+3(t+1))$. Combining like terms inside the parentheses and factoring out a $-1$ gives $12-27(t+1)^2 = -3(3t+1)(3t+5)$.

**62.** First, write the first three terms as a perfect square giving $x^2+8x+16-y^2 = (x+4)^2 - y^2$. The new expression is the difference of squares, which factors into $((x+4)-y)((x+4)+y)$. This is the same as $(x+4-y)(x+4+y)$.

**63.** Factoring out a common $q^4$ term gives $q^4(1-q^4)$. Now we factor the difference of squares to give $q^4(1+q^2)(1-q^2)$. Since $(1-q^2)$ can still be factored, we obtain $q^4(1+q^2)(1+q)(1-q)$.

**64.** First factor out the common term $(t+1)$ to give $(t+1)((t+1)^2-25)$. Factoring the difference of squares gives $(t+1)((t+1)-5)((t+1)+5)$. This simplifies to $(t+1)(t-4)(t+6)$.

**65.** First factor out a common $2w$. This gives $2w(w^2-8w+16)$. Since $(w^2-8w+16)$ is a perfect square, we obtain $2w(w-4)^2$.

**66.** Factoring out the constant 3 gives $3(4a^2+20a+25)$. The first and last terms are the squares $(2a)^2$ and $5^2$. Twice the product of $2a$ and 5 is the middle term $20a$, so we obtain $3(2a+5)^2$.

**67.** Factoring out $st$ gives $st(16s^2-24st+9t^2)$. The first and third terms are squares. Twice the product of $4s$ and $-3t$ is $-24st$ so we have $st(4s-3t)^2$.

**68.** The first and third terms are squares. Twice the product of $(r+1)$ and $6t$ is $12t(r+1)$ so we have $((r+1)+6t)^2 = (r+6t+1)^2$.

**69.**

$$\begin{aligned}((x+h)+1)((x+h)-1) &= (x+h)^2 - 1\\ &= x^2+2xh+h^2-1.\end{aligned}$$

**70.** We have

$$\begin{aligned}(2+3(a+b))^2 &= 4+12(a+b)+9(a+b)^2\\ &= 4+12a+12b+9a^2+18ab+9b^2.\end{aligned}$$

**71.** We see that $8y\left(\frac{y^3}{2}-\frac{1}{4}\right)\left(\frac{y^3}{2}+\frac{1}{4}\right)$ is a difference of squares in factored form, so

$$\begin{aligned}8y\left(\frac{y^3}{2}-\frac{1}{4}\right)\left(\frac{y^3}{2}+\frac{1}{4}\right) &= 8y\left(\frac{y^6}{4}-\frac{1}{16}\right)\\ &= 2y^7-\frac{y}{2}.\end{aligned}$$

We could also multiply out.

## Solutions for Section 2.4

### EXERCISES

**1.** We have

$$\frac{m}{2}+\frac{m}{3} = \frac{3m}{6}+\frac{2m}{6} = \frac{5m}{6}.$$

**2.** We have

$$2+\frac{3}{x} = \frac{2x}{x}+\frac{3}{x} = \frac{2x+3}{x}.$$

**3.** We have

$$\frac{1}{x-2}-\frac{1}{x-3}=\frac{x-3}{(x-2)(x-3)}-\frac{x-2}{(x-2)(x-3)}=\frac{x-3-(x-2)}{(x-2)(x-3)}=\frac{-1}{(x-2)(x-3)}.$$

**4.** We have

$$\frac{3}{x}+\frac{4}{x-1}=\frac{3(x-1)}{x(x-1)}+\frac{4x}{x(x-1)}=\frac{3x-3+4x}{x(x-1)}=\frac{7x-3}{x(x-1)}.$$

**5.** We have

$$\frac{-1}{x}-\frac{1}{-x}+\frac{-1}{-x}=-\frac{1}{x}+\frac{1}{x}+\frac{1}{x}=\frac{1}{x}.$$

**6.** We have

$$\begin{aligned} z+\frac{z}{2}+\frac{2}{z} &= \frac{2z\cdot z}{2z}+\frac{z\cdot z}{z\cdot 2}+\frac{2\cdot 2}{2\cdot z} \\ &= \frac{2z^2+z^2+4}{2z} \\ &= \frac{3z^2+4}{2z}. \end{aligned}$$

**7.** We have

$$\frac{1}{4}(e/2)=\frac{1}{4}\cdot\frac{e}{2}=\frac{e}{8}.$$

**8.** We have

$$\frac{1}{a}+\frac{1}{b}=\frac{b}{ab}+\frac{a}{ab}=\frac{b+a}{ab}.$$

**9.** We have

$$\frac{1}{a-b}+\frac{1}{a+b}=\frac{a+b}{(a-b)(a+b)}+\frac{a-b}{(a-b)(a+b)}=\frac{a+b+a-b}{(a-b)(a+b)}=\frac{2a}{(a-b)(a+b)}.$$

**10.** We have

$$\frac{1}{x-a}-\frac{1}{x-b}=\frac{x-b}{(x-a)(x-b)}-\frac{x-a}{(x-a)(x-b)}=\frac{x-b-(x-a)}{(x-a)(x-b)}=\frac{a-b}{(x-a)(x-b)}.$$

**11.** We have

$$\begin{aligned} \frac{\frac{1}{3}r+r/4}{2r/5-\frac{1}{11}(3r)} &= \frac{\frac{r}{3}+\frac{r}{4}}{\frac{2r}{5}-\frac{3r}{11}} \\ &= \frac{\frac{4\cdot r}{4\cdot 3}+\frac{3\cdot r}{3\cdot 4}}{\frac{11\cdot 2r}{11\cdot 5}-\frac{5\cdot 3r}{5\cdot 11}} \\ &= \frac{\frac{4r+3r}{12}}{\frac{22r-15r}{55}} \\ &= \frac{\frac{7r}{12}}{\frac{7r}{55}} \\ &= \frac{7r}{12}\cdot\frac{55}{7r} \\ &= \frac{55}{12}. \end{aligned}$$

**12.** We have

$$\begin{aligned}
1+\frac{1}{1+\frac{1}{x}} &= 1+\frac{1}{\frac{x}{x}+\frac{1}{x}}\\
&= 1+\frac{1}{\frac{x+1}{x}}\\
&= 1+\frac{x}{x+1} \text{ since } x\neq 0\\
&= \frac{x+1}{x+1}+\frac{x}{x+1}\\
&= \frac{x+1+x}{x+1}\\
&= \frac{2x+1}{x+1}.
\end{aligned}$$

**13.** Multiply the two numerators and the two denominators and cancel like terms:

$$\frac{5p}{6q^2}\cdot\frac{3pq}{5p}=\frac{15p^2q}{30pq^2}=\frac{\cancel{15}p^{\cancel{2}\,1}\cancel{q}}{\cancel{30}^2\cancel{p}q^{\cancel{2}\,1}}=\frac{p}{2q}.$$

**14.** Multiply the two numerators and the two denominators:

$$\frac{3xy^2}{4x^2z}\cdot\frac{8xy^3z}{6xy^5}=\frac{24x^2y^5z}{24x^3y^5z}.$$

We can simplify the answer by canceling like terms:

$$\frac{24x^2y^5z}{24x^3y^5z}=\frac{\cancel{24x^2y^5z}}{\cancel{24}x^{\cancel{3}\,1}\cancel{y^5z}}=\frac{1}{x}.$$

**15.** Multiply the two numerators and the two denominators:

$$\frac{2ab}{5b}\cdot\frac{10a^2b^2}{6a}=\frac{20a^3b^3}{30ab}.$$

We can simplify the answer by canceling like terms:

$$\frac{20a^3b^3}{30ab}=\frac{\cancel{20}^2a^{\cancel{3}\,2}b^{\cancel{3}\,2}}{\cancel{30}^3\cancel{ab}}=\frac{2a^2b^2}{3}.$$

**16.** Multiply the two numerators and the two denominators:

$$\frac{4}{6x+12y}\cdot\frac{3x+6y}{10}=\frac{4(3x+6y)}{10(6x+12y)}.$$

Factor the numerator and denominator and cancel like terms:

$$\frac{4(3x+6y)}{10(6x+12y)}=\frac{4\cdot 3(x+2y)}{10\cdot 6(x+2y)}=\frac{12(x+2y)}{60(x+2y)}=\frac{\cancel{12(x+2y)}}{\cancel{60}^5\cancel{(x+2y)}}=\frac{1}{5}.$$

**17.** Multiply the two numerators and the two denominators:

$$\frac{2r+3s}{4s}\cdot\frac{6r}{6r+9s}=\frac{6r(2r+3s)}{4s(6r+9s)}.$$

Factor the 3 out in the denominator, multiply 4 by 3 and cancel like terms:

$$\frac{6r(2r+3s)}{4s(6r+9s)}=\frac{6r(2r+3s)}{4s\cdot 3(2r+3s)}=\frac{\cancel{6}r\cancel{(2r+3s)}}{\cancel{12}^2s\cancel{(2r+3s)}}=\frac{r}{2s}.$$

**18.** Multiply the two numerators and the two denominators:

$$\frac{x+3}{x+4}\cdot\frac{2x+8}{4x+12}=\frac{(x+3)(2x+8)}{(x+4)(4x+12)}.$$

Factor the numerator and denominator and cancel like terms:

$$\frac{(x+3)(2x+8)}{(x+4)(4x+12)}=\frac{2(x+3)(x+4)}{4(x+4)(x+3)}=\frac{\cancel{2(x+3)(x+4)}}{{}^{2}\cancel{4(x+4)(x+3)}}=\frac{1}{2}.$$

**19.** Make a fraction out of $c(a+b)+(a+b)$ by writing it with a denominator of 1.

$$\frac{1}{ab+abc}\cdot(c(a+b)+(a+b))=\frac{1}{ab+abc}\cdot\frac{c(a+b)+(a+b)}{1}.$$

Multiply the two numerators and the two denominators:

$$\frac{1}{ab+abc}\cdot\frac{c(a+b)+(a+b)}{1}=\frac{c(a+b)+(a+b)}{ab+abc}.$$

Factor the numerator and denominator and cancel like terms:

$$\frac{c(a+b)+(a+b)}{ab+abc}=\frac{(a+b)\cancel{(c+1)}}{ab\cancel{(1+c)}}=\frac{a+b}{ab}.$$

**20.** Multiply the two numerators and the two denominators:

$$\frac{p^2+4p}{p^2-2p}\cdot\frac{3p-6}{3p+12}=\frac{(p^2+4p)(3p-6)}{(p^2-2p)(3p+12)}.$$

Factor the numerator and denominator, and cancel like terms:

$$\frac{(p^2+4p)(3p-6)}{(p^2-2p)(3p+12)}=\frac{p(p+4)\cdot 3(p-2)}{p(p-2)\cdot 3(p+4)}=\frac{\cancel{3p(p+4)(p-2)}}{\cancel{3p(p-2)(p+4)}}=1.$$

**21.** Multiply the two numerators and the two denominators:

$$\frac{w^2r+4wr}{2r^2w+2wr}\cdot\frac{r+r^2}{4w+16}=\frac{(w^2r+4wr)(r+r^2)}{(2r^2w+2wr)(4w+16)}.$$

Factor the numerator and denominator, and cancel like terms:

$$\frac{(w^2r+4wr)(r+r^2)}{(2r^2w+2wr)(4w+16)}=\frac{wr(w+4)\cdot r(1+r)}{2wr(r+1)\cdot 4(w+4)}=\frac{wr^2(w+4)(1+4)r}{8wr(r+1)(w+4)}=\frac{\cancel{wr}^{1}\cancel{{}^{2}}\cancel{(w+4)(1+r)}}{8\cancel{wr(r+1)(w+4)}}=\frac{r}{8}.$$

**22.** Multiply the two numerators and the two denominators:

$$\frac{cd+c}{cd-8d}\cdot\frac{16-2c}{4+4d}=\frac{(cd+c)(16-2c)}{(cd-8d)(4+4d)}.$$

Factor the numerator and denominator, and cancel like terms:

$$\frac{(cd+c)(16-2c)}{(cd-8d)(4+4d)}=\frac{c(d+1)\cdot 2(8-c)}{-d(-c+8)\cdot 4(1+d)}=-\frac{2c(d+1)(8-c)}{4d(8-c)(1+d)}=-\frac{\cancel{2}c\cancel{(d+1)(8-c)}}{{}^{2}\cancel{4}d\cancel{(8-c)(1+d)}}=-\frac{c}{2d}.$$

**23.** Multiply the two numerators and the two denominators:

$$\frac{5v^2+15v}{vw-v}\cdot\frac{3w+3}{5v+15}=\frac{(5v^2+15v)(3w+3)}{(vw-v)(5v+15)}.$$

Factor the numerator and denominator, and cancel like terms:

$$\frac{(5v^2+15v)(3w+3)}{(vw-v)(5v+15)}=\frac{5v(v+3)\cdot 3(w+1)}{v(w-1)\cdot 5(v+3)}=\frac{15v(v+3)(w+1)}{5v(w-1)(v+3)}=\frac{\cancel{15}^{3}\cancel{v(v+3)}(w+1)}{\cancel{5v}(w-1)\cancel{(v+3)}}=\frac{3(w+1)}{w-1}.$$

**24.** Multiply the two numerators and the two denominators:

$$\frac{ab+b}{2b^2+6b}\cdot\frac{3a^2+6a}{a+a^2}=\frac{(ab+b)(3a^2+6a)}{(2b^2+6b)(a+a^2)}.$$

Factor the numerator and denominator, and cancel like terms:

$$\frac{(ab+b)(3a^2+6a)}{(2b^2+6b)(a+a^2)}=\frac{b(a+1)\cdot 3a(a+2)}{2b(b+3)\cdot a(1+a)}=\frac{3ab(a+1)(a+2)}{2ba(b+3)(1+a)}=\frac{3\cancel{ab(a+1)}(a+2)}{2\cancel{ba}(b+3)\cancel{(1+a)}}=\frac{3(a+2)}{2(b+3)}.$$

**25.** $\frac{z+1}{2}=\frac{z}{2}+\frac{1}{2}$.

**26.** $\frac{w}{10}=\frac{1}{10}w$, which can be thought of as a product of fractions by writing it as $\frac{1}{10}\frac{w}{1}$.

**27.** $\frac{4}{c+2}=\frac{2}{c+2}+\frac{2}{c+2}$

**28.** $\frac{5}{6p}=\frac{5}{6}\cdot\frac{1}{p}$.

**29.** $\frac{6p-3}{6}=\frac{6p}{6}-\frac{3}{6}=p-\frac{1}{2}$, which can be thought of as a difference of fractions by writing it as $\frac{p}{1}-\frac{1}{2}$.

**30.** $\frac{1}{xyz}=\frac{1}{x}\cdot\frac{1}{y}\cdot\frac{1}{z}$.

**31.** $\frac{8-5x}{-2}=\frac{8}{-2}-\frac{5x}{-2}=-4+\frac{5x}{2}$, which can be thought of as a sum of two fractions by writing it as $\frac{-4}{1}+\frac{5x}{2}$.

**32.** $\frac{-3t}{9s}=\frac{-3}{9}\cdot\frac{t}{s}=\frac{-1}{3}\cdot\frac{t}{s}$.

**33.** $\frac{2xh+h^2}{h}=\frac{2xh}{h}+\frac{h^2}{h}=2x+h$, which can be thought of as a sum of two fractions by writing it as $\frac{2x}{1}+\frac{h}{1}$.

**34.** $\frac{c}{ab}=\left(\frac{c}{a}\right)\left(\frac{1}{b}\right)$.

**35.** $\frac{c}{a+b}=\frac{c+1}{a+b}+\frac{-1}{a+b}$.

**36.** $\frac{h(B+b)}{2}=\frac{h}{2}(B+b)$, which can be thought of as a product of two fractions by writing it as $\frac{h}{2}\frac{(B+b)}{1}$.

**37.** $\frac{3}{t(r+s)}=\frac{3}{t}\cdot\frac{1}{r+s}$.

**38.** $\frac{4}{y+x}=\frac{2}{y+x}+\frac{2}{y+x}$.

**39.** $\frac{1+2a+3b}{4}=\frac{1}{4}+\frac{2a}{4}+\frac{3b}{4}=\frac{1}{4}+\frac{a}{2}+\frac{3b}{4}$.

**40.** $\frac{p+prt}{p}=1+rt$, which can be thought of as a sum of two fractions by writing it as $=\frac{1}{1}+\frac{rt}{1}$.

**41.** $\frac{(x+1)^2-y}{xy}=\frac{(x+1)^2}{xy}-\frac{y}{xy}=\frac{(x+1)^2}{xy}-\frac{1}{x}$.

**42.** $\frac{1}{(p+2)^2+b}=\frac{2}{(p+2)^2+b}-\frac{1}{(p+2)^2+b}$

**43.** $\frac{x}{x-1}=\frac{x+1}{x-1}-\frac{1}{x-1}$

## PROBLEMS

**44.** We first need to simplify the denominator:

$$\frac{1}{A}+\frac{1}{B}=\frac{B+A}{AB}.$$

Using the rule for dividing fractions,

$$\frac{1}{1/A+1/B}=\frac{1}{(B+A)/(AB)}=\frac{1}{1}\cdot\frac{AB}{B+A}=\frac{AB}{B+A}.$$

**45.** We have a common denominator, so we can write

$$\frac{1}{x}+\frac{1}{x}=\frac{1+1}{x}=\frac{2}{x}.$$

This matches expression (c).

**46.** We can multiply the numerator and denominator by 2, giving

$$\frac{x}{0.5}=\frac{x}{0.5}\cdot\frac{2}{2}=\frac{2x}{1}=2x.$$

This matches expression (a).

**47.** Instead of dividing $1/x$ by 2, we can multiply by $1/2$:

$$\frac{1/x}{2}=\frac{1}{2}\cdot\frac{1}{x}=\frac{1}{2x}.$$

This matches expression (b).

**48.** We first rewrite the expression as follows:

$$-\frac{1}{-1-x}=\frac{-1}{1}\cdot\frac{1}{-1-x}=\frac{-1}{-1-x}.$$

Next we multiply the numerator and denominator by $-1$:

$$\frac{-1}{-1}\cdot\frac{-1}{-1-x}=\frac{1}{-1(-1-x)}=\frac{1}{1+x}=\frac{1}{x+1}.$$

This matches expression (f).

**49.** This expression cannot be rewritten to match any of the expressions (a)–(f).

**50.** We first rewrite this as

$$-\frac{1}{x-1}=\frac{-1}{1}\cdot\frac{1}{x-1}=\frac{-1}{x-1}.$$

Multiplying the numerator and denominator by $-1$ gives

$$\frac{-1}{-1}\cdot\frac{-1}{x-1}=\frac{-1(-1)}{-1(x-1)}=\frac{1}{-x+1}=\frac{1}{1-x}.$$

This matches expression (e).

**51.** Inverting the denominator and multiplying, we have

$$\frac{\frac{4ab^3}{3}}{\frac{2b}{a^2}}=\frac{4ab^3}{3}\cdot\frac{a^2}{2b}$$
$$=\frac{2a^3b^2}{3}.$$

**52.** First write the numerator as a single fraction. $\frac{\frac{1}{s}+\frac{1}{t}}{st}=\frac{\frac{t+s}{st}}{st}$. Therefore, if we invert the denominator and multiply we obtain $\frac{\frac{1}{s}+\frac{1}{t}}{st}=\frac{t+s}{st}\cdot\frac{1}{st}=\frac{t+s}{(st)^2}$.

**53.** By first writing the denominator of the original fraction as a single fraction whose denominator is 36, we obtain $\dfrac{p+q}{\dfrac{p}{12}+\dfrac{q}{18}} = \dfrac{p+q}{\dfrac{3p+2q}{36}}$. Inverting the denominator and multiplying, gives $\dfrac{36(p+q)}{3p+2q}$.

**54.** First invert the denominator and multiply. $\dfrac{\dfrac{m+2}{3}}{\dfrac{m^2-4}{6m}} = \dfrac{m+2}{3}\cdot\dfrac{6m}{m^2-4}$.

If $m^2-4$ is factored into $(m+2)(m-2)$ we can cancel the $m+2$ term and also cancel a common factor of 3. Therefore, $\dfrac{\dfrac{m+2}{3}}{\dfrac{m^2-4}{6m}} = \dfrac{m+2}{3}\cdot\dfrac{6m}{(m+2)(m-2)} = \dfrac{2m}{m-2}$.

**55.** Writing the numerator as a single fraction gives $\dfrac{\dfrac{2}{x}-3}{\dfrac{2-3x}{2}} = \dfrac{\dfrac{2-3x}{x}}{\dfrac{2-3x}{2}}$.

Inverting the denominator, multiplying and canceling like terms produces

$$\frac{\dfrac{2}{x}-3}{\dfrac{2-3x}{2}} = \frac{\dfrac{2-3x}{x}}{\dfrac{2-3x}{2}} = \frac{2-3x}{x}\cdot\frac{2}{2-3x} = \frac{2}{x}.$$

**56.** Inverting the denominator and multiplying produces

$$\frac{\dfrac{t}{t-3}}{\dfrac{4}{4t-12}} = \frac{t}{t-3}\cdot\frac{4t-12}{4}.$$

Factoring out a 4 from the numerator of the second factor gives

$$\frac{t}{t-3}\cdot\frac{4t-12}{4} = \frac{t}{t-3}\cdot\frac{4(t-3)}{4}.$$

Canceling the common factors of 4 and $(t-3)$ gives $\dfrac{t}{(t-3)}\cdot\dfrac{4(t-3)}{4} = t$.

**57.** Writing the numerator and denominator as single fractions gives

$$\frac{\dfrac{1}{25}-\dfrac{1}{x^2}}{\dfrac{1}{x}-\dfrac{1}{5}} = \frac{\dfrac{x^2-25}{25x^2}}{\dfrac{5-x}{5x}}.$$

Inverting the denominator and multiplying fractions gives

$$\frac{\dfrac{x^2-25}{25x^2}}{\dfrac{5-x}{5x}} = \frac{x^2-25}{25x^2}\cdot\frac{5x}{5-x}.$$

Since the numerator $x^2-25 = (x+5)(x-5)$ and the denominator $(5-x) = -(x-5)$, we have

$$\frac{x^2-25}{25x^2}\cdot\frac{5x}{5-x} = \frac{(x-5)(x+5)}{25x^2}\cdot\frac{5x}{-(x-5)}.$$

Canceling out common factors of 5, $x$, and $(x-5)$ gives

$$\frac{(x-5)(x+5)}{25x^2}\cdot\frac{5x}{-(x-5)} = -\frac{x+5}{5x}.$$

**58.** Writing both the numerator and denominator as single fractions gives

$$\frac{\frac{x}{y}-\frac{y}{x}}{\frac{1}{y}+\frac{1}{x}}=\frac{\frac{x^2-y^2}{xy}}{\frac{x+y}{xy}}.$$

We invert the denominator and multiply the fractions, noting that $x^2-y^2=(x+y)(x-y)$, to obtain

$$\frac{\frac{x^2-y^2}{xy}}{\frac{x+y}{xy}}=\frac{(x+y)(x-y)}{xy}\cdot\frac{xy}{x+y}.$$

Canceling common factors of $xy$ and $x+y$ gives $\frac{(x+y)(x-y)}{xy}\cdot\frac{xy}{x+y}=x-y$

**59.** Writing the numerator as a single fraction gives

$$\frac{\frac{1}{c^2}-\frac{1}{d^2}}{\frac{d-c}{c^2d}}=\frac{\frac{d^2-c^2}{c^2d^2}}{\frac{d-c}{c^2d}}.$$

We invert the denominator and multiply the fractions producing

$$\frac{\frac{d^2-c^2}{c^2d^2}}{\frac{d-c}{c^2d}}=\frac{d^2-c^2}{c^2d^2}\cdot\frac{c^2d}{d-c}.$$

Canceling common terms $c^2d$ and recognizing that $d^2-c^2=(d-c)(d+c)$ gives

$$\frac{(d-c)(d+c)}{c^2d^2}\cdot\frac{c^2d}{d-c}=\frac{d+c}{d}.$$

**60.** Writing the numerator and denominator as single fractions gives

$$\frac{\frac{1}{k+1}-1}{\frac{1}{k+1}+1}=\frac{\frac{1-(k+1)}{k+1}}{\frac{1+(k+1)}{k+1}}.$$

We invert the denominator and multiply the fractions producing

$$\frac{\frac{1-(k+1)}{k+1}}{\frac{1+(k+1)}{k+1}}=\frac{-k}{k+1}\cdot\frac{k+1}{k+2}=\frac{-k}{(k+2)}.$$

**61.** Writing the numerator a single fraction, we have

$$\frac{\frac{1}{m-1}+\frac{2}{m+2}}{\frac{3}{m+2}}=\frac{\frac{(m+2)+2(m-1)}{(m-1)(m+2)}}{\frac{3}{m+2}}.$$

We invert the denominator and multiply the fractions producing $\frac{(m+2)+2(m-1)}{(m-1)(m+2)}\cdot\frac{m+2}{3}$. Since the numerator $(m+2)+2(m-1)$ is equal to $3m$, and since there are a common factors of $m+2$ and 3, we have

$$\frac{(m+2)+2(m-1)}{(m-1)(m+2)}\cdot\frac{m+2}{3}=\frac{3m}{(m-1)(m+2)}\cdot\frac{m+2}{3}=\frac{m}{m-1}.$$

**62.** We have

$$
\begin{aligned}
2-\frac{2}{2+\frac{2}{2+2}} &= 2-\frac{2}{2+\frac{1}{2}} \\
&= 2-\frac{2}{5/2} \\
&= 2-\frac{4}{5} \\
&= \frac{6}{5}.
\end{aligned}
$$

**63.** Writing both the numerator and denominator over the common denominator $w^2$ produces

$$\frac{1+\frac{2}{w}-\frac{24}{w^2}}{1-\frac{1}{w}-\frac{12}{w^2}} = \frac{\frac{w^2+2w-24}{w^2}}{\frac{w^2-w-12}{w^2}}.$$

Inverting the denominator, and factoring we get $\dfrac{(w+6)(w-4)}{w^2}\cdot\dfrac{w^2}{(w-4)(w+3)}$. Canceling like terms gives

$$\frac{(w+6)(w-4)}{w^2}\cdot\frac{w^2}{(w-4)(w+3)} = \frac{w+6}{w+3}.$$

**64.** Writing the numerator and denominator as single fractions we have

$$\frac{12+\frac{12}{a}+\frac{3}{a^2}}{12+\frac{6}{a}} = \frac{\frac{12a^2+12a+3}{a^2}}{\frac{12a+6}{a}}.$$

Now invert the denominator and factor both the numerator and denominator to produce

$$\frac{\frac{12a^2+12a+3}{a^2}}{\frac{12a+6}{a}} = \frac{3(2a+1)^2}{a^2}\cdot\frac{a}{6(2a+1)}.$$

Canceling the $2a+1$ and 3 terms in the numerator and denominator produces

$$\frac{3(2a+1)^2}{a^2}\cdot\frac{a}{6(2a+1)} = \frac{2a+1}{2a}.$$

## Solutions for Chapter 2 Review

### EXERCISES

**1.** Yes. The expression $x(5x)$ can be reordered to $5x \cdot x$, which is equivalent to $5x^2$.

**2.** Yes. The order of the addition is switched.

**3.** No. The expression $(2x)(5y)$ can be reordered as $2 \cdot 5 \cdot x \cdot y$ which is equivalent to $10xy$ not $7xy$.

**4.** Yes. The expression $(a+3)+(b+2)$ can be reordered as $a+b+2+3$ which is equivalent to $(a+b)+5$.

**5.** We have

$$\begin{aligned}3(z+r)^2+6rz+\frac{r-z-2}{r+z} &= 3(2-5)^2+6(-5)\cdot 2+\frac{-5-2-2}{-5+2}\\ &= 3(-3)^2-60+\frac{-9}{-3}\\ &= 3\cdot 9-60+3\\ &= -30.\end{aligned}$$

**6.** We have

$$\begin{aligned}3(z+r)^2+6rz+\frac{r-z-2}{r+z} &= 3(5k-3k)^2+6(5k)(-3k)+\frac{5k-(-3k)-2}{5k+(-3k)}\\ &= 3(2k)^2-90k^2+\frac{8k-2}{2k}\\ &= 12k^2-90k^2+\frac{4k-1}{k}\\ &= -78k^2+\frac{4k-1}{k}\\ &= -78k^2+4-\frac{1}{k}.\end{aligned}$$

**7.** We can combine the $p^2$ terms in $3p^2-2q^2+6pq-p^2$ to get $2p^2-2q^2+6pq$.

**8.** We can combine the $y^3$ terms and the $xy$ terms in $y^3+2xy-4y^3+x-2xy$ to get $-3y^3+x$.

**9.** We combine all three terms to find $(5/12)A$.

**10.** We have $(a+4)/3+(2a-4)/3=a/3+(4/3)+(2a)/3-(4/3)=a$.

**11.** We have $3(2t-4)-t(3t-2)+16=6t-12-3t^2+2t+16=-3t^2+8t+4$.

**12.** We can combine the $z^4$ terms in $5z^4+5z^3-3z^4$ to get $2z^4+5z^3$.

**13.** We have

$$\begin{aligned}2(x+5)+3(x-4) &= 2x+10+3x-12\\ &= 5x-2.\end{aligned}$$

**14.** We have

$$\begin{aligned}7(x-2)-5(2x-5) &= 7x-14-10x+25\\ &= -3x+11.\end{aligned}$$

**15.** We have

$$\begin{aligned}3(2x-5)-2(5x+4)+5(10-3x) &= 6x-15-10x-8+50-15x\\ &= -19x+27.\end{aligned}$$

**16.** We have

$$\begin{aligned}6(2x+1)-5(3x-4)+6x-10 &= 12x+6-15x+20+6x-10\\ &= 3x+16.\end{aligned}$$

**17.** We have

$$\begin{aligned} 2x(3x+4)+3(x^2-5x+6) &= 6x^2+8x+3x^2-15x+18 \\ &= 9x^2-7x+18. \end{aligned}$$

**18.** We have

$$\begin{aligned} 3x(x+5)-4x(3x-1)+5(6x-3) &= 3x^2+15x-12x^2+4x+30x-15 \\ &= -9x^2+49x-15. \end{aligned}$$

**19.** We have

$$\begin{aligned} 2a(a+b)-5b(a+b) &= 2a^2+2ab-5ab-5b^2 \\ &= 2a^2-3ab-5b^2. \end{aligned}$$

**20.** We have

$$\begin{aligned} 5a(a+2b)-3b(2a-5b)+ab(2a-1) &= 5a^2+10ab-6ab+15b^2+2a^2b-ab \\ &= 5a^2+3ab+15b^2+2a^2b. \end{aligned}$$

**21.** We have

$$\begin{aligned} mn(m+2n)+3mn(2m+n)+5m^2n &= m^2n+2mn^2+6m^2n+3mn^2+5m^2n \\ &= 12m^2n+5mn^2. \end{aligned}$$

**22.** We have

$$\begin{aligned} 5(x^2-3x-5)-2x(x^2-5x-7) &= 5x^2-15x-25-2x^3+10x^2+14x \\ &= -2x^3+15x^2-x-25. \end{aligned}$$

**23.** This could be equivalent to an expression of the form $(z+r)(z+s)$. If so, then $r+s$ must equal $-5$ and $rs$ must equal $6$. The numbers $r=-3$ and $s=-2$ satisfy both conditions.

Therefore, $z^2-5z+6=(z-3)(z-2)$.

**24.** This could be equivalent to an expression of the form $(a+r)(a+s)$. If so, then $r+s$ must equal $8$ and $rs$ must equal $-48$. The numbers $r=12$ and $s=-4$ satisfy both conditions.

Therefore, $a^2+8a-48=(a+12)(a-4)$.

**25.** This could be equivalent to an expression of the form $(v+r)(v+s)$. If so, then $r+s$ must equal $-4$ and $rs$ must equal $-32$. The numbers $r=-8$ and $s=4$ satisfy both conditions.

Therefore, $v^2-4v-32=(v-8)(v+4)$.

**26.** This could be equivalent to an expression of the form $(b+r)(b+s)$. If so, then $r+s$ must equal $-23$ and $rs$ must equal $-50$. The numbers $r=-25$ and $s=2$ satisfy both conditions.

Therefore, $b^2-23b-50=(b-25)(b+2)$.

**27.** This could be equivalent to an expression of the form $(w+r)(w+s)$ where $r$ and $s$ are integers. If so, then $r+s$ must equal $2$ and $rs$ must equal $24$. There are no integers that satisfy both conditions.

Therefore, $w^2+2w+24$ cannot be factored this way.

**28.** This could be equivalent to an expression of the form $(x+r)(x+s)$. If so, then $r+s$ must equal 1 and $rs$ must equal $-72$. The numbers $r=9$ and $s=-8$ satisfy both conditions.

Therefore, $x^2+x-72=(x+9)(x-8)$.

**29.** There is no common factor. We multiply the coefficient of the $x^2$ term by the constant term: $6\cdot(-6)=-36$.

Now we try to write $5x$ as a sum of two terms whose coefficients multiply to $-36$. Writing $5x=9x-4x$ works since $9\cdot(-4)=-36$.

So, we write $6x^2+5x-6$ as $6x^2+9x-4x-6$ and factor by grouping.

$$\begin{aligned}6x^2+5x-6&=6x^2+9x-4x-6\\&=(6x^2+9x)+(-4x-6)\\&=3x(2x+3)-2(2x+3)\\&=(2x+3)(3x-2).\end{aligned}$$

**30.** Factor out the common factor of 2.

$$30t^2+26t+4=2(15t^2+13t+2).$$

We multiply the coefficient of the $t^2$ term by the constant term: $15\cdot 2=30$.

Now we try to write $13t$ as a sum of two terms whose coefficients multiply to 30. Writing $13t=10t+3t$ works since $10\cdot 3=30$.

So, we write $15t^2+13t+2$ as $15t^2+10t+3t+2$ and factor by grouping.

$$\begin{aligned}30t^2+26t+4&=2(15t^2+13t+2)\\&=2(15t^2+10t+3t+2)\\&=2(5t(3t+2)+(3t+2))\\&=2(3t+2)(5t+1).\end{aligned}$$

**31.** This could be equivalent to an expression of the form $(q+rz)(q+sz)$. If so, then $r+s$ must equal $-4$ and $rs$ must equal 3. The numbers $r=-3$ and $s=-1$ satisfy both conditions.

Therefore, $q^2-4qz+3z^2=(q-3z)(q-z)$.

**32.** There is no common factor. We multiply the coefficient of the $s^2$ term by the constant term: $12\cdot(-5)=-60$.

Now we try to write $17s$ as a sum of two terms whose coefficients multiply to $-60$. Writing $17s=20s-3s$ works since $20\cdot(-3)=-60$.

So, we write $12s^2+17s-5$ as $12s^2+20s-3s-5$ and factor by grouping.

$$\begin{aligned}12s^2+17s-5&=12s^2+20s-3s-5\\&=(12s^2+20s)+(-3s-5)\\&=4s(3s+5)-(3s+5)\\&=(3s+5)(4s-1).\end{aligned}$$

**33.** Factor out the common factor of 2.

$$12w^2-10w-8=2(6w^2-5w-4).$$

We multiply the coefficient of the $w^2$ term by the constant term: $6\cdot(-4)=-24$.

Now we try to write $-5w$ as a sum of two terms whose coefficients multiply to $-24$. Writing $-8w+3w$ works since $(-8)\cdot 3=-24$.

So, we write $6w^2-5w-4$ as $6w^2-8w+3w-4$ and factor by grouping.

$$\begin{aligned}12w^2-10w-54&=2(6w^2-5w-4)\\&=2(6w^2-8w+3w-4)\\&=2(2w(3w-4)+(3w-4))\\&=2(3w-4)(2w+1).\end{aligned}$$

**34.** There is no common factor. We multiply the coefficient of the $z^2$ term by the constant term: $10 \cdot (-10) = -100$.

Now we try to write $-21z$ as a sum of two terms whose coefficients multiply to $-100$. Writing $-21z = -25z + 4z$ works since $25 \cdot (-4) = -100$.

So, we write $10z^2 - 21z - 10 = 10z^2 - 25z + 4z - 10$ and factor by grouping.

$$\begin{aligned} 10z^2 - 21z - 10 &= 10z^2 - 25z + 4z - 10 \\ &= (10z^2 - 25z) + (4z - 10) \\ &= 5z(2z - 5) + 2(2z - 5) \\ &= (2z - 5)(5z + 2). \end{aligned}$$

**35.** This is the sum of two squares, so cannot be factored as a difference of two squares.

**36.** This is the difference of two squares.

$$(c+3)^2 - d^4 = (c + 3 + d^2)(c + 3 - d^2).$$

**37.** We note a common factor of $x$ in the two terms, so we factor it out:

$$x^3 + 4x = x(x^2 + 4).$$

**38.** We note that this is a difference of squares. Thus, we know that

$$x^2 - 81 = (\sqrt{x^2} + \sqrt{81})(\sqrt{x^2} - \sqrt{81}) = (x+9)(x-9).$$

**39.** We note that this is a difference of squares. Thus, we know that

$$x^2 - 144 = (\sqrt{x^2} + \sqrt{144})(\sqrt{x^2} - \sqrt{144}) = (x+12)(x-12).$$

**40.** This could be equivalent to an expression of the form $(x+r)(x+s)$. If so, then $r+s$ must equal $-14$ and $rs$ must equal $49$. The numbers $r = -7$ and $s = -7$ satisfy both conditions. Therefore, $x^2 - 14x + 49 = (x-7)(x-7)$.

**41.** This could be equivalent to an expression of the form $(x+r)(x+s)$. If so, then $r+s$ must equal $-22$ and $rs$ must equal $121$. The numbers $r = -11$ and $s = -11$ satisfy both conditions. Therefore, $x^2 - 22x + 121 = (x-11)(x-11)$.

**42.** This could be equivalent to an expression of the form $(x+r)(x+s)$. If so, then $r+s$ must equal $-3$ and $rs$ must equal $-54$. The numbers $r = -9$ and $s = 6$ satisfy both conditions. Therefore, $x^2 - 3x - 54 = (x-9)(x+6)$.

**43.** We multiply the coefficient of the $x^2$ term by the constant term: $3 \cdot 35 = 105$. Now we try to write $22x$ as a sum of two terms whose coefficients multiply to 105. Writing $22x = 15x + 7x$ works, since $15 \cdot 7 = 105$. So we write $3x^2 + 22x + 35$ as $3x^2 + 15x + 7x + 35$, and factor by grouping.

$$\begin{aligned} 3x^2 + 22x + 35 &= 3x^2 + 15x + 7x + 35 \\ &= (3x^2 + 15x) + (7x + 35) \\ &= 3x(x+5) + 7(x+5) \\ &= (3x+7)(x+5). \end{aligned}$$

**44.** We can group the first two terms and the second two terms as $(x^2 + x) + (3x + 3)$. Factoring out an $x$ from the first group and a 3 from the second group gives us

$$x^2 + x + 3x + 3 = (x^2 + x) + (3x + 3) = x(x+1) + 3(x+1) = (x+1)(x+3).$$

**45.** We can group the first two terms and the second two terms as $(ax + bx) + (-ay - by)$. Factoring out an $x$ from the first group and a $-y$ from the second group gives us

$$ax + bx - ay - by = (ax + bx) + (-ay - by) = x(a+b) - y(a+b) = (a+b)(x-y).$$

**46.** Although a 2 can be factored out of the first two terms and an $a$ can be factored out from the second two terms, after that there is no other common factor. So we rearrange the terms as $12a^2 + 24a + ab + 2b$, then group the first two terms and the second two terms as $(12a^2 + 24a) + (ab + 2b)$. Factoring out a $12a$ from the first group and a $b$ from the second group gives us

$$12a^2 + 24a + ab + 2b = 12a(a + 2) + b(a + 2) = (a + 2)(12a + b).$$

**47.** We first take out a common factor of $x$, giving

$$x^3 + 23x^2 + 132x = x(x^2 + 23x + 132).$$

This could be equivalent to an expression of the form $x(x+r)(x+s)$. If so, then $r + s$ must equal 23 and $rs$ must equal 132. The numbers $r = 12$ and $s = 11$ satisfy both conditions. Therefore, $x^3 + 23x^2 + 132x = x(x + 12)(x + 11)$.

**48.** We first take out a common factor of $x^2$, giving

$$x^4 - 18x^3 + 81x^2 = x^2(x^2 - 18x + 81).$$

This could be equivalent to an expression of the form $x^2(x + r)(x + s)$. If so, then $r + s$ must equal $-18$ and $rs$ must equal 81. The numbers $r = -9$ and $s = -9$ satisfy both conditions. Therefore, $x^4 - 18x^3 + 81x^2 = x^2(x - 9)(x - 9)$.

**49.** First, factor out the common factor $y$.

$$y^3 + 7y^2 - 18y = y(y^2 + 7y - 18).$$

The remaining factor is a quadratic expression. This could be equivalent to an expression of the form $(y + r)(y + s)$. If so, then $r + s$ must equal 7 and $rs$ must equal $-18$. The numbers $r = 9$ and $s = -2$ satisfy both conditions.

Therefore,

$$y^3 + 7y^2 - 18y = y(y^2 + 7y - 18) = y(y + 9)(y - 2).$$

**50.** First, factor out the common factor $7y$.

$$7y^5 - 28yz^6 = 7y(y^4 - 4z^6).$$

The remaining expression is the difference of two squares.

$$7y^5 - 28yz^6 = 7y(y^2 - 2z^3)(y^2 + 2z^3).$$

**51.** Factor out the common factor $-3t^3$.

$$-3t^7 + 24t^5v^2 - 48t^3v^4 = -3t^3(t^4 - 8t^2v^2 + 16v^4).$$

The first and last terms of $t^4 - 8t^2v^2 + 16v^4$ are both squares. If $t^4 - 8t^2v^2 + 16v^4$ is a square of a sum of two terms, the two terms could be $\pm t^2$ and $\pm 4v^2$. Twice the product of $t^2$ and $-4v^2$ is $-8t^2v^2$, which is the middle term of $t^4 - 8t^2v^2 + 16v^4$. Thus,

$$-3t^7 + 24t^5v^2 - 48t^3v^4 = -3t^3(t^2 - 4v^2)^2.$$

The expression $t^2 - 4v^2$ is the difference of two squares and can be factored as $(t + 2v)(t - 2v)$. Thus,

$$-3t^7 + 24t^5v^2 - 48t^3v^4 = -3t^3(t^2 - 4v^2)^2 = -3t^3((t + 2v)(t - 2v))^2 = -3t^3(t + 2v)^2(t - 2v)^2.$$

**52.** The first and last terms are both squares. If $n^2 + 10n + 25q^2$ is a square of a sum of two terms, the two terms could be $\pm n$ and $\pm 5q$. However, twice the product of $n$ and $5q$ is $10nq$, which is not the middle term of $n^2 + 10n + 25q^2$. Thus, $n^2 + 10n + 25q^2$ cannot be factored as a perfect square.

**53.** $\dfrac{4a - 8}{16} = \dfrac{4(a - 2)}{16} = \dfrac{a - 2}{4}$.

**54.** $\dfrac{10y^3 - 5y^2}{15y} = \dfrac{5y^2(2y - 1)}{15y} = \dfrac{y(2y - 1)}{3}$.

**55.** $\dfrac{12w - 36w^3}{24w^2} = \dfrac{12w(1 - 3w^2)}{24w^2} = \dfrac{1 - 3w^2}{2w}$.

**56.** $\dfrac{4x-8}{10x-20} = \dfrac{4(x-2)}{10(x-2)} = \dfrac{2}{5}.$

**57.** $\dfrac{3t^3+12t}{4t^2+16} = \dfrac{3t(t^2+4)}{4(t^2+4)} = \dfrac{3t}{4}.$

**58.** We have

$$\begin{aligned}\frac{3s^3-12s}{s-2} &= \frac{3s(s^2-4)}{s-2}\\ &= \frac{3s(s+2)(s-2)}{s-2}\\ &= 3s(s+2).\end{aligned}$$

**59.** We have

$$\begin{aligned}\frac{(x-y)^2}{x^2-y^2} &= \frac{(x-y)(x-y)}{(x+y)(x-y)}\\ &= \frac{x-y}{x+y}.\end{aligned}$$

**60.** We have

$$\begin{aligned}\frac{2x^2-32}{x^2-2x-8} &= \frac{2(x^2-16)}{(x-4)(x+2)}\\ &= \frac{2(x+4)(x-4)}{(x-4)(x+2)}\\ &= \frac{2(x+4)}{x+2}.\end{aligned}$$

**61.** $\dfrac{p^2q-pq^2}{(p-q)^2} = \dfrac{pq(p-q)}{(p-q)(p-q)} = \dfrac{pq}{p-q}.$

**62.** $\dfrac{2(y-4)}{4(4-y)} = \dfrac{2(y-4)}{-4(y-4)} = -\dfrac{1}{2}.$

**63.** We have

$$\begin{aligned}\frac{9z-3z^2}{z^2-9} &= \frac{3z(3-z)}{(z+3)(z-3)}\\ &= -\frac{3z(z-3)}{(z+3)(z-3)}\\ &= -\frac{3z}{z+3}.\end{aligned}$$

**64.** We have

$$\begin{aligned}\frac{6a-2a^2}{a^2-a-6} &= \frac{2a(3-a)}{(a-3)(a+2)}\\ &= -\frac{2a(a-3)}{(a-3)(a+2)}\\ &= -\frac{2a}{a+2}.\end{aligned}$$

**65.** We have

$$\begin{aligned}\frac{r^4-1}{r^3p-rp} &= \frac{(r^2-1)(r^2+1)}{rp(r^2-1)}\\ &= \frac{r^2+1}{rp}.\end{aligned}$$

**66.** $\dfrac{9k^2+12k+4}{12k+8} = \dfrac{(3k+2)^2}{4(3k+2)} = \dfrac{3k+2}{4}.$

**67.** $\dfrac{r(r+s)-s(r+s)}{r^2s+rs^2} = \dfrac{(r+s)(r-s)}{rs(r+s)} = \dfrac{r-s}{rs}.$

## PROBLEMS

**68.** **(a)** To decrease a quantity by 10%, we multiply by $1-0.10 = 0.90$. The cost of the four items without the sale is $a+b+c+d$, so with the sale

$$\text{Total cost } = 0.90(a+b+c+d).$$

**(b)** We multiply each price by 0.90 and then add them up:

$$\text{Total cost } = 0.90a + 0.90b + 0.90c + 0.90d.$$

**(c)** Yes. We use the distributive law to multiply through by 0.90.

**(d)** No, it does not matter. The price is the same either way.

**69.** We have

$$\frac{3a}{2b} = \frac{3}{2}\cdot\frac{a}{b} = \frac{3}{2}\cdot\frac{2}{3} = 1.$$

**70.** We have

$$2(m+n)-(s-2)+(1-s) = 2(s+2)-(s-2)+(1-s) = 2s+4-s+1-s = 5.$$

**71.** If we let $r$ denote the radius of the original circle, then we have $\pi r^2 = 30$. The radius of the new circle is $3r$, so we have

$$\begin{aligned}\text{Area } &= \pi(\text{radius})^2\\ &= \pi\cdot(3r)^2\\ &= \pi\cdot(9r^2)\\ &= 9\cdot(\pi r^2)\\ &= 9\cdot(30)\\ &= 270.\end{aligned}$$

The area of the new circle is 270 square meters.

**72.** If we let $r$ denote the radius of the original cylinder and $h$ denote the height of the original cylinder, then we have $\pi r^2 h = 100$. The radius of the new cylinder is $2r$ and the height of the new cylinder is $3h$, so we have

$$\begin{aligned}\text{Volume } &= \pi\cdot(\text{radius})^2\cdot(\text{height})\\ &= \pi\cdot(2r)^2\cdot(3h)\\ &= \pi\cdot(4r^2)\cdot(3h)\\ &= 12\cdot(\pi r^2 h)\\ &= 12\cdot(100)\\ &= 1200.\end{aligned}$$

The volume of the new cylinder is 1200 cubic centimeters.

**73.** We have

$$\begin{aligned}6x^2+12 &= 2\underbrace{(3x^2+6)}_{r}\\ &= 2r,\end{aligned}$$

so $r = 3x^2+6$.

**74.** We have

$$\begin{aligned} 6x^2+12 &= 6x^2+9+3 \\ &= 3\underbrace{(2x^2+3)}_{v}+3, \end{aligned}$$

so $v = 2x^2+3$.

**75.** One possibility is

$$6x^2+12 = \underbrace{2}_{r}\cdot\underbrace{3x^2}_{v}+\underbrace{(2+1)}_{r+1}\cdot\underbrace{4}_{w},$$

so $r=2, v=3x^2, w=4$. Another possibility is

$$6x^2+12 = \underbrace{3}_{r}\cdot\underbrace{2x^2}_{v}+\underbrace{(3+1)}_{r+1}\cdot\underbrace{3}_{w},$$

so in this case $r=3, v=2x^2, w=3$. There are many other possibilities.

**76.** We have

$$\begin{aligned} \frac{4x}{2x-6}-5 &= -5+\frac{4x}{2x-6} && \text{re-order} \\ &= -5+\frac{4x}{2(x-3)} && \text{factor denominator} \\ &= -5+\frac{2x}{x-3} && \text{cancel} \\ &= -5-(-1)\cdot\frac{2x}{x-3} && \text{write addition as subtraction} \\ &= -5-\frac{-2x}{x-3}, \end{aligned}$$

so $k=-5, m=-2, n=3$.

**77.** We have

$$\begin{aligned} \frac{3(x+5)-7x}{x+5} &= \frac{3(x+5)-7x}{x+5} \\ &= \frac{3(x+5)}{x+5}-\frac{7x}{x+5} \\ &= 3-\frac{7x}{x+5} \\ &= 3-\frac{7x}{x-(-5)}, \end{aligned}$$

so $k=3, m=7, n=-5$.

**78.** We have

$$\begin{aligned} 2w^3+3v^2+3w^2v-3(v-w)^2 &= 2(-2)^3+3(k-1)^2+3(-2)^2(k-1)-3\left(k-1-(-2)\right)^2 \\ &= 2(-8)+3\left(k^2-2k+1\right)+12(k-1)-3(k+1)^2 \\ &= -16+3k^2-6k+3+12k-12-3\left(k^2+2k+1\right) \\ &= -25+3k^2+6k-3k^2-6k-3 \\ &= -28. \end{aligned}$$

**79. (a)** We add the amounts from the two jobs, so

$$\text{Earnings per week } = A+B.$$

**(b)** To increase a quantity by 5%, we multiply by 1.05, so

$$\text{Earnings per week } = 1.05(A+B).$$

**(c)** We multiply $A$ by 1.05 and leave $B$ the same, so

$$\text{Earnings per week } = 1.05A+B.$$

**(d)** We leave $A$ the same and multiply $B$ by 1.05, so

$$\text{Earnings per week } = A+1.05B.$$

**(e)** Both $A$ and $B$ are multiplied by 1.05, so

$$\text{Earnings per week } = 1.05A+1.05B.$$

**(f)** The expressions in (b) and (e) are equivalent.

**80.** We have

$$\begin{aligned}
\left(x^2+x+6\right)\left(3x^3+6x^2\right) &= \left(3x^3+6x^2\right)\left(x^2+(x+6)\right) && \text{reorder and regroup}\\
&= \underbrace{\left(3x^3+6x^2\right)}_{a}\left(\underbrace{x^2}_{b}+\underbrace{(x+6)}_{c}\right) && \text{identify } a,b,c\\
&= \underbrace{\left(3x^3+6x^2\right)}_{a}\underbrace{x^2}_{b}+\underbrace{\left(3x^3+6x^2\right)}_{a}\underbrace{(x+6)}_{c} && \text{distributive law}\\
&= x^2(3x^3+6x^2)+(x+6)(3x^3+6x^2), && \text{reorder as required}
\end{aligned}$$

$$\begin{aligned}
\text{so}\quad a &= 3x^3+6x^2\\
b &= x^2\\
c &= x+6.
\end{aligned}$$

**81.** We have

$$\begin{aligned}
\left(x^2+x+6\right)\left(3x^3+6x^2\right) &= \left(x^2+x+6\right)\left(3x^2\cdot x+3x^2\cdot 2\right) && \text{regroup}\\
&= \left(x^2+x+6\right)\left(\underbrace{3x^2}_{a}\cdot\underbrace{x}_{b}+\underbrace{3x^2}_{a}\cdot\underbrace{2}_{c}\right) && \text{identify } a,b,c\\
&= \left(x^2+x+6\right)\cdot\underbrace{3x^2}_{a}\cdot\left(\underbrace{x}_{b}+\underbrace{2}_{c}\right) && \text{factor (distribute law)}\\
&= 3x^2\left(x^2+x+6\right)(x+2), && \text{reorder as required}
\end{aligned}$$

$$\begin{aligned}
\text{so}\quad a &= 3x^2\\
b &= x\\
c &= 2.
\end{aligned}$$

Note that here, we used the distributive law to factor part of this expression, writing $3x^3+6x^2$ as $3x^2(x+2)$. The other part of the expression, $x^2+x+6$, was not involved.