Chapter 2: Linear Equations and Inequalities

2.1: Introduction to Equations

Concepts

1. equal
2. solution
3. solution set
4. Equivalent
5. solutions
6. given
7. $b + c$
8. bc
9. 1
10. 2

The Addition Property of Equality

11. 22
12. $-\frac{1}{6}$
13. $x + 5 = 0 \Rightarrow x + 5 - 5 = 0 - 5 \Rightarrow x + 0 = -5 \Rightarrow x = -5$; To check you answer, substitute -5 for x in the original equation, $-5 + 5 = 0$. This statement is true. Thus, the solution $x = -5$ is correct.
14. $x + 3 = 7 \Rightarrow x + 3 - 3 = 7 - 3 \Rightarrow x + 0 = 4 \Rightarrow x = 4$; To check you answer, substitute 4 for x in the original equation, $4 + 3 = 7$. This statement is true. Thus, the solution $x = 4$ is correct.
15. $x - 7 = 1 \Rightarrow x - 7 + 7 = 1 + 7 \Rightarrow x + 0 = 8 \Rightarrow x = 8$
16. $x - 23 = 0 \Rightarrow x - 23 + 23 = 0 + 23 \Rightarrow x + 0 = 23 \Rightarrow x = 23$
17. $9 = y - 8 \Rightarrow 9 + 8 = y - 8 + 8 \Rightarrow 17 = y + 0 \Rightarrow 17 = y \Rightarrow y = 17$
18. $97 = -23 + y \Rightarrow 97 + 23 = 23 + (-23) + y \Rightarrow 120 = 0 + y \Rightarrow y = 120$
19. $\frac{1}{2} = z - \frac{3}{2} \Rightarrow \frac{1}{2} + \frac{3}{2} = z - \frac{3}{2} + \frac{3}{2} \Rightarrow \frac{4}{2} = z + 0 \Rightarrow 2 = z \Rightarrow z = 2$
20. $\frac{3}{4} + z = -\frac{1}{2} \Rightarrow \frac{3}{4} + \left(-\frac{3}{4}\right) + z = -\frac{1}{2} + \left(-\frac{3}{4}\right) \Rightarrow 0 + z = -\frac{5}{4} \Rightarrow z = -\frac{5}{4}$
21. $t - 0.8 = 4.2 \Rightarrow t - 0.8 + 0.8 = 4.2 + 0.8 \Rightarrow t + 0 = 5 \Rightarrow t = 5$
22. $4 = -9 + t \Rightarrow 4 + 9 = -9 + 9 + t \Rightarrow 13 = 0 + t \Rightarrow 13 = t \Rightarrow t = 13$
23. $25 + x = 10 \Rightarrow 25 - 25 + x = 10 - 25 \Rightarrow 0 + x = -15 \Rightarrow x = -15$
24. $85 = x - 20 \Rightarrow 85 + 20 = x - 20 + 20 \Rightarrow 105 = x + 0 \Rightarrow 105 = x \Rightarrow x = 105$
25. $1989 = 26 + y \Rightarrow 1989 + (-26) = 26 + (-26) + y \Rightarrow 1963 = 0 + y \Rightarrow 1963 = y \Rightarrow y = 1963$

26. $y - 1.23 = -0.02 \Rightarrow y - 1.23 + 1.23 = -0.02 + 1.23 \Rightarrow y + 0 = 1.21 \Rightarrow y = 1.21$

The Multiplication Property of Equality

27. 5

28. $\frac{3}{4}$

29. $5x = 15 \Rightarrow \frac{5x}{5} = \frac{15}{5} \Rightarrow x = 3$

30. $-2x = 8 \Rightarrow \frac{-2x}{-2} = \frac{8}{-2} \Rightarrow x = -4$

31. $-7x = 0 \Rightarrow \frac{-7x}{-7} = \frac{0}{-7} \Rightarrow x = 0$

32. $25x = 0 \Rightarrow \frac{25x}{25} = \frac{0}{25} \Rightarrow x = 0$

33. $\frac{1}{2}x = \frac{3}{2} \Rightarrow \frac{2}{1} \cdot \frac{1}{2}x = \frac{3}{2} \cdot \frac{2}{1} \Rightarrow x = 3$

34. $\frac{3}{4}x = \frac{5}{8} \Rightarrow \frac{4}{3} \cdot \frac{3}{4}x = \frac{5}{8} \cdot \frac{4}{3} \Rightarrow x = \frac{5}{6}$

35. $\frac{1}{2} = \frac{2}{5}z \Rightarrow \frac{5}{2} \cdot \frac{1}{2} = \frac{5}{2} \cdot \frac{2}{5}z \Rightarrow \frac{5}{4} = z \Rightarrow z = \frac{5}{4}$

36. $-\frac{3}{4} = -\frac{1}{8}z \Rightarrow -\frac{3}{4} \cdot \left(-\frac{8}{1}\right) = -\frac{8}{1} \cdot \left(-\frac{1}{8}\right)z \Rightarrow 6 = z \Rightarrow z = 6$

37. $25 = 5z \Rightarrow \frac{25}{5} = \frac{5z}{5} \Rightarrow 5 = z \Rightarrow z = 5$

38. $-10 = -4z \Rightarrow \frac{-10}{-4} = \frac{-4z}{-4} \Rightarrow \frac{5}{2} = z \Rightarrow z = \frac{5}{2}$

39. $0.5t = 3.5 \Rightarrow \frac{0.5t}{0.5} = \frac{3.5}{0.5} \Rightarrow t = 7$

40. $2.2t = -9.9 \Rightarrow \frac{2.2t}{2.2} = \frac{-9.9}{2.2} \Rightarrow t = -\frac{9.9}{2.2} \Rightarrow t = -4.5$

41. $\frac{3}{8} = \frac{1}{4}y \Rightarrow \frac{3}{8} \cdot \frac{4}{1} = \frac{4}{1} \cdot \frac{1}{4}y \Rightarrow \frac{3}{2} = y \Rightarrow y = \frac{3}{2}$

42. $1.2 = 0.3y \Rightarrow \frac{1.2}{0.3} = \frac{0.3y}{0.3} \Rightarrow 4 = y \Rightarrow y = 4$

Applications

43. (a) See Figure 43.

(b) Let R represent total rainfall and let x represent the number of hours past noon. Start with 3 inches of rain and then add $\frac{1}{2}$, or 0.5, inches per hour after noon, $3 + 0.5x = R$, or equivalently $R = 0.5x + 3$.

(c) At 3 pm, $x = 3$. Substituting x with 3 in the formula, $R = 0.5 \cdot 3 + 3 \Rightarrow R = 4.5$ inches. This answer agrees with the table from part (a).

(d) At 2:15 pm, $x = 2.25$. Substituting x with 2.25 in the formula, $R = 0.5 \cdot 2.25 + 3 \Rightarrow R = 4.125$ inches.

Hours (x)	0	1	2	3	4	5	6
Rainfall (R)	3	3.5	4	4.5	5	5.5	6

Figure 43

Hours (x)	0	1	2	3	4	5	6	7
Temp. (T)	0	10	20	30	40	50	60	70

Figure 44

44. (a) See Figure 44.

(b) Let T represent the temperature and let x represent the number of hours past midnight. Since the temperature increases 10ºF per hour, $T = 10x$.

(c) At 5 am, $x = 5$. Substituting x with 5 in the formula, $T = 10 \cdot 5 \Rightarrow T = 50$°F. This agrees with the table from part (a).

(d) At 2:45 am, $x = 2.75$. Substituting x with 2.75 in the formula, $T = 10 \cdot 2.75 \Rightarrow T = 27.5$°F.

45. (a) Let L be the length of the football fields and x be the number of fields. Because each field x contains 300 feet, $L = 300x$.

(b) Substitute L with 870. Then $870 = 300x$.

(c) $870 = 300x \Rightarrow \frac{870}{300} = \frac{300x}{300} \Rightarrow x = \frac{870}{300} \Rightarrow x = 2.9$

46. (a) Let A represent the number of acres and let S represent the number of square feet. Because each acre contains 43,560 square feet, let $S = 43{,}560A$.

(b) Substitute S with 871,200. Then, $871{,}200 = 43{,}560A$.

(c) $871{,}200 = 43{,}560A \Rightarrow \frac{871{,}200}{43{,}560} = \frac{43{,}560A}{43{,}560} \Rightarrow A = \frac{871{,}200}{43{,}560} \Rightarrow A = 20$

47. Let x represent the total number of cubic miles that have melted and let y represent the number of years. Because the glaciers are melting at a rate of 24 cubic miles per year, $x = 24y$. Substitute x with 420. Then, $420 = 24y \Rightarrow \frac{420}{24} = \frac{24y}{24} \Rightarrow y = \frac{420}{24} \Rightarrow y = 17.5$. Thus, it takes 17.5 years for 420 cubic miles of the glacier to melt.

48. Let x represent the total number of cubic miles of ice and let y represent the number of years to obtain the equation $x = 50y$. Then, substitute x with 680,000 to obtain the equation $680{,}000 = 50y$. Then the solution is $680{,}000 = 50y \Rightarrow \frac{680{,}000}{50} = \frac{50y}{50} \Rightarrow y = \frac{680{,}000}{50} \Rightarrow y = 13{,}600$. Thus, it will take 13,600 years for 680,000 cubic miles of ice to melt.

49. Let x represent the cost of the car to obtain the equation $0.07x = 1750$. Then the solution is $\frac{0.07x}{0.07} = \frac{1750}{0.07} \Rightarrow x = \frac{1750}{0.07} \Rightarrow x = 25{,}000$. Thus, the cost of the car is \$25,000.

50. Let S represent the employee's current salary to obtain the equation $1.06S = 58{,}300$. Then the solution is $\frac{1.06S}{1.06} = \frac{58{,}300}{1.06} \Rightarrow S = \frac{58{,}300}{1.06} \Rightarrow S = 55{,}000$. Thus, the employee's current salary is \$55,000.

2.2: Linear Equations

Concepts

1. $ax + b = 0$

2. is not
3. addition, multiplication
4. distributive
5. LCD
6. 100
7. Infinitely many
8. None

Identifying Linear Equations

9. $3x - 7$ is a linear equation. $a = 3$ and $b = -7$.
10. $-2x + 1 = 4$ is a linear equation. $-2x + 1 = 4 \Rightarrow -2x + 1 - 4 = 4 - 4 \Rightarrow -2x - 3 = 0$.
 $a = -2$ and $b = -3$.
11. $\frac{1}{2}x = 0$ is a linear equation. $a = \frac{1}{2}$ and $b = 0$.
12. $-\frac{3}{4}x = 0$ is a linear equation. $a = -\frac{3}{4}$ and $b = 0$.
13. $4x^2 - 6 = 11$ is not a linear equation because in cannot be written in the form $ax + b = 0$. It has a non-zero term containing x^2.
14. $-2x^2 + x = 4$ is not a linear equation because in cannot be written in the form $ax + b = 0$. It has a non-zero term containing x^2.
15. $1.1x = 0.9$ is a linear equation. $1.1x = 0.9 \Rightarrow 1.1x - 0.9 = 0.9 - 0.9 \Rightarrow 1.1x - 0.9 = 0$.
 $a = 1.1$ and $b = -0.9$.
16. $-5.7x = -3.4$ is a linear equation. $-5.7x = -3.4 \Rightarrow -5.7x + 3.4 = -3.4 + 3.4 \Rightarrow -5.7x + 3.4 = 0$.
 $a = -5.7$ and $b = 3.4$.
17. $2(x - 3) = 0$ is a linear equation. Use the distributive property to obtain $2x - 6 = 0$. $a = 2$ and $b = -6$.
18. $\frac{1}{2}(x + 4) = 0$ is a linear equation. Use the distributive property to obtain $\frac{1}{2}x + 2 = 0$. $a = \frac{1}{2}$ and $b = 2$.
19. $6x - x^2 = 0$ is not a linear equation because in cannot be written in the form $ax + b = 0$. It has a non-zero term containing x^2.
20. $5 = 4x^3$ is not a linear equation because in cannot be written in the form $ax + b = 0$. It has a non-zero term containing x^3.
21. For $x = 0$, substitute 0 for x and solve: $-3(0) + 7 = 0 + 7 = 7$.
 For $x = 1$, substitute 1 for x and solve: $-3(1) + 7 = -3 + 7 = 4$.
 For $x = 2$, substitute 2 for x and solve: $-3(2) + 7 = -6 + 7 = 1$.
 For $x = 3$, substitute 3 for x and solve: $-3(3) + 7 = -9 + 7 = -2$.
 For $x = 4$, substitute 4 for x and solve: $-3(4) + 7 = -12 + 7 = -5$.
 See Figure 21. From the table, we see that the equation $-3x + 7 = 1$ is true when $x = 2$. Therefore, the solution to the equation $-3x + 7 = 1$ is $x = 2$.

x	0	1	2	3	4
$-3x+7$	7	4	1	−2	−5

Figure 21

x	−1	0	1	2	3
$5x-2$	−7	−2	3	8	13

Figure 22

22. For $x = -1$, substitute -1 for x and solve: $5(-1) - 2 = -5 - 2 = -7$.

    For $x = 0$, substitute 0 for x and solve: $5(0) - 2 = 0 - 2 = -2$.

    For $x = 1$, substitute 1 for x and solve: $5(1) - 2 = 5 - 2 = 3$.

    For $x = 2$, substitute 2 for x and solve: $5(2) - 2 = 10 - 2 = 8$.

    For $x = 3$, substitute 3 for x and solve: $5(3) - 2 = 15 - 2 = 13$.

    See Figure 22. From the table, we see that the equation $5x - 2 = 3$ is true when $x = 1$. Therefore, the solution to the equation $5x - 2 = 3$ is $x = 1$.

23. For $x = -2$, substitute -2 for x and solve: $4 - 2(-2) = 4 + 4 = 8$.

    For $x = -1$, substitute -1 for x and solve: $4 - 2(-1) = 4 + 2 = 6$.

    For $x = 0$, substitute 0 for x and solve: $4 - 2(0) = 4 + 0 = 4$.

    For $x = 1$, substitute 1 for x and solve: $4 - 2(1) = 4 - 2 = 2$.

    For $x = 2$, substitute 2 for x and solve: $4 - 2(2) = 4 - 4 = 0$.

    See Figure 23. From the table, we see that the equation $4 - 2x = 6$ is true when $x = -1$. Therefore, the solution to the equation $4 - 2x = 6$ is $x = -1$.

x	−2	−1	0	1	2
$4-2x$	8	6	4	2	0

Figure 23

x	−2	−1	0	1	2
$9-(x+3)$	8	7	6	5	4

Figure 24

24. For $x = -2$, substitute -2 for x and solve: $9 - (-2 + 3) = 9 - 1 = 8$.

    For $x = -1$, substitute -1 for x and solve: $9 - (-1 + 3) = 9 - 2 = 7$.

    For $x = 0$, substitute 0 for x and solve: $9 - (0 + 3) = 9 - 3 = 6$.

    For $x = 1$, substitute 1 for x and solve: $9 - (1 + 3) = 9 - 4 = 5$.

    For $x = 2$, substitute 2 for x and solve: $9 - (2 + 3) = 9 - 5 = 4$.

    See Figure 24. From the table, we see that the equation $9 - (x + 3) = 4$ is true when $x = 2$. Therefore, the solution to the equation $9 - (x + 3) = 4$ is $x = 2$.

Solving Linear Equations

25. $11x = 3 \Rightarrow \frac{11x}{11} = \frac{3}{11} \Rightarrow x = \frac{3}{11}$

26. $-5x = 15 \Rightarrow \frac{-5x}{-5} = \frac{15}{-5} \Rightarrow x = -3$

27. $x - 18 = 5 \Rightarrow x - 18 + 18 = 5 + 18 \Rightarrow x = 23$

28. $8 = 5 + 3x \Rightarrow 8 - 5 = 5 - 5 + 3x \Rightarrow 3 = 3x \Rightarrow \frac{3}{3} = \frac{3x}{3} \Rightarrow 1 = x \Rightarrow x = 1$

29. $2x - 1 = 13 \Rightarrow 2x - 1 + 1 = 13 + 1 \Rightarrow 2x = 14 \Rightarrow \frac{2x}{2} = \frac{14}{2} \Rightarrow x = 7$

30. $4x + 3 = 39 \Rightarrow 4x + 3 - 3 = 39 - 3 \Rightarrow 4x = 36 \Rightarrow \frac{4x}{4} = \frac{35}{4} \Rightarrow x = 9$

31. $5x + 5 = -6 \Rightarrow 5x + 5 - 5 = -6 - 5 \Rightarrow 5x = -11 \Rightarrow \frac{5x}{5} = \frac{-11}{5} \Rightarrow x = -\frac{11}{5}$

32. $-7x - 4 = 31 \Rightarrow -7x - 4 + 4 = 31 + 4 \Rightarrow -7x = 35 \Rightarrow \frac{-7x}{-7} = \frac{35}{-7} \Rightarrow x = -5$

33. $3z + 2 = z - 5 \Rightarrow 3z + 2 - 2 = z - 5 - 2 \Rightarrow 3z = z - 7 \Rightarrow 3z - z = z - z - 7 \Rightarrow 2z = -7 \Rightarrow$
$\frac{2z}{2} = \frac{-7}{2} \Rightarrow z = -\frac{7}{2}$

34. $z - 5 = 5z - 3 \Rightarrow z - 5 + 5 = 5z - 3 + 5 \Rightarrow z = 5z + 2 \Rightarrow z - 5z = 5z - 5z + 2 \Rightarrow -4z = 2 \Rightarrow$
$\frac{-4z}{-4} = \frac{2}{-4} \Rightarrow z = -\frac{1}{2}$

35. $12y - 6 = 33 - y \Rightarrow 12y - 6 + 6 = 33 + 6 - y \Rightarrow 12y = 39 - y \Rightarrow 12y + y = 39 - y + y \Rightarrow$
$13y = 39 \Rightarrow \frac{13y}{13} = \frac{39}{13} \Rightarrow y = 3$

36. $-13y + 2 = 22 - 3y \Rightarrow +13y + 2 - 2 = 22 - 2 - 3y \Rightarrow -13y = 20 - 3y \Rightarrow$
$-13y + 3y = 20 - 3y + 3y \Rightarrow -10y = 20 \Rightarrow \frac{-10y}{-10} = \frac{20}{-10} \Rightarrow y = -2$

37. $4(x - 1) = 5 \Rightarrow 4x - 4 = 5 \Rightarrow 4x - 4 + 4 = 5 + 4 \Rightarrow 4x = 9 \Rightarrow \frac{4x}{4} = \frac{9}{4} \Rightarrow x = \frac{9}{4}$

38. $-2(2x + 7) = 1 \Rightarrow -4x - 14 = 1 \Rightarrow -4x - 14 + 14 = 1 + 14 \Rightarrow -4x = 15 \Rightarrow \frac{-4x}{-4} = \frac{15}{-4} \Rightarrow$
$x = -\frac{15}{4}$

39. $1 - (3x + 1) = 5 - x \Rightarrow 1 - 3x - 1 = 5 - x \Rightarrow -3x = 5 - x \Rightarrow -3x + x = 5 - x + x \Rightarrow$
$-2x = 5 \Rightarrow \frac{-2x}{-2} = \frac{5}{-2} \Rightarrow x = -\frac{5}{2}$

40. $6 + 2(x - 7) = 10 - 3(x - 3) \Rightarrow 6 + 2x - 14 = 10 - 3x + 9 \Rightarrow 2x - 8 = 19 - 3x \Rightarrow$
$2x + 3x - 8 = 19 - 3x + 3x \Rightarrow 5x - 8 = 19 \Rightarrow 5x - 8 + 8 = 19 + 8 \Rightarrow 5x = 27 \Rightarrow$
$\frac{5x}{5} = \frac{27}{5} \Rightarrow x = \frac{27}{5}$

41. $5t - 6 + 2(t + 1) = 0 \Rightarrow 5t - 6 + 2t + 2 = 0 \Rightarrow 7t - 4 = 0 \Rightarrow 7t - 4 + 4 = 0 + 4 \Rightarrow$
$7t = 4 \Rightarrow \frac{7t}{7} = \frac{4}{7} \Rightarrow t = \frac{4}{7}$

42. $-2(t - 7) - (t + 5) = 5 \Rightarrow -2t + 14 - t - 5 = 5 \Rightarrow -3t + 9 = 5 \Rightarrow -3t + 9 - 9 = 5 - 9 \Rightarrow$
$-3t = -4 \Rightarrow \frac{-3t}{-3} = \frac{-4}{-3} \Rightarrow t = \frac{4}{3}$

43. $3(4z - 1) - 2(z + 2) = 2(z + 1) \Rightarrow 12z - 3 - 2z - 4 = 2z + 2 \Rightarrow 10z - 7 = 2z + 2 \Rightarrow$
$10z - 7 - 2 = 2z + 2 - 2 \Rightarrow 10z - 9 = 2z \Rightarrow 10z - 9 + 9 = 2z + 9 \Rightarrow 10z = 2z + 9 \Rightarrow$
$10z - 2z = 2z - 2z + 9 \Rightarrow 8z = 9 \Rightarrow \frac{8z}{8} = \frac{9}{8} \Rightarrow z = \frac{9}{8}$

44. $-(z + 4) + (3z + 1) = -2(z + 1) \Rightarrow -z - 4 + 3z + 1 = -2z - 2 \Rightarrow 2z - 3 = -2z - 2 \Rightarrow$
$2z - 3 + 3 = -2z - 2 + 3 \Rightarrow 2z = -2z + 1 \Rightarrow z + 2z = -2z + 2z + 1 \Rightarrow 4z = 1 \Rightarrow$
$\frac{4z}{4} = \frac{1}{4} \Rightarrow z = \frac{1}{4}$

45. $7.3x - 1.7 = 5.6 \Rightarrow 7.3x - 1.7 + 1.7 = 5.6 + 1.7 \Rightarrow 7.3x = 7.3 \Rightarrow \frac{7.3x}{7.3} = \frac{7.3}{7.3} \Rightarrow x = 1$

46. $5.5x + 3x = 51 \Rightarrow 8.5x = 51 \Rightarrow \frac{8.5x}{8.5} = \frac{51}{8.5} \Rightarrow x = 6$

47. $-9.5x - 0.05 = 10.5x + 1.05 \Rightarrow -9.5x - 10.5x - 0.05 = 10.5x - 10.5x + 1.05 \Rightarrow$
$-20x - 0.05 = 1.05 \Rightarrow -20x - 0.05 + 0.05 = 1.05 + 0.05 \Rightarrow -20x = 1.1 \Rightarrow \frac{-20x}{-20} = \frac{1.1}{-20} \Rightarrow$
$x = -0.055$

48. $0.04x + 0.03 = 0.02x - 0.1 \Rightarrow 0.04x - 0.02x + 0.03 = 0.02x - 0.02x - 0.01 \Rightarrow$
$0.02x + 0.03 = -0.1 \Rightarrow 0.02x - 0.03 - 0.03 = -0.1 - 0.03 \Rightarrow 0.02x = -0.13 \Rightarrow$
$\frac{0.02x}{0.02} = \frac{-0.13}{0.02} \Rightarrow x = -6.5$

49. $\frac{1}{2}x - \frac{3}{2} = \frac{5}{2} \Rightarrow \frac{1}{2}x - \frac{3}{2} + \frac{3}{2} = \frac{5}{2} + \frac{3}{2} \Rightarrow \frac{1}{2}x = 4 \Rightarrow 2 \cdot \frac{1}{2}x = 4 \cdot 2 \Rightarrow x = 8$

50. $-\frac{1}{4}x + \frac{5}{4} = \frac{3}{4} \Rightarrow -\frac{1}{4}x + \frac{5}{4} - \frac{5}{4} = \frac{3}{4} - \frac{5}{4} \Rightarrow -\frac{1}{4}x = -\frac{2}{4} \Rightarrow (-4) \cdot \left(-\frac{1}{4}x\right) = -\frac{2}{4} \cdot (-4) \Rightarrow x = 2$

51. $-\frac{3}{8}x + \frac{1}{4} = \frac{1}{2}x + \frac{1}{8} \Rightarrow -\frac{3}{8}x - \frac{1}{2}x + \frac{1}{4} = \frac{1}{2}x - \frac{1}{2}x + \frac{1}{8} \Rightarrow \frac{7}{8}x + \frac{1}{4} = \frac{1}{8} \Rightarrow$
$-\frac{7}{8}x + \frac{1}{4} - \frac{1}{4} = \frac{1}{8} - \frac{1}{4} \Rightarrow -\frac{7}{8}x = -\frac{1}{8} \Rightarrow \left(-\frac{8}{7}\right)\left(-\frac{7}{8}\right)x = \left(-\frac{1}{8}\right)\left(-\frac{8}{7}\right) \Rightarrow x = \frac{8}{56} \Rightarrow x = \frac{1}{7}$

52. $\frac{1}{3}x + \frac{1}{4} = \frac{1}{6} - x \Rightarrow \frac{1}{3}x + x + \frac{1}{4} = \frac{1}{6} - x + x \Rightarrow \frac{4}{3}x + \frac{1}{4} = \frac{1}{6} \Rightarrow \frac{4}{3}x + \frac{1}{4} - \frac{1}{4} = \frac{1}{6} - \frac{1}{4} \Rightarrow$
$\frac{4}{3}x = -\frac{1}{12} \Rightarrow \frac{3}{4} \cdot \frac{4}{3}x = -\frac{1}{12} \cdot \frac{3}{4} \Rightarrow x = -\frac{3}{48} \Rightarrow x = -\frac{1}{16}$

53. $4y - 2(y + 1) = 0 \Rightarrow 47 - 2y - 2 = 0 \Rightarrow 2y - 2 = 0 \Rightarrow 2y - 2 + 2 = 0 + 2 \Rightarrow 2y = 2 \Rightarrow$
$\frac{2y}{2} = \frac{2}{2} \Rightarrow y = 1$

54. $(15y + 20) - 5y = 5 - 10y \Rightarrow 10y + 20 = 5 - 10y \Rightarrow 10y + 10y + 20 = 5 - 10y + 10y \Rightarrow$
$20y + 20 = 5 \Rightarrow 20y + 20 - 20 = 5 - 20 \Rightarrow 20y = -15 \Rightarrow \frac{20y}{20} = \frac{-15}{20} \Rightarrow y = -\frac{3}{4}$

55. $5x = 5x + 1 \Rightarrow 5x - 5x = 5x - 5x + 1 \Rightarrow 0 = 1$
Because the equation $0 = 1$ is always false, there are zero solutions.

56. $2(x - 3) = 2x - 6 \Rightarrow 2x - 6 = 2x - 6 \Rightarrow 2x - 2x - 6 = 2x - 2x - 6 \Rightarrow -6 = -6$
Since the equation $-6 = -6$ is always true, there are infinitely many solutions.

57. $8x = 0 \Rightarrow \frac{8x}{8} = \frac{0}{8} \Rightarrow x = 0$ Thus, there is only one solution.

58. $9x = x + 1 \Rightarrow 9x - x = x - x + 1 \Rightarrow 8x = 1 \Rightarrow \frac{8x}{8} = \frac{1}{8} \Rightarrow x = \frac{1}{8}$ Thus, there is only one solution.

59. $4(x + 2) - 2(2x + 3) = 10 \Rightarrow 4x + 8 - 4x - 6 = 10 \Rightarrow 8 - 6 = 10 \Rightarrow 2 = 10$
Because the equation $2 = 10$ is always false, there are zero solutions.

60. $5(2x + 7) - (10x + 5) \Rightarrow 10x + 35 - 10x - 5 = 30 \Rightarrow 30 = 30$

Since the equation $30 = 30$ is always true, there are infinitely many solutions.

61. $4x = 5(x + 3) - x \Rightarrow 4x = 5x + 15 - x \Rightarrow 4x = 4x + 15 \Rightarrow 43x - 4x = 4x - 4x + 15 \Rightarrow 0 = 15$

Because the equation $0 = 15$ is always false, there are zero solutions.

62. $x - (3x + 2) = 15 - 2x \Rightarrow x - 3x - 2 = 15 - 2x \Rightarrow -2x - 2 = 15 - 2x \Rightarrow$

$-2x + 2x - 2 = 15 - 2x + 2x \Rightarrow -2 = 15$

Because the equation $-2 = 15$ is always false, there are zero solutions.

63. $2x - (x + 5) = x - 5 \Rightarrow 2x - x - 5 = x - 5 \Rightarrow x - 5 = x - 5 \Rightarrow x - x - 5 = x - x - 5 \Rightarrow$

$-5 = -5$ Since the equation $-5 = -5$ is always true, there are infinitely many solutions.

64. $5x = 15 - 2(x + 7) \Rightarrow 5x = 15 - 2x - 14 \Rightarrow 5x - 2x = 15 - 2x + 2x - 14 \Rightarrow 7x = 1 \Rightarrow$

$\frac{7x}{7} = \frac{1}{7} \Rightarrow x = \frac{1}{7}$ Thus, there is only one solution.

Applications

65. (a) See Figure 65.

(b) Let D represent the distance from home and x represent the number of hours. Then $D = 4 + 8x$.

(c) Substitute 3 for x. Then, $D = 4 + 8(3) = 28$ miles. This agrees with the value found in the table.

(d) Using the formula $D = 4 + 8x$, substitute 22 for D. Then, $22 = 4 + 8x$. Then, solving for x:

$22 - 4 = 4 - 4 + 8x \Rightarrow 18 = 8x \Rightarrow \frac{18}{8} = \frac{8x}{8} \Rightarrow \frac{9}{4} = x \Rightarrow x = 2.25$ miles. Thus, the bicyclist is 22 miles from home after 2 hours and 15 minutes.

Hours (x)	0	1	2	3	4
Distance (D)	4	12	20	28	36

Figure 65

66. (a) Let D represent distance from home and x represent hours. Note that each hour spent running decreases the distance from home by 6 miles. Thus, set the formula to $D = 16 - 6x$.

(b) Using the formula, substitute 1.5 for x and solve for D. $D = 16 - 6(1.5) \Rightarrow D = 16 - 9 \Rightarrow D = 7$ miles.

(c) Substitute 5.5 for D and solve for x. $5.5 = 16 - 6x \Rightarrow 5.5 - 16 = 16 - 16 - 6x \Rightarrow -10.5 = -6x \Rightarrow$

$\frac{-10.5}{-6} = \frac{-6x}{-6} \Rightarrow 1.75 = x \Rightarrow x = 1.75$ Thus, the athlete is 5.5 miles from home after 1 hour and 45 minutes of running.

67. Using the formula, substitute 815 for N and solve for x. $815 = 40x - 79{,}065 \Rightarrow$

$815 + 79{,}065 = 40x - 79{,}065 + 79{,}065 \Rightarrow 79{,}880 = 40x \Rightarrow \frac{79{,}880}{40} = \frac{40x}{40} \Rightarrow 1997 = x \Rightarrow x = 1997$

68. Using the formula, substitute 85 for I and solve for x. $85 = \frac{44}{3}x - 29{,}219 \Rightarrow$

$85 + 29{,}219 = \frac{44}{3}x - 29{,}219 + 29{,}219 \Rightarrow 29{,}304 = \frac{44}{3}x \Rightarrow 29{,}304 \cdot \frac{3}{44} = \frac{44}{3}x \Rightarrow \frac{87{,}912}{44} = x \Rightarrow$

$1998 = x \Rightarrow x = 1998$

69. Using the formula, substitute 908 for N and solve for x. $908 = 70x - 138{,}532 \Rightarrow$

$908 + 138{,}532 = 70x - 138{,}532 + 138{,}532 \Rightarrow 139{,}440 = 70x \Rightarrow \frac{139{,}440}{70} = \frac{70x}{70} \Rightarrow 1992 = x \Rightarrow$

$x = 1992$

70. Using the formula, substitute 6.6 for C and solve for x. $6.6 = .035x + 684 \Rightarrow$

$6.6 + 684 = 0.35x - 684 + 684 \Rightarrow 690.6 = 0.35x \Rightarrow \frac{690.6}{0.35} = \frac{0.35x}{0.35} \Rightarrow 1973.143 \approx x \Rightarrow$

$x \approx 1973.143$ Thus, the cost reached \$6.6 billion sometime in the year 1973.

Checking Basic Concepts for Sections 2.1 & 2.2

1. For $x = 3$, substitute 3 for x and solve: $4(3) - 3 = 12 - 3 = 9$

For $x = 3.5$, substitute 3.5 for x and solve: $4(3.5) - 3 = 14 - 3 = 11$

For $x = 4$, substitute 4 for x and solve: $4(4) - 3 = 16 - 3 = 13$

For $x = 4.5$, substitute 4.5 for x and solve: $4(4.5) - 3 = 18 - 3 = 15$

For $x = 5$, substitute 5 for x and solve: $4(5) - 3 = 20 - 3 = 17$

See Figure 1. To solve $4x - 3 = 13$, the table tells us that when $x = 4$, $4x - 3 = 13$.

x	3	3.5	4	4.5	5
$4x - 3$	9	11	13	15	17

Figure 1

2. (a) $x - 12 = 6 \Rightarrow x - 12 + 12 = 6 + 12 \Rightarrow x = 18$ To check the answer, substitute 18 for x in the original equation $x - 12 = 6$. $18 - 12 = 6 \Rightarrow 6 = 6$. Since this is true $x = 18$ is correct.

(b) $\frac{3}{4}z = \frac{1}{8} \Rightarrow \frac{4}{3} \cdot \frac{3}{4}z = \frac{1}{8} \cdot \frac{4}{3} \Rightarrow z = \frac{4}{24} \Rightarrow z = \frac{1}{6}$

(c) $0.6t + 0.4 = 2 \Rightarrow 0.6t + 0.4 - 0.4 = 2 - 0.4 \Rightarrow 0.6t = 1.6 \Rightarrow \frac{0.6t}{0.6} = \frac{1.6}{0.6} \Rightarrow t = 2.\overline{6}$

(d) $5 - 2(x - 2) = 3(4 - x) \Rightarrow 5 - 2x + 4 = 12 - 3x \Rightarrow 9 - 2x = 12 - 3x \Rightarrow$

$9 - 2x + 3x = 12 - 3x + 3x \Rightarrow 9 + x = 12 \Rightarrow 9 - 9 + x = 12 - 9 \Rightarrow x = 3$

3. (a) $x - 5 = 6x \Rightarrow x - x - 5 = 6x - x \Rightarrow -5 = 5x \Rightarrow \frac{-5}{5} = \frac{5x}{5} \Rightarrow -1 = x$. Thus, the equation has 1 solution.

(b) $-2(x - 5) = 10 - 2x \Rightarrow -2x + 10 = 10 - 2x \Rightarrow -2x + 2x + 10 = 10 - 2x + 2x \Rightarrow 10 = 10$

Since $10 = 10$ is always true, the equation has infinitely many solutions.

(c) $-(x - 1) = -x - 1 \Rightarrow -x + 1 = -x - 1 \Rightarrow -x + x + 1 = -x + x - 1 \Rightarrow 1 = -1$

Since this is never true, the equation has zero solutions.

4. (a) Let D represent distance from home and x represent hours driven. Note that the driver is initially 300 miles from home and that each hour driven the driver gets closer to home by 75 miles. Thus, the formula is $D = 300 - 75x$.

(b) Since the distance from home, when the drive is home, is 0, use the formula and set D equal to 0. Thus, $0 = 300 - 75x$.

(c) $0 = 300 - 75x \Rightarrow 0 + 75x = 300 - 75x + 75x \Rightarrow 75x = 300 \Rightarrow \frac{75x}{75} = \frac{300}{75} \Rightarrow x = 4$ hours.

2.3: Introduction to Problem Solving

Concepts

1. Check your solution
2. $n + 1$ and $n + 2$
3. $\frac{x}{100}$
4. 0.01
5. 50
6. 75
7. $\frac{P_2 - P_1}{P_1} \times 100$
8. rt

Number Problems

9. Let x represent the number. $2 + x = 12 \Rightarrow 2 - 2 + x = 12 - 2 \Rightarrow x = 10$
10. $2x + 7 = 9 \Rightarrow 2x + 7 - 7 = 9 - 7 \Rightarrow 2x = 2 \Rightarrow \frac{2x}{2} = \frac{2}{2} \Rightarrow x = 1$
11. $\frac{x}{5} = x - 24 \Rightarrow \frac{x}{5} \cdot 5 = 5(x - 24) \Rightarrow x = 5x - 120 \Rightarrow x - 5x = 5x - 5x - 120 \Rightarrow -4x = -120 \Rightarrow$
$\frac{-4x}{-4} = \frac{-120}{-4} \Rightarrow x = 30$
12. $25x = 125 \Rightarrow \frac{25x}{25} = \frac{125}{25} \Rightarrow x = 5$
13. $\frac{x + 5}{2} = 7 \Rightarrow \frac{x + 2}{2} \cdot 2 = 7 \cdot 2 \Rightarrow x + 5 = 14 \Rightarrow x + 5 - 5 = 14 - 5 \Rightarrow x = 9$
14. $8 - x = 5 \Rightarrow 8 - 8 - x = 5 - 8 \Rightarrow -x = -3 \Rightarrow -1(-x) = -1(-3) \Rightarrow x = 3$
15. $\frac{x}{2} = 17 \Rightarrow \frac{x}{2} \cdot 2 = 17 \cdot 2 \Rightarrow x = 34$
16. $5x = 95 \Rightarrow \frac{5x}{5} = \frac{95}{5} \Rightarrow x = 19$
17. Let the smallest natural number be represented by x. $x + (x + 1) + (x + 2) = 96 \Rightarrow 3x + 3 = 96 \Rightarrow$
$3x + 3 - 3 = 96 - 3 \Rightarrow 3x = 93 \Rightarrow \frac{3x}{3} = \frac{93}{3} \Rightarrow x = 31$ Thus, the numbers are 31, 32 and 33.
18. Let x represent the smallest integer, $x + (x + 1) + (x + 2) = -123 \Rightarrow 3x + 3 = -123 \Rightarrow$
$3x + 3 - 3 = -123 - 3 \Rightarrow 3x = -126 \Rightarrow \frac{3x}{3} = \frac{-126}{3} \Rightarrow x = -42$
Thus, the numbers are $-42, -41$ and -40.
19. $3x = 102 \Rightarrow \frac{3x}{3} = \frac{102}{3} \Rightarrow x = 34$
20. $x + 18 = 2x \Rightarrow x - x + 18 = 2x - x \Rightarrow 18 = x \Rightarrow x = 18$
21. $5x = 2x + 24 \Rightarrow 5x - 2x = 2x - 2x + 24 \Rightarrow 3x = 24 \Rightarrow \frac{3x}{3} = \frac{24}{3} \Rightarrow x = 8$

22. $3x = x - 18 \Rightarrow 3x - x = x - x - 18 \Rightarrow 2x = -18 \Rightarrow \frac{2x}{2} = \frac{-18}{2} \Rightarrow x = -9$

23. $\frac{6x}{7} = 18 \Rightarrow \frac{6x}{7} \cdot 7 = 18 \cdot 7 \Rightarrow 6x = 126 \Rightarrow \frac{6x}{6} = \frac{126}{6} \Rightarrow x = 21$

24. $\frac{2x - 2}{5} = 4 \Rightarrow \frac{2x - 2}{5} \cdot 5 = 4 \cdot 5 \Rightarrow 2x - 2 = 20 \Rightarrow 2x - 2 + 2 = 20 + 2 \Rightarrow 2x = 22 \Rightarrow$
$\frac{2x}{2} = \frac{22}{2} \Rightarrow x = 11$

25. $4(x + 5) = 64 \Rightarrow 4x + 20 = 64 \Rightarrow 4x + 20 - 20 = 64 - 20 \Rightarrow 4x = 44 \Rightarrow \frac{4x}{4} = \frac{44}{4} \Rightarrow x = 11$

26. $-(x + (-5)) = 24 \Rightarrow -(x - 5) = 24 \Rightarrow -x + 5 = 24 \Rightarrow -x + 5 - 5 = 24 - 5 \Rightarrow -x = 19 \Rightarrow$
$-1(-x) = -1(19) \Rightarrow x = -19$

Percent Problems

27. $37\% = \frac{37}{100}$
$37\% = 37 \times 0.01 = 0.37$

28. $52\% = \frac{52}{100} = \frac{13 \cdot 4}{25 \cdot 4} = \frac{13}{25}$
$52\% = 52 \times 0.01 = 0.52$

29. $148\% = \frac{148}{100} = \frac{37 \cdot 4}{25 \cdot 4} = \frac{37}{25}$
$148\% = 148 \times 0.01 = 1.48$

30. $252\% = \frac{252}{100} = \frac{63 \cdot 4}{25 \cdot 4} = \frac{63}{25}$
$252\% = 252 \times 0.01 = 2.52$

31. $6.9\% = \frac{6.9}{100} = \frac{6.9}{100} \cdot \frac{10}{10} = \frac{69}{1000}$
$6.9\% = 6.9 \times 0.01 = 0.069$

32. $8.1\% = \frac{8.1}{100} = \frac{8.1}{100} \cdot \frac{10}{10} = \frac{81}{1000}$
$8.1\% = 8.1 \times 0.01 = 0.081$

33. $0.05\% = \frac{0.05}{100} = \frac{0.05}{100} \cdot \frac{100}{100} = \frac{5}{10{,}000} = \frac{1 \cdot 5}{2000 \cdot 5} = \frac{1}{2000}$
$0.05\% = 0.05 \times 0.01 = 0.0005$

34. $0.12\% = \frac{0.12}{100} = \frac{0.12}{100} \cdot \frac{100}{100} = \frac{12}{10{,}000} = \frac{3 \cdot 4}{2500 \cdot 4} = \frac{3}{2500}$
$0.12\% = 0.12 \times 0.01 = 0.0012$

35. $0.45 = 0.45 \times 100 = 45\%$

36. $0.08 = 0.08 \times 100 = 8\%$

37. $1.8 = 1.8 \times 100 = 180\%$

38. $2.97 = 2.97 \times 100 = 297\%$

39. $\frac{2}{5} = 0.4 = 0.4 \times 100 = 40\%$

40. $\frac{1}{3} = 0.33\overline{3} = 0.33\overline{3} \times 100 = 33.\overline{3}\%$

41. $\frac{3}{4} = 0.75 = 0.75 \times 100 = 75\%$

42. $\frac{7}{20} = 0.35 = 0.35 \times 100 = 35\%$

43. $\frac{5}{6} = 0.83\overline{3} = 0.83\overline{3} \times 100 = 83.\overline{3}\%$

44. $\frac{53}{50} = 1.06 = 1.06 \times 100 = 106\%$

45. Let P_2 represent voters in 2000 and let P_1 represent voters in 1980. Then, the percent change in the number of voters is $\frac{P_2 - P_1}{P_1} = \frac{105.4 - 86.5}{86.5} = \frac{18.9}{86.5} \approx .218$ or about 21.8%.

46. Let P_2 represent the number of bachelor's degrees received in 1998 and let P_1 represent the number of bachelor's degrees received in 1960. Then, the percent change in the number of bachelor's degrees received is $\frac{P_2 - P_1}{P_1} = \frac{1{,}184{,}000 - 392{,}000}{392{,}000} = \frac{792{,}000}{392{,}000} \approx 2.02$ or about 202%.

47. Calculate the value of 4% of 950, then add the value to 950. Thus, 4% of 950 = .04(950) = 38. Then, 950 + 38 = $988 per credit.

48. Calculate the value of 8% of 125, then add the value to 125. Thus, 8% of 125 = .08(125) = 10. Then, 125 + 10 = $135 per credit.

49. Let x represent the number of returns in 1998. Then, $\frac{125 - x}{x} = 0.0162 \Rightarrow \frac{125 - x}{x} \cdot x = 0.0162 \cdot x \Rightarrow$ $125 - x = 0.0162x \Rightarrow 125 - x + x = 0.0162x + x \Rightarrow 125 = 1.0162x \Rightarrow \frac{125}{1.0162} = \frac{1.0162x}{1.0162} \Rightarrow$ $123 \approx x \Rightarrow x \approx 123$ Thus, about 123 million returns were processed in 1998.

50. Let x represent the number of people enrolled in Medicare in 1995. Then, $\frac{39.3 - x}{x} = 0.062 \Rightarrow$ $\frac{39.3 - x}{x} \cdot x = 0.062 \cdot x \Rightarrow 39.3 - x = 0.062x \Rightarrow 39.3 - x + x = 0.062x + x \Rightarrow 39.3 = 1.062x \Rightarrow$ $\frac{39.3}{1.062} = \frac{1.062x}{1.062} \Rightarrow 37 \approx x \Rightarrow x \approx 37$. Thus, about 37 million people were enrolled in Medicare in 1995.

51. Let x represent the number of AIDS deaths in 1995. Then, $0.345x = 17{,}047 \Rightarrow \frac{0.345x}{0.345} = \frac{17{,}047}{0.345} \Rightarrow$ $x \approx 49{,}412$ Thus, there were about 49,412 AIDS deaths in 1995.

52. Let x represent the percentage change. Then, $\frac{1.50 - 1.20}{1.20} = x \Rightarrow \frac{1.50 - 1.20}{1.20} \cdot 1.20 = x \cdot 1.20 \Rightarrow$ $1.50 - 1.20 = 1.20x \Rightarrow 0.3 = 1.20x \Rightarrow \frac{0.3}{1.20} = \frac{1.20x}{1.20} \Rightarrow 0.25 = x \Rightarrow x = 0.25$ Thus, the percentage change from \$1.20 to \$1.50 is 25%.

To calculate the percentage change from \$1.50 to \$1.20, let P_1 represent \$1.50 and let P_2 represent \$1.20 and let x represent percentage change. Then, $\frac{P_2 - P_1}{P_1} = x \Rightarrow \frac{1.20 - 1.50}{1.50} = x \Rightarrow$ $\frac{1.20 - 1.50}{1.50} \cdot 1.50 = x \cdot 1.50 \Rightarrow 1.20 - 1.50 = 1.5x \Rightarrow -0.30 = 1.5x \Rightarrow \frac{-0.30}{1.5} = \frac{1.5x}{1.5} \Rightarrow$ $-0.2 = x \Rightarrow x = -0.2$. Thus, the percentage change from \$1.50 to \$1.20 is −20%.

53. To calculate the total area of Wisconsin, let x represent the unknown number and note that 13.6 is 38% of the unknown number. Then, $0.38x = 13.6 \Rightarrow \frac{0.38x}{0.38} = \frac{13.6}{0.38} \Rightarrow x \approx 35.8$. Thus, the total area of Wisconsin is about 35.8 million acres.

54. Let x represent the minimum wage in 1980. Note that 60.2% of the known minimum wage of \$5.15 in 2003 is the minimum wage for 1980. Then, $x = 0.602(5.15) \approx 3.10$. Thus, the minimum wage in 1980 was \$3.10.

Distance Problems

55. $d = rt \Rightarrow d = 4 \cdot 2 \Rightarrow d = 8$ miles

56. $d = rt \Rightarrow d = 70 \cdot 2.5 \Rightarrow d = 175$ miles

57. $d = rt \Rightarrow 1000 = r \cdot 50 \Rightarrow \frac{1000}{50} = \frac{r \cdot 50}{50} \Rightarrow 20 = r \Rightarrow r = 20$ feet/second

58. $d = rt \Rightarrow 1250 = r \cdot 5 \Rightarrow \frac{1250}{5} = \frac{r \cdot 5}{5} \Rightarrow 250 = r \Rightarrow r = 250$ miles/day

59. $d = rt \Rightarrow 200 = 40t \Rightarrow \frac{200}{40} = \frac{40t}{40} \Rightarrow 5 = t \Rightarrow t = 5$ hours

60. $d = rt \Rightarrow 1700 = 10t \Rightarrow \frac{1700}{10} = \frac{10t}{10} \Rightarrow 170 = t \Rightarrow t = 170$ seconds

61. Given that the distance traveled (d) is 255 and that the time spent traveling (t) is 4.25 hours, calculate the speed of the car (r). Then, $d = rt \Rightarrow 255 = r \cdot 4.25 \Rightarrow \frac{255}{4.25} = \frac{r \cdot 4.25}{4.25} \Rightarrow 60 = r \Rightarrow r = 60$ miles/hour.

62. Given the distance and the time, calculate the rate of speed. Then, $715 = r \cdot 5.5 \Rightarrow \frac{715}{5.5} = \frac{r \cdot 5.5}{5.5} \Rightarrow$ $130 = r \Rightarrow r = 130$ miles/hour.

63. Let the slower runner be standing still. Then, the faster runner will be traveling at 0 + 2 mph. Then, this problem is equivalent to solving how long it takes the faster runner to travel $\frac{3}{4}$ of a mile. Using the $d = rt$ formula: $\frac{3}{4} = 2t \Rightarrow \frac{3}{4} \cdot \frac{1}{2} = \frac{2}{1} \cdot \frac{1}{2}t \Rightarrow \frac{3}{8} = t \Rightarrow t = \frac{3}{8}$. So, in $\frac{3}{8}$ hour the faster runner will be $\frac{3}{4}$ mile ahead of the slower runner.

64. Since the athlete runs $\frac{1}{3}$ of an hour at 6 mph, $\frac{1}{3} \cdot 6 = 2$ equals the miles the athlete runs in the first $\frac{1}{3}$ of an hour. Since the athlete runs a total of 8 miles, $8 - 2 = 6$ is the number of miles the athlete runs in the remaining $1 - \frac{1}{3} = \frac{2}{3}$ of an hour. Using the $d = rt$ formula: $6 = r \cdot \frac{2}{3} \Rightarrow 6 \cdot \frac{3}{2} = r \cdot \frac{2}{3} \cdot \frac{3}{2} \Rightarrow 9 = r \Rightarrow r = 9.$ Therefore, the second speed is 9 miles/hour.

65. Let x represent the amount of time spent running 5 mph. Since the total time spent running was 1.1 hours, let $1.1 - x$ represent the amount of time running at 8 mph. Using the $d = rt$ formula, the distance run will equal the sum of $5x$ and $8(1.1 - x)$. Thus, $7 = 5x + 8(1.1 - x) \Rightarrow 7 = 5x + 8.8 - 8x \Rightarrow 7 = 7.8 - 3x \Rightarrow$ $3x + 7 = 8.8 - 3x + 3x \Rightarrow 3x + 7 = 8.8 \Rightarrow 3x + 7 - 7 = 8.8 - 7 \Rightarrow 3x = 1.8 \Rightarrow x = \frac{1.8}{3} = 0.6.$ Therefore, the athlete ran at 5 mph for 0.6 hour and ran at 8 mph for $(1.1 - 0.6) = 0.5$ hour.

66. The distance the bus travels is $160 + 295 = 455$ miles. The bus travels at 70 mph. Then, $455 = 70t \Rightarrow \frac{455}{70} = \frac{70t}{70} \Rightarrow 6.5 = t \Rightarrow t = 6.5.$ Therefore, it will take the bus 6.5 hours to be 295 miles west of the border.

67. Since the plane is already 300 miles west of Chicago, it will have to fly $2175 - 300 = 1875$ miles to be 2175 miles west of Chicago. The plane is traveling at 500 mph. Then, $1875 = 500t \Rightarrow \frac{1875}{500} = \frac{500t}{500} \Rightarrow 3.75 = t \Rightarrow t = 3.75.$ Therefore, it will take the plane 3.75 hours to be 2175 miles west of Chicago.

68. Let x represent the speed of the slower car and $x + 6$ represent the speed of the faster car. Then, the distance of 171 miles will equal the speed of the slower car multiplied by the time and then added to the speed of the faster car multiplied by the time. Therefore, $171 = x(1.5) + (x + 6)(1.5) \Rightarrow 171 = 1.5x + 1.5x + 9 \Rightarrow$ $171 = 3x + 9 \Rightarrow 171 - 9 = 3x + 9 - 9 \Rightarrow 162 = 3x \Rightarrow \frac{162}{3} = \frac{3x}{3} \Rightarrow 54 = x \Rightarrow x = 54.$ Thus, 54 mph is the speed of the slower car and $54 + 6 = 60$ mph is the speed of the faster car.

Other Types of Problems

69. Let x represent the amount of water that should be added. Note that there is no salt in pure water and that we will add the amount of pure water to the 3% salt solution to obtain a 1.2% solution. Therefore, set up the equation so that the amount of salt on both sides of the equation is equal. Thus, $x(0.00) + 20(0.03) = (x + 20)(0.012) \Rightarrow 0.00x + 0.6 = 0.012x + 0.24 \Rightarrow$ $0.6 - 0.24 = 0.012x + 0.24 - 0.24 \Rightarrow 0.36 = 0.012x \Rightarrow \frac{0.36}{0.012} = \frac{0.012x}{0.012} \Rightarrow 30 = x \Rightarrow x = 30.$ Therefore, 30 ounces of water should be added.

70. Let x represent the amount of water that should be added. Note that there is no hydrochloric acid in pure water and that we will add the amount of pure water to the 15% solution to obtain a 2% solution. Therefore, set up the equation so that the amount of hydrochloric acid on both sides of the equation is equal. Thus, $x(0.00) + 50(0.15) = (x + 50)(0.02) \Rightarrow 0.00x + 7.5 = 0.02x + 1 \Rightarrow 7.5 - 1 = 0.02x + 1 - 1 \Rightarrow$ $6.5 = 0.02x \Rightarrow \frac{6.5}{0.02} = \frac{0.02x}{0.02} \Rightarrow 325 = x \Rightarrow x = 325.$ Therefore, 325 ml of water should be added.

71. Let x represent the amount of the loan at 6% and let $x + 1000$ represent the amount of the loan at 5%. The total interest for one year is \$215 and this is the sum of the interest paid on the two loans. Therefore: $0.06x + 0.05(x + 1000) = 215 \Rightarrow 0.06x + 0.05x + 50 = 215 \Rightarrow 0.11x + 50 - 50 = 215 - 50 \Rightarrow$ $0.11x = 165 \Rightarrow \frac{0.11x}{0.11} = \frac{165}{0.11} \Rightarrow x = 1500$. Therefore, the amount of the loan at 6% interest is \$1500 and the amount of the loan at 5% interest is $1500 + 1000 = \$2500$.

72. Let x represent the interest rate for the \$3000 loan and let $x - 0.03$ represent the interest rate for the \$5000 loan. The total interest cost for the year will equal the sum of the interest for each loan. Therefore: $3000x + 5000(x - 0.03) = 550 \Rightarrow 3000x + 5000x - 150 = 550 \Rightarrow 8000x - 150 + 150 = 550 + 150 \Rightarrow$ $8000x = 700 \Rightarrow \frac{8000x}{8000} = \frac{700}{8000} \Rightarrow x = 0.0875$. Therefore, the interest rate for the \$3000 loan is 8.75% and the interest rate for the for the \$5000 loan is $0.0875 - 0.03 = 0.0575$ or 5.75%.

73. Let x represent the amount of 70% antifreeze. Then, the 45% antifreeze mixture is the sum of the 70% mixture and the 30% mixture. Therefore: $0.7x + 10(0.3) = (x + 10)(0.45) \Rightarrow 0.7x + 3 = 0.45x + 4.5 \Rightarrow$ $0.7x + 3 - 3 = 0.45x + 4.5 - 3 \Rightarrow 0.7x = 0.45x + 1.5 \Rightarrow 0.7x - 0.45x = 0.45x - 0.45x + 1.5 \Rightarrow$ $0.25x = 1.5 \Rightarrow \frac{0.25x}{0.25} = \frac{1.5}{0.25} \Rightarrow x = 6$. Therefore, 6 gallons of 70% antifreeze should be mixed with 10 gallons of 30% antifreeze to obtain the 45% mixture.

74. Since there is a total of 50 gallons of mixture, let x represent the gallons of 65% antifreeze and let $50 - x$ represent the 20% antifreeze. Then: $0.65x + 0.20(50 - x) = 50(0.56) \Rightarrow 0.65x + 10 - 0.20x = 28 \Rightarrow$ $0.45x + 10 - 10 = 28 - 10 \Rightarrow 0.45x = 18 \Rightarrow \frac{0.45x}{0.45} = \frac{18}{0.45} \Rightarrow x = 40$. Therefore, 40 gallons of 65% antifreeze should be mixed with 10 gallons of 20% antifreeze.

Group Activity Solutions

1. Area $(A) =$ length $(l) \times$ width (w). Then, $A = lw = (12)(11) = 132\text{ cm}^2$.
2. Volume $(V) =$ length $(l) \times$ width $(w) \times$ height (h). Then, $V = lwh = (12)(11)(5) = 660\text{ cm}^3$.
3. Start by solving for width (w) in terms of length (l). Then, insert is value for width into the formula for volume. The area of the bottom of the box is 50 square centimeters and the formula for area is $A = lw$. Thus, $50 = lw \Rightarrow \frac{50}{l} = \frac{lw}{l} \Rightarrow \frac{50}{l} = w \Rightarrow w = \frac{50}{l}$. Then, use the formula for volume $V = lwh$ to solve for h. $V = lwh \Rightarrow 100 = l\left(\frac{50}{l}\right)h \Rightarrow 100 = 50h \Rightarrow \frac{100}{50} = \frac{50h}{50} \Rightarrow 2 = h \Rightarrow h = 2$ cm.
4. Because one cubic centimeter weighs 2.7 grams, $\frac{5.4}{2.7} = 2\text{ cm}^3$. Thus, the volume of the aluminum foil is 2 cm^3.
5. Because the sheet of foil weighs 5.4 grams, from the previous problem the volume of the foil is 2 cm^3. Then, insert this information into the volume formula $V = lwh$. Thus, $V = lwh \Rightarrow 2 = (50)(20)h \Rightarrow 2 = 1000h \Rightarrow$ $\frac{2}{1000} = \frac{1000h}{1000} \Rightarrow \frac{2}{1000} = h \Rightarrow h = \frac{2}{1000} = 0.002$ cm.

2.4: Formulas

Concepts

1. formula
2. lw
3. $\frac{1}{2}bh$
4. $\frac{1}{360}$
5. 360
6. 180
7. $2\pi r$
8. πr^2
9. lwh
10. $\frac{1}{2}(a + b)h$

Formulas from Geometry

11. $A = lw$. Thus, $A = 6 \cdot 3 = 18 \text{ ft}^2$.
12. $A = lw$. Thus, $A = 4 \cdot 2.5 = 10 \text{ yd}^2$.
13. $A = \frac{1}{2}bh$. Thus, $A = \frac{1}{2} \cdot 6 \cdot 3 = 9 \text{ in}^2$.
14. $A = \frac{1}{2}bh$. Thus, $A = \frac{1}{2} \cdot 3 \cdot 1 = \frac{3}{2} \text{ mi}^2$.
15. $A = \pi r^2$. Thus, $A = \pi 4^2 \approx 50.3 \text{ ft}^2$.
16. $A = \pi r^2$. Thus, $A = \pi 3^2 \approx 28.3 \text{ in}^2$.
17. $A = \frac{1}{2}(a + b)h$. Thus, $A = \frac{1}{2}(5 + 6)2 = \frac{1}{2}(11)2 = 11 \text{ ft}^2$.
18. $A = \frac{1}{2}(a + b)h$. Thus, $A = \frac{1}{2}(4 + 3)2 = \frac{1}{2}(7)2 = 7 \text{ mi}^2$.
19. $A = lw$. Thus, $A = 13 \cdot 7 = 91 \text{ in}^2$.
20. Convert yards to feet in order to keep the units consistent. Therefore, 7 yards $= 7 \cdot 3 = 21$ feet. $A = lw$. Thus, $A = 21 \cdot 5 = 105 \text{ ft}^2$. Or, convert feet to yards to obtain $\frac{5}{3}$ yards. Then $A = lw = 7 \cdot \frac{5}{3} = \frac{35}{3} \text{ yd}^2$.
21. $A = \frac{1}{2}bh$. Thus, $A = \frac{1}{2} \cdot 12 \cdot 6 = 36 \text{ in}^2$.
22. Convert feet to inches in order to keep the units consistent. Thus, 9 feet $= 9 \cdot 12 = 108$ inches. Then, $A = \frac{1}{2}bh = \frac{1}{2} \cdot 108 \cdot 72 = 3888 \text{ in}^2$. Or convert inches to feet to obtain 72 inches $= \frac{72}{12} = 6$ feet. Then, $A = \frac{1}{2}bh = \frac{1}{2} \cdot 9 \cdot 6 = 27 \text{ ft}^2$.

23. $C = 2\pi r$. Because the circle has a diameter of 8 inches, the radius is $= \frac{8}{2} = 4$ inches. Thus, $C = 2\pi r = 2\pi 4 = 8\pi \approx 25.1$ inches.

24. $A = \pi r^2$. Thus, $A = \pi 9^2 = 81\pi \approx 254.5\ \text{ft}^2$.

25. The total area of the lot is the sum of the area of the square and the area of the triangle. The area of the square is $lw = 52 \cdot 52 = 2704\ \text{ft}^2$. The area of the triangle is $\frac{1}{2}bh = \frac{1}{2} \cdot 73 \cdot 52 = 1898\ \text{ft}^2$. Thus, the area of the lot is $1898 + 2704 = 4602\ \text{ft}^2$.

26. The total area of the lot is the sum of the area of the rectangle and the areas of the two triangles. The area of the rectangle is $lw = 60 \cdot 20 = 1200\ \text{yd}^2$. The area of the left triangle is $\frac{1}{2}bh = \frac{1}{2} \cdot 10 \cdot 20 = 100\ \text{yd}^2$. The area of the right triangle is $\frac{1}{2}bh = \frac{1}{2} \cdot 30 \cdot 20 = 300\ \text{yd}^2$. Thus, the total area is $1200 + 100 + 300 = 1600\ \text{yd}^2$.

27. The sum of the angles of a triangle is 180º. Let the unknown angle be represented by x. Then, $x + 75 + 40 = 180 \Rightarrow x + 75 + 40 - 75 - 40 = 180 - 75 - 40 \Rightarrow x = 65$. Thus, the third angle is 65º.

28. The sum of the angles of a triangle is 180º. Let the unknown angle be represented by x. Then, $x + 30 + 25 = 180 \Rightarrow x + 30 + 25 - 30 - 25 = 180 - 30 - 25 \Rightarrow x = 125$. Thus, the third angle is 125º.

29. The sum of the angles of a triangle is 180º. Let the unknown angle be represented by x. Then, $x + 23 + 76 = 180 \Rightarrow x + 23 + 76 - 23 - 76 = 180 - 23 - 76 \Rightarrow x = 81$. Thus, the third angle is 81º.

30. Because there are three angles in any triangle and because the measures of the angles in a equilateral triangle are equal, $x + x + x = 180 \Rightarrow 3x = 180 \Rightarrow \frac{3x}{3} = \frac{180}{3} \Rightarrow x = 60$. Thus, the measure of each angle is 60º.

31. Because the sum of the angles of a triangle is 180, $x + 2x + 3x = 180 \Rightarrow 6x = 180 \Rightarrow \frac{6x}{6} = \frac{180}{6} \Rightarrow$ $x = 30$. Thus, the value of x is 30º.

32. $3x + 4x + 11x = 180 \Rightarrow 18x = 180 \Rightarrow \frac{18x}{18} = \frac{180}{18} \Rightarrow x = 10$. Thus, the value of x is 10º.

33. Let x represent the largest angle. Then, $x + \frac{1}{3}x + \frac{1}{3}x = 180 \Rightarrow \frac{5}{3}x = 180 \Rightarrow \frac{3}{5} \cdot \frac{5}{3}x = \frac{180}{1} \cdot \frac{3}{5} \Rightarrow$ $x = \frac{540}{5} \Rightarrow x = 108$. Thus, the largest angle has a measure of 108º and the two small angles each have measure $\frac{1}{3} \cdot \frac{108}{1} = \frac{108}{3} = 36$º.

34. Let x represent the largest angle. Then, $x - 10$ represents the second largest angle and $x - 50$ represents the smallest angle. Then, $x + (x - 10) + (x - 50) = 180 \Rightarrow x + x - 10 + x - 50 = 180 \Rightarrow$ $3x - 60 = 180 \Rightarrow 3x - 60 + 60 = 180 + 60 \Rightarrow 3x = 240 \Rightarrow \frac{3x}{3} = \frac{240}{3} \Rightarrow x = 80$. Thus, the largest angle has measure 80º, the second largest angle has measure $80 - 10 = 70$º and the smallest angle has measure $80 - 50 = 30$º.

35. $C = 2\pi r$. Since the diameter of the circle is 12 inches, the radius is $\frac{12}{2} = 6$ inches. Then, $C = 2\pi 6 = 12\pi \approx 37.7$ inches. Then, $A = \pi r^2 = \pi 6^2 = 36\pi \approx 113.1\ \text{in}^2$.

36. $C = 2\pi r$. Then, $C = 2\pi\left(\frac{5}{4}\right) = \frac{5}{2}\pi \approx 7.9$ feet. Then, $A = \pi r^2 = \pi\left(\frac{5}{4}\right)^2 = \frac{25}{16}\pi \approx 4.9 \text{ ft}^2$.

37. $C = 2\pi r$. Then, set C equal to 2π and solve for r. $2\pi = 2\pi r \Rightarrow \frac{2\pi}{2\pi} = \frac{2\pi r}{2\pi} \Rightarrow 1 = r \Rightarrow r = 1$ inch. Then, $A = \pi r^2$. Because $r = 1$, substitute 1 for r and solve for A. $A = \pi(1)^2 = \pi$. Thus, the area is equal to π, which is approximately equal to 3.1 in^2.

38. $C = 2\pi r$. Then, set C equal to 13π and solve for r. $13\pi = 2\pi r \Rightarrow \frac{13\pi}{2\pi} = \frac{2\pi r}{2\pi} \Rightarrow \frac{13}{2} \cdot \frac{\pi}{\pi} = r \Rightarrow \frac{13}{2} = r \Rightarrow$ $r = \frac{13}{2}$. Thus, the radius is $\frac{13}{2} = 6.5$ feet. Then, $A = \pi r^2$. Thus, $A = \pi(6.5)^2 = 42.25\pi \approx 132.7 \text{ ft}^2$.

39. $V = lwh$. Thus, $V = 22 \cdot 12 \cdot 10 = 2640 \text{ in}^3$. Surface area equals $2lw + 2lh + 2wh$. Thus, $S = 2 \cdot 22 \cdot 12 + 2 \cdot 22 \cdot 10 + 2 \cdot 12 \cdot 10 = 528 + 440 + 240 = 1208 \text{ in}^2$.

40. $V = lwh$. Thus, $V = 5 \cdot 3 \cdot 6 = 90 \text{ ft}^3$. Surface area equals $2lw + 2lh + 2wh$. Thus, $S = 2 \cdot 5 \cdot 3 + 2 \cdot 5 \cdot 6 + 2 \cdot 3 \cdot 6 = 30 + 60 + 36 = 126 \text{ ft}^2$.

41. Convert yards to feet. Then, $\frac{2}{3}$ yard $= \frac{2}{3} \cdot 3 = 2$ feet. Then, $V = lwh = 2 \cdot \frac{2}{3} \cdot \frac{3}{2} = 2 \text{ ft}^3$. Surface area equals $2lw + 2lh + 2wh$. Thus, $S = 2 \cdot 2 \cdot \frac{2}{3} + 2 \cdot 2 \cdot \frac{3}{2} + 2 \cdot \frac{2}{3} \cdot \frac{3}{2} = \frac{8}{3} + 6 + 2 = 8\frac{8}{3} = 10\frac{2}{3} \text{ ft}^2$.

42. $V = lwh$. Thus, $V = 1.2 \cdot 0.8 \cdot 0.6 = 0.576 \text{ m}^3$. Surface area equals $2lw + 2lh + 2wh$. Thus, $S = 2 \cdot 1.2 \cdot 0.8 + 2 \cdot 1.2 \cdot 0.6 + 2 \cdot 0.8 \cdot 0.6 = 1.92 + 1.44 + 0.96 = 4.32 \text{ m}^2$.

43. $V = \pi r^2 h$. Thus, $V = \pi 2^2 \cdot 5 = \pi \cdot 4 \cdot 5 = 20\pi \text{ in}^3$.

44. $V = \pi r^2 h$. Thus, $V = \pi\left(\frac{1}{2}\right)^2\left(\frac{3}{2}\right) = \pi\left(\frac{1}{4}\right)\left(\frac{3}{2}\right) = \frac{3}{8}\pi \text{ in}^3$.

45. Convert feet to inches to obtain $h = 2$ feet $= 2 \cdot 12 = 24$ inches. Then, $V = \pi r^2 h$. Thus, $V = \pi 5^2 \cdot 24 =$ $\pi \cdot 25 \cdot 24 = 600\pi \text{ in}^3$.

46. Convert yards to feet to obtain $h = 1.5$ yards $= 1.5(3) = 4.5$ feet. Then, $V = \pi r^2 h$. Thus, $V = \pi(2.5)^2(4.5) =$ $\pi \cdot 6.25 \cdot 4.5 = 28.125\pi \text{ ft}^3$.

47. The volume formula for a cylindrical container is given by $V = \pi r^2 h$. Because the diameter of the barrel is $1\frac{3}{4} = \frac{7}{4}$ feet, the radius is $\left(\frac{1}{2}\right)\left(\frac{7}{4}\right) = \frac{7}{8}$ feet. Thus, $V = \pi r^2 h = \pi\left(\frac{7}{8}\right)^2(3) = \pi\left(\frac{49}{64}\right)(3) = \pi\left(\frac{147}{64}\right) =$ $\frac{147}{64}\pi \approx 7.2 \text{ ft}^3$.

48. (a) $V = \pi r^2 h$. Thus, $V = \pi\left(\frac{3}{4}\right)^2\left(\frac{5}{2}\right) = \pi\left(\frac{9}{16}\right)\left(\frac{5}{2}\right) = \pi\left(\frac{45}{32}\right) = \frac{45}{32}\pi \approx 4.4 \text{ in}^3$.

(b) Because there is about 4.4 cubic inches of volume in the can, the number of fluid ounces is $(4.4)(0.554) \approx 2.4$ oz.

Solving for a Variable

49. The formula is given as $A = lw$. To solve for W, proceed as follows: $A = lw \Rightarrow \frac{A}{l} = \frac{lw}{l} \Rightarrow w = \frac{A}{l}$.

50. The formula is given as $A = \frac{1}{2}bh$. To solve for b, proceed as follows: $A = \frac{1}{2}bh \Rightarrow 2A = \frac{1}{2}(2)bh \Rightarrow$

$2A = bh \Rightarrow \frac{2A}{h} = \frac{bh}{h} \Rightarrow b = \frac{2A}{h}$.

51. The formula is given as $V = \pi r^2 h$. To solve for h, proceed as follows: $V = \pi r^2 h \Rightarrow \frac{V}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2} \Rightarrow$

$h = \frac{V}{\pi r^2}$.

52. The formula is given as $V = \frac{1}{3}\pi r^2 h$. To solve for h, proceed as follows: $V = \frac{1}{3}\pi r^2 h \Rightarrow 3V = \frac{1}{3}(3)\pi r^2 h \Rightarrow$

$3V = \pi r^2 h \Rightarrow \frac{3V}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2} \Rightarrow h = \frac{3v}{\pi r^2}$.

53. The formula is given as $\frac{1}{2}(a + b)h = A$. To solve for a, proceed as follows: $A = \frac{1}{2}(a + b)h \Rightarrow$

$2A = \frac{1}{2}(2)(a + b)h \Rightarrow 2A = (a + b)h \Rightarrow \frac{2A}{h} = \frac{(a + b)h}{h} \Rightarrow \frac{2A}{h} = a + b \Rightarrow \frac{2A}{h} - b = a + b - b \Rightarrow$

$a = \frac{2A}{h} - b$.

54. The formula is given as $C = 2\pi r$. To solve for r, proceed as follows: $C = 2\pi r \Rightarrow \frac{C}{2\pi} = \frac{2\pi r}{2\pi} \Rightarrow r = \frac{C}{2\pi}$.

55. The formula is given as $V = lwh$. To solve for W, proceed as follows: $V = lwh \Rightarrow \frac{V}{lh} = \frac{lwh}{lh} \Rightarrow w = \frac{V}{lh}$.

56. $P = 2x + 2y \Rightarrow P - 2x = 2x - 2x + 2y \Rightarrow P - 2x = 2y \Rightarrow \frac{P - 2x}{2} = \frac{2y}{2} \Rightarrow y = \frac{P - 2x}{2}$

57. $s = \frac{a + b + c}{2} \Rightarrow 2s = \frac{a + b + c}{2} \cdot 2 \Rightarrow 2s = a + b + c \Rightarrow 2s - a - c = a + b + c - a - c \Rightarrow$

$2s - a - c = b \Rightarrow b = 2s - a - c$

58. $t = \frac{x - y}{3} \Rightarrow 3t = \frac{x - y}{3} \cdot 3 \Rightarrow 3t = x - y \Rightarrow 3t + y = x - y + y \Rightarrow 3t + y = x \Rightarrow x = 3t + y$

59. $\frac{a}{b} - \frac{c}{b} = 1 \Rightarrow b\left(\frac{a}{b} - \frac{c}{b}\right) = 1(b) \Rightarrow a - c = b \Rightarrow b = a - c$

60. $\frac{x}{y} + \frac{z}{y} = 5 \Rightarrow y\left(\frac{x}{y} + \frac{z}{y}\right) = 5y \Rightarrow x + z = 5y \Rightarrow x - x + z = 5y - x \Rightarrow z = 5y - x$

61. $ab = cd + ad \Rightarrow ab - ad = cd + ad - ad \Rightarrow ab - ad = cd \Rightarrow a(b - d) = cd \Rightarrow$

$\frac{a(b - d)}{(b - d)} = \frac{cd}{(b - d)} \Rightarrow a = \frac{cd}{b - d}$

62. $S = 2lw + 2lh + 2wh \Rightarrow S - 2lh = 2lw + 2lh + 2wh - 2lh \Rightarrow S - 2lh = 2lw + 2wh \Rightarrow$

$S - 2lh = w(2l + 2h) \Rightarrow \frac{S - 2lh}{(2l + 2h)} = \frac{w(2l + 2h)}{(2l + 2h)} \Rightarrow \frac{S - 2lh}{(2l + 2h)} = w \Rightarrow w = \frac{S - 2lh}{(2l + 2h)}$

63. Because the perimeter equals the lengths of the four sides, $P = 2w + 2l$. Thus, $P = 2w + 2l \Rightarrow$

$40 = 2(5) + 2l \Rightarrow 40 = 10 + 2l \Rightarrow 40 - 10 = 10 + 2l - 10 \Rightarrow 2l = 30 \Rightarrow \frac{2l}{2} = \frac{30}{2} \Rightarrow l = 15$

Thus, the length of the rectangle is 15 inches.

64. Let x represent the length of the third side. Thus, $5 + 7 + x = 21 \Rightarrow 5 + 7 + x - 5 - 7 = 21 - 5 - 7 \Rightarrow$

$x = 9$. Thus, the length of the third side is 9 feet.

Other Formulas and Applications

65. The formula for GPA is given by $\frac{4a + 3b + 2c + d}{a + b + c + d + f}$.

$\frac{4(30) + 3(45) + 2(12) + 1(4)}{30 + 45 + 12 + 4 + 4} = \frac{120 + 135 + 24 + 4}{95} = \frac{283}{95} \approx 2.98$. Thus, the GPA is 2.98.

66. The formula for GPA is given by $\frac{4a + 3b + 2c + d}{a + b + c + d + f}$.

$\frac{4(70) + 3(35) + 2(5) + 1(0)}{70 + 35 + 5 + 0 + 0} = \frac{280 + 105 + 10 + 0}{110} = \frac{395}{110} \approx 3.59$. Thus, the GPA is 3.59.

67. The formula for GPA is given by $\frac{4a + 3b + 2c + d}{a + b + c + d + f}$.

$\frac{4(0) + 3(60) + 2(80) + 1(10)}{0 + 60 + 80 + 10 + 6} = \frac{0 + 180 + 160 + 10}{156} = \frac{350}{156} \approx 2.24$. Thus, the GPA is 2.24.

68. The formula for GPA is given by $\frac{4a + 3b + 2c + d}{a + b + c + d + f}$.

$\frac{4(3) + 3(5) + 2(8) + 1(0)}{3 + 5 + 8 + 0 + 22} = \frac{12 + 15 + 16 + 0}{38} = \frac{43}{38} \approx 1.13$. Thus, the GPA is 1.13.

69. To convert Celsius to Fahrenheit temperature, the formula given is $\frac{9}{5}C + 32 = F$.

$\frac{9}{5}(25) + 32 = F \Rightarrow \frac{225}{5} + \frac{160}{5} = F \Rightarrow \frac{385}{5} = F \Rightarrow F = 77°\text{F}$.

70. To convert Celsius to Fahrenheit temperature, the formula given is $\frac{9}{5}C + 32 = F$.

$\frac{9}{5}(100) + 32 = F \Rightarrow \frac{900}{5} + \frac{160}{5} = F \Rightarrow \frac{1060}{5} = F \Rightarrow F = 212°\text{F}$.

71. To convert Celsius to Fahrenheit temperature, the formula given is $\frac{9}{5}C + 32 = F$.

$\frac{9}{5}(-40) + 32 = F \Rightarrow \frac{-360}{5} + \frac{160}{5} = F \Rightarrow \frac{-200}{5} = F \Rightarrow F = -40°\text{F}$.

72. To convert Celsius to Fahrenheit temperature, the formula given is $\frac{9}{5}C + 32 = F$.

$\frac{9}{5}(0) + 32 = F \Rightarrow \frac{0}{5} + \frac{160}{5} = F \Rightarrow \frac{160}{5} = F \Rightarrow F = 32°\text{F}$.

73. To convert Fahrenheit to Celsius temperature, the formula given is $C = \frac{5}{9}(F - 32)$.

$C = \frac{5}{9}(23 - 32) \Rightarrow C = \frac{5}{9}(-9) \Rightarrow C = \frac{-45}{9} \Rightarrow C = -5°\text{C}$.

74. To convert Fahrenheit to Celsius temperature, the formula given is $C = \frac{5}{9}(F - 32)$.

$C = \frac{5}{9}(98.6 - 32) \Rightarrow C = \frac{5}{9}(66.6) \Rightarrow C = \frac{333}{9} \Rightarrow C = 37°\text{C}$.

75. To convert Fahrenheit to Celsius temperature, the formula given is $C = \frac{5}{9}(F - 32)$.

$C = \frac{5}{9}(-4 - 32) \Rightarrow C = \frac{5}{9}(-36) \Rightarrow C = \frac{-180}{9} \Rightarrow C = -20°\text{C}$.

76. To convert Fahrenheit to Celsius temperature, the formula given is $C = \frac{5}{9}(F - 32)$.

$C = \frac{5}{9}(-31 - 32) \Rightarrow C = \frac{5}{9}(-63) \Rightarrow C = \frac{-315}{9} \Rightarrow C = -35°\text{C}$.

77. The formula given for calculating the delay between seeing lightning and hearing the thunder is $D = \frac{x}{5}$ where D represents the distance form the lightning and x represents the delay. Therefore, $D = \frac{x}{5} \Rightarrow D = \frac{12}{5} \Rightarrow$ $D = 2\frac{2}{5} = 2.4$ miles.

78. The formula given for calculating the delay between seeing lightning and hearing the thunder is $D = \frac{x}{5}$ where D represents the distance form the lightning and x represents the delay. Therefore, $D = \frac{x}{5} \Rightarrow 2.5 = \frac{x}{5} \Rightarrow$ $5(2.5) = \frac{x}{5} \cdot 5 \Rightarrow 12.5 = x \Rightarrow x = 12.5$ seconds.

Checking Basic Concepts for Sections 2.3 & 2.4

1. (a) $3x = 36 \Rightarrow \frac{3x}{3} = \frac{36}{3} \Rightarrow x = 12$

   (b) $35 - x = 43 \Rightarrow 35 - 35 - x = 43 - 35 \Rightarrow -x = 8 \Rightarrow -1(x) = 8(-1) \Rightarrow x = -8$

2. $x + (x + 1) + (x + 2) = -93 \Rightarrow 3x + 3 = -93 \Rightarrow 3x + 3 - 3 = -93 - 3 \Rightarrow 3x = -96 \Rightarrow$ $\frac{3x}{3} = \frac{-96}{3} \Rightarrow x = -32$. The three consecutive integers are $-32, -31, -30$.

3. $9.5\% = 0.095$

4. $\frac{5}{4} = 1\frac{1}{4} = 1.25 = 125\%$

5. Convert 8% to the decimal 0.08 and let x represent the unknown rate. Therefore, $x - 0.08x = 3850$. Thus, $x - 0.08x = 3850 \Rightarrow 0.92x = 3850 \Rightarrow \frac{0.92x}{0.92} = \frac{3850}{0.92} \Rightarrow x \approx 4285$. Thus, the rate in 1998 was about 4185.

6. Use the formula $D = rt$, where D is distance, r is the speed and t is the time. Thus, $390 = 60t \Rightarrow$ $\frac{390}{60} = \frac{60t}{60} \Rightarrow 6.5 = t$. Thus, the travel time is 6.5 hours.

7. Let x represent the amount of the loan at 7% and $x + 2000$ represent the amount of the loan at 6%. Thus, $0.07x + 0.06(x + 2000) = 510 \Rightarrow 0.07x + 0.06x + 120 = 510 \Rightarrow 0.13x + 120 - 120 = 510 - 120 \Rightarrow$ $0.13x = 390 \Rightarrow \frac{0.13x}{0.13} = \frac{390}{0.13} \Rightarrow x = 3000$. Thus, the loan at 7% was \$3000 and the loan at 6% was \$5000.

8. The area of a triangle is given by $A = \frac{1}{2}bh$. Thus, $A = \frac{1}{2}bh \Rightarrow 36 = \frac{1}{2}(6)h \Rightarrow 36 = 3h \Rightarrow \frac{36}{3} = \frac{3h}{3} \Rightarrow$ $12 = h$. Thus, the height of the triangle is 12 inches.

9. The area of a circle is given by $A = \pi r^2$. Thus, $A = \pi r^2 \Rightarrow A = \pi(3)^2 \Rightarrow A = 9\pi \approx 28.3 \text{ ft}^2$.

   The circumference of a circle is given by $C = 2\pi r$. Thus, $C = 2\pi r \Rightarrow C = 2\pi 3 \Rightarrow C = 6\pi \approx 18.8$ feet.

10. Notice that the angle denoted by $3x$ is a right angle, that is, it is an angle measuring 90º. Thus,

$3x = 90 \Rightarrow \frac{3x}{3} = \frac{90}{3} \Rightarrow x = 30$. Thus, the value of x is 30º.

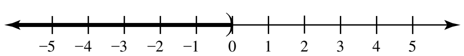
11. $A = \pi r^2 + \pi rl \Rightarrow A - \pi r^2 = \pi r^2 - \pi r^2 + \pi rl \Rightarrow A - \pi r^2 = \pi rl \Rightarrow \frac{A - \pi r^2}{\pi r} = \frac{\pi rl}{\pi r} \Rightarrow l = \frac{A - \pi r^2}{\pi r}$

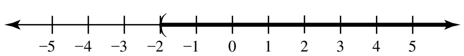
2.5: Linear Inequalities

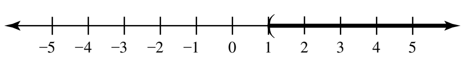
Concepts

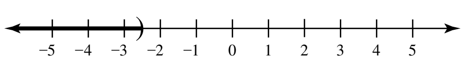
1. equals sign; $<$; $\leq$; $>$; $\geq$
2. equivalent
3. one
4. infinitely many
5. number line
6. is not; $3(5) < 10$ is not true
7. $>$
8. $<$
9. $>$
10. They are not equivalent inequalities because, $-4x < 8 \Rightarrow \frac{-4x}{-4} > \frac{8}{-4} \Rightarrow x > -2$, which is not equivalent to $x < -2$.

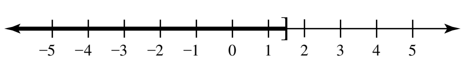
Solutions and Number Line Graphs

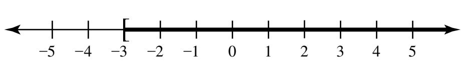
11. 

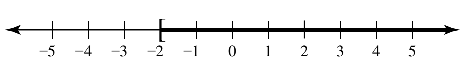
12. 

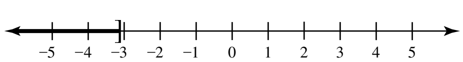
13. 

14. 

15. 

16. 

17. 

18. 

19. $x < 0$
20. $x \geq -2$

21. $x \le 3$

22. $x > -5$

23. $x \ge 10$

24. $x < -20$

25. Substitute x with 4 in order to test the inequality. Thus, $x + 5 > 5 \Rightarrow 4 + 5 > 5 \Rightarrow 9 > 5$. Because this inequality statement is true, $x = 4$ is a solution.

26. Substitute x with 6 in order to test the inequality. Thus, $x - 7 < 0 \Rightarrow 6 - 7 < 0 \Rightarrow -1 < 0$. Because this inequality statement is true, $x = 6$ is a solution.

27. First, isolate x on one side of the statement. Thus, $5x \ge 25 \Rightarrow \frac{5x}{5} \ge \frac{25}{5} \Rightarrow x \ge 5$. Then substitute x with 5 in order to test the inequality. Thus, $x \ge 5 \Rightarrow 5 \ge 5$. Because this inequality statement is true, $x = 5$ is a solution.

28. First, isolate x on one side of the statement. Thus, $-3x \le -8 \Rightarrow \frac{-3x}{-3} \ge \frac{-8}{-3} \Rightarrow x \ge \frac{8}{3}$. Then substitute x with -2 in order to test the inequality. Thus, $x \ge \frac{8}{3} \Rightarrow -2 \ge \frac{8}{3}$. Because this inequality statement is not true, $x = -2$ is not a solution.

29. First, isolate y on one side of the statement. Thus, $4y - 3 \le 5 \Rightarrow 4y - 3 + 3 \le 5 + 3 \Rightarrow 4y \le 8 \Rightarrow \frac{4y}{4} \le \frac{8}{4} \Rightarrow y \le 2$. Then substitute y with -3 in order to test the inequality. Thus, $y \le 2 \Rightarrow -3 \le 2$. Because this inequality statement is true, $y = -3$ is a solution.

30. First, isolate y on one side of the statement. Thus, $3y + 5 \ge -8 \Rightarrow 3y + 5 - 5 \ge -8 - 5 \Rightarrow 3y \ge -13 \Rightarrow \frac{3y}{3} \ge \frac{-13}{3} \Rightarrow y \ge -4\frac{1}{3}$. Then substitute y with -3 in order to test the inequality. Thus, $y \ge -4\frac{1}{3} \Rightarrow -3 \ge -4\frac{1}{3}$. Because this inequality statement is true, $y = -3$ is a solution.

31. First, isolate z on one side of the statement. Thus, $5(z + 1) < 3z - 7 \Rightarrow 5z + 5 < 3z - 7 \Rightarrow 5z - 3z + 5 < 3z - 3z - 7 \Rightarrow 2z + 5 < -7 \Rightarrow 2z + 5 - 5 < -7 - 5 \Rightarrow 2z < -12 \Rightarrow \frac{2z}{2} < \frac{-12}{2} \Rightarrow z \le -6$. Then substitute z with -7 in order to test the inequality. Thus, $z < -6 \Rightarrow -7 < -6$. Because this inequality statement is true, $z = -7$ is a solution.

32. First, isolate z on one side of the statement. Thus, $-(z + 7) > 3(6 - z) \Rightarrow -z - 7 > 18 - 3z \Rightarrow -z + 3z - 7 > 18 - 3z + 3z \Rightarrow 2z - 7 > 18 \Rightarrow 2z - 7 + 7 > 18 + 7 \Rightarrow 2z > 25 \Rightarrow \frac{2z}{2} > \frac{25}{2} \Rightarrow z > 12\frac{1}{2}$. Then substitute z with 2 in order to test the inequality. Thus, $z > 12\frac{1}{2} \Rightarrow 2 > 12\frac{1}{2}$. Because this inequality statement is not true, $z = 2$ is not a solution.

33. First, isolate t on one side of the statement. Thus, $\frac{3}{2}t - \frac{1}{2} \geq 1 - t \Rightarrow \frac{3}{2}t + t - \frac{1}{2} \geq 1 - t + t \Rightarrow$ $\frac{5}{2}t - \frac{1}{2} \geq 1 \Rightarrow \frac{5}{2}t - \frac{1}{2} + \frac{1}{2} \geq 1 + \frac{1}{2} \Rightarrow \frac{5}{2}t \geq \frac{3}{2} \Rightarrow \frac{2}{5} \cdot \frac{5}{2}t \geq \frac{3}{2} \cdot \frac{2}{5} \Rightarrow t \geq \frac{3}{5}$. Then substitute t with -2 in order to test the inequality. Thus, $t \geq \frac{3}{5} \Rightarrow -2 \geq \frac{3}{5}$. Because this inequality statement is not true, $t = -2$ is not a solution.

34. First, isolate t on one side of the statement. Thus, $2t - 3 > 5t - (2t + 1) \Rightarrow 2t - 3 > 3t - 1 \Rightarrow$ $2t - 3t - 3 > 3t - 3t - 1 \Rightarrow -t - 3 > -1 \Rightarrow -t - 3 + 3 > -1 + 3 \Rightarrow -t > 2 \Rightarrow$ $-1(-t) < (-2)(-1) \Rightarrow t < 2$. Then substitute t with 5 in order to test the inequality. Thus, $t < 2 \Rightarrow 5 < 2$. Because this inequality statement is not true, $t = 5$ is not a solution.

Tables and Linear Inequalities

35. $x > -2$

36. $x \geq 2$

37. $x < 1$

38. $x \leq 3.8$

39. To complete the table, insert the x value into $-2x + 6$ whenever there is a missing value in the table. Thus,
$-2x + 6 \Rightarrow -2(2) + 6 \Rightarrow -4 + 6 = 2; -2x + 6 \Rightarrow -2(3) + 6 \Rightarrow -6 + 6 = 0;$
$-2x + 6 \Rightarrow -2(4) + 6 \Rightarrow -8 + 6 = -2$. Thus, the missing values in the table are 2, 0 and -2. See Figure 39.
From the table, we see that $-2x + 6 \leq 0$ whenever $x \geq 3$. Thus, the solution to the inequality is $x \geq 3$.

x	1	2	3	4	5
$-2x + 6$	4	2	0	-2	-4

Figure 39

x	0	1	2	3	4
$3x - 1$	-1	2	5	8	11

Figure 40

40. To complete the table, insert the x value into $3x - 1$ whenever there is a missing value in the table. Thus,
$3x - 1 \Rightarrow 3(1) - 1 \Rightarrow 3 - 1 = 2; 3x - 1 \Rightarrow 3(2) - 1 \Rightarrow 6 - 1 = 5;$
$3x - 1 \Rightarrow 3(3) - 1 \Rightarrow 9 - 1 = 8; 3x - 1 \Rightarrow 3(4) - 1 \Rightarrow 12 - 1 = 11.$
Thus, the missing values in the table are 2, 5, 8 and 11. See Figure 40.
From the table, we see that $3x - 1 < 8$ whenever $x < 3$. Thus, the solution to the inequality is $x < 3$.

41. To complete the table, insert the x value into $5 - x$ and $x + 7$ whenever there is a missing value in the table.
$5 - x \Rightarrow 5 - (-2) = 7; 5 - x \Rightarrow 5 - (-1) = 6; 5 - x \Rightarrow 5 - (0) = 5.$
Thus, the missing values in the table that correspond to $5 - x$ are 7, 6 and 5.
$x + 7 \Rightarrow (-2) + 7 = 5; x + 7 \Rightarrow (-1) + 7 = 6; x + 7 \Rightarrow (0) + 7 = 7.$
Thus, the missing values in the table that correspond to $x + 7$ are 5, 6 and 7. See Figure 41.
From the table, we see that $5 - x > x + 7$ whenever $x < -1$. Thus, the solution to the inequality is $x < -1$.

x	-3	-2	-1	0	1
$5 - x$	8	7	6	5	4
$x + 7$	4	5	6	7	8

Figure 41

x	-2	-1	0	1	2
$2(3 - x)$	10	8	6	4	2
$-3(x - 2)$	12	9	6	3	0

Figure 42

42. To complete the table, insert the x value into $2(3 - x)$ and $-3(x - 2)$ whenever there is a missing value in the table.

$2(3 - x) \Rightarrow 2(3 - (-2)) \Rightarrow 2(5) = 10; 2(3 - x) \Rightarrow 2(3 - (-1)) \Rightarrow 2(4) = 8;$

$2(3 - x) \Rightarrow 2(3 - (0)) \Rightarrow 2(3) = 6; 2(3 - x) \Rightarrow 2(3 - 1) \Rightarrow 2(2) = 4;$

$2(3 - x) \Rightarrow 2(3 - 2) \Rightarrow 2(1) = 2$. Thus, the missing values that correspond to $2(3 - x)$ are 10, 8, 6, 4 and 2.

$-3(x - 2) \Rightarrow -3(-2 - 2) \Rightarrow -3(-4) = 12; -3(x - 2) \Rightarrow -3(-1 - 2) \Rightarrow -3(-3) = 9;$

$-3(x - 2) \Rightarrow -3(0 - 2) \Rightarrow -3(-2) = 6; -3(x - 2) \Rightarrow -3(1 - 2) \Rightarrow -3(-1) = 3;$

$-3(x - 2) \Rightarrow -3(2 - 2) \Rightarrow -3(0) = 0.$

Thus, the missing values that correspond to $-3(x - 2)$ are 12, 9, 6, 3 and 0. See Figure 42. From the table, we see that $2(3 - x) \geq -3(x - 2)$ whenever $x \geq 0$. Thus, the solution to the inequality is $x \geq 0$.

Solving Linear Inequalities

43. $x - 3 > 0 \Rightarrow x - 3 + 3 > 0 + 3 \Rightarrow x > 3$. See Figure 43.

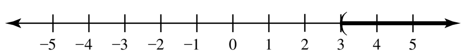


Figure 43

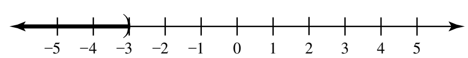


Figure 44

44. $x + 6 < 3 \Rightarrow x + 6 - 6 < 3 - 6 \Rightarrow x < -3$. See Figure 44.

45. $3 - y \leq 5 \Rightarrow 3 - 3 - y \leq 5 - 3 \Rightarrow -y \leq 2 \Rightarrow -1(-y) \geq 2(-1) \Rightarrow y \geq -2$. See Figure 45.

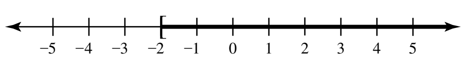


Figure 45

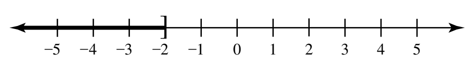


Figure 46

46. $8 - y \geq 10 \Rightarrow 8 - 8 - y \geq 10 - 8 \Rightarrow -y \geq 2 \Rightarrow -1(-y) \leq 2(-1) \Rightarrow y \leq -2$. See Figure 46.

47. $12 < 4 + z \Rightarrow 12 - 4 < 4 - 4 + z \Rightarrow 8 < z \Rightarrow z > 8$. See Figure 47.

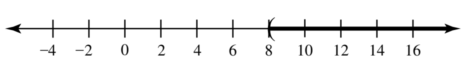


Figure 47

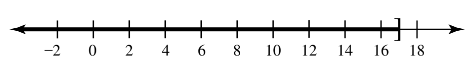


Figure 48

48. $2z \leq z + 17 \Rightarrow 2z - z \leq z - z + 17 \Rightarrow z \leq 17$. See Figure 48.

49. $5 - 2t \geq 10 - t \Rightarrow 5 - 2t + t \geq 10 - t + t \Rightarrow 5 - t \geq 10 \Rightarrow 5 - 5 - t \geq 10 - 5 \Rightarrow -t \geq 5 \Rightarrow$ $-1(-t) \leq 5(-1) \Rightarrow t \leq -5$. See Figure 49.

Figure 49

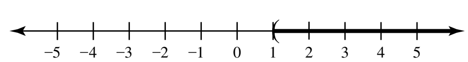


Figure 50

50. $-2t > -3t + 1 \Rightarrow -2t + 3t > -3t + 3t + 1 \Rightarrow t > 1$. See Figure 50.

51. $2x < 10 \Rightarrow \frac{2x}{2} < \frac{10}{2} \Rightarrow x < 5$. See Figure 51.

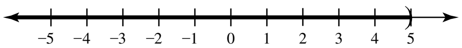


Figure 51

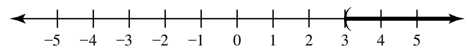


Figure 52

52. $3x > 9 \Rightarrow \frac{3x}{3} > \frac{9}{3} \Rightarrow x > 3$. See Figure 52.

53. $-\frac{1}{2}t \geq 1 \Rightarrow \frac{-\frac{1}{2}t}{-\frac{1}{2}} \leq \frac{1}{-\frac{1}{2}} \Rightarrow t \leq -2$. See Figure 53.

Figure 53

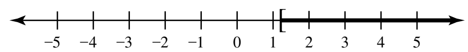


Figure 54

54. $-5t \leq -6 \Rightarrow \frac{-5t}{-5} \geq \frac{-6}{-5} \Rightarrow t \geq \frac{6}{5}$. See Figure 54.

55. $\frac{3}{4} > -5y \Rightarrow -5y < \frac{3}{4} \Rightarrow \frac{-5y}{-5} < \frac{\frac{3}{4}}{-5} \Rightarrow y < \frac{3}{4} \cdot \left(-\frac{1}{5}\right) \Rightarrow y > -\frac{3}{20}$. See Figure 55.

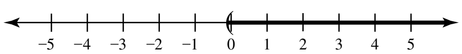


Figure 55

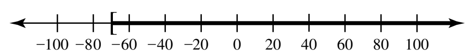


Figure 56

56. $10 \geq -\frac{1}{7}y \Rightarrow -\frac{1}{7}y \leq 10 \Rightarrow -\frac{7}{1}\left(-\frac{1}{7}y\right) \geq \frac{10}{1}\left(-\frac{7}{1}\right) \Rightarrow y \geq -70$. See Figure 56.

57. $-\frac{2}{3} \leq \frac{1}{7}z \Rightarrow \frac{1}{7}z \geq -\frac{2}{3} \Rightarrow \frac{7}{1}\left(\frac{1}{7}z\right) \geq -\frac{2}{3}\left(\frac{7}{1}\right) \Rightarrow z \geq -\frac{14}{3}$. See Figure 57.

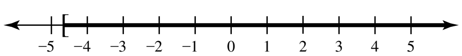


Figure 57

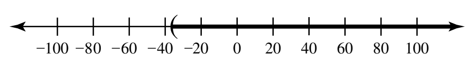


Figure 58

58. $-\frac{3}{10}z < 11 \Rightarrow -\frac{10}{3}\left(-\frac{3}{10}\right) > 11\left(-\frac{10}{3}\right) \Rightarrow z > -\frac{110}{3}$. See Figure 58.

59. $3x + 1 < 22 \Rightarrow 3x + 1 - 1 < 22 - 1 \Rightarrow 3x < 21 \Rightarrow \frac{3x}{3} < \frac{21}{3} \Rightarrow x < 7 \Rightarrow \{x \mid x < 7\}$

60. $4 + 5x \leq 9 \Rightarrow 4 - 4 + 5x \leq 9 - 4 \Rightarrow 5x \leq 5 \Rightarrow \frac{5x}{5} \leq \frac{5}{5} \Rightarrow x \leq 1 \Rightarrow \{x \mid x \leq 1\}$

61. $5 - \frac{3}{4}x \geq 6 \Rightarrow 5 - 5 - \frac{3}{4}x \geq 6 - 5 \Rightarrow -\frac{3}{4}x \geq 1 \Rightarrow -\frac{4}{3}\left(-\frac{3}{4}x\right) \leq 1\left(-\frac{4}{3}\right) \Rightarrow x \leq -\frac{4}{3} \Rightarrow \left\{x \,\middle|\, x \leq -\frac{4}{3}\right\}$

62. $10 - \frac{2}{5}x > 0 \Rightarrow 10 - 10 - \frac{2}{5}x > 0 - 10 \Rightarrow -\frac{2}{5}x > -10 \Rightarrow -\frac{5}{2}\left(-\frac{2}{5}x\right) < -10\left(-\frac{5}{2}\right) \Rightarrow x < \frac{50}{2} \Rightarrow$
$x < 25 \Rightarrow \{x \mid x < 25\}$

63. $45 > 6 - 2x \Rightarrow 6 - 2x < 45 \Rightarrow 6 - 6 - 2x < 45 - 6 \Rightarrow -2x < 39 \Rightarrow \frac{-2x}{-2} > \frac{39}{-2} \Rightarrow x > -\frac{39}{2} \Rightarrow$
$\left\{x \,\middle|\, x > -\frac{39}{2}\right\}$

64. $69 \geq 3 - 11x \Rightarrow 3 - 11x \leq 69 \Rightarrow 3 - 3 - 11x \leq 69 - 3 \Rightarrow -11x \leq 66 \Rightarrow \frac{-11x}{-11} \geq \frac{66}{-11} \Rightarrow x \geq -6 \Rightarrow$ $\{x \mid x \geq -6\}$

65. $5x - 2 \leq 3x + 1 \Rightarrow 5x - 3x - 2 \leq 3x - 3x + 1 \Rightarrow 2x - 2 \leq 1 \Rightarrow 2x - 2 + 2 \leq 1 + 2 \Rightarrow 2x \leq 3 \Rightarrow$ $\frac{2x}{2} \leq \frac{3}{2} \Rightarrow x \leq \frac{3}{2} \Rightarrow \left\{x \,\middle|\, x \leq \frac{3}{2}\right\}$

66. $12x + 1 < 25 - 3x \Rightarrow 12x + 3x + 1 < 25 - 3x + 3x \Rightarrow 15x + 1 < 25 \Rightarrow 15x + 1 - 1 < 25 - 1 \Rightarrow$ $15x < 24 \Rightarrow \frac{15x}{15} < \frac{24}{15} \Rightarrow x < \frac{8}{5} \Rightarrow \left\{x \,\middle|\, x < \frac{8}{5}\right\}$

67. $-x + 24 < x + 23 \Rightarrow -x - x + 24 < x - x + 23 \Rightarrow -2x + 24 < 23 \Rightarrow -2x + 24 - 24 < 23 - 24 \Rightarrow$ $-2x < -1 \Rightarrow \frac{-2x}{-2} > \frac{-1}{-2} \Rightarrow x > \frac{1}{2} \Rightarrow \left\{x \,\middle|\, x > \frac{1}{2}\right\}$

68. $6 - 4x \leq x + 1 \Rightarrow 6 - 4x - x \leq x - x + 1 \Rightarrow 6 - 5x \leq 1 \Rightarrow 6 - 6 - 5x \leq 1 - 6 \Rightarrow -5x \leq -5 \Rightarrow$ $\frac{-5x}{-5} \geq \frac{-5}{-5} \Rightarrow x \geq 1 \Rightarrow \{x \mid x \geq 1\}$

69. $-(x + 1) \geq 3(x - 2) \Rightarrow -x - 1 \geq 3x - 6 \Rightarrow -x - 3x - 1 \geq 3x - 3x - 6 \Rightarrow -4x - 1 \geq -6 \Rightarrow$ $-4x - 1 + 1 \geq -6 + 1 \Rightarrow -4x \geq -5 \Rightarrow \frac{-4x}{-4} \leq \frac{-5}{-4} \Rightarrow x \leq \frac{5}{4} \Rightarrow \left\{x \,\middle|\, x \leq \frac{5}{4}\right\}$

70. $5(x + 2) > -2(x - 3) \Rightarrow 5x + 10 > -2x + 6 \Rightarrow 5x + 2x + 10 > -2x + 2x + 6 \Rightarrow 7x + 10 > 6 \Rightarrow$ $7x + 10 - 10 > 6 - 10 \Rightarrow 7x > -4 \Rightarrow \frac{7x}{7} > \frac{-4}{7} \Rightarrow x > -\frac{4}{7} \Rightarrow \left\{x \,\middle|\, x > -\frac{4}{7}\right\}$

71. $3(2x - 1) > -(5 - 3x) \Rightarrow 6x + 3 > -5 + 3x \Rightarrow 6x - 3x + 3 > -5 + 3x - 3x \Rightarrow 3x + 3 > -5 \Rightarrow$ $3x + 3 - 3 > -5 - 3 \Rightarrow 3x > -8 \Rightarrow \frac{3x}{3} > \frac{-8}{3} \Rightarrow x > -\frac{8}{3} \Rightarrow \left\{x \,\middle|\, x > -\frac{8}{3}\right\}$

72. $4x \geq -3(7 - 2x) + 1 \Rightarrow 4x \geq -21 + 6x + 1 \Rightarrow 4x \geq 6x - 20 \Rightarrow 4x - 6x \geq 6x - 6x - 20 \Rightarrow$ $-2x \geq -20 \Rightarrow \frac{-2x}{-2} \leq \frac{-20}{-2} \Rightarrow x \leq 10 \Rightarrow \{x \mid x \leq 10\}$

73. $-(7x + 5) + 1 \geq 3x - 1 \Rightarrow -7x - 5 + 1 \geq 3x - 1 \Rightarrow -7x - 4 \geq 3x - 1 \Rightarrow$ $-7x - 3x - 4 \geq 3x - 3x - 1 \Rightarrow -10x - 4 \geq -1 \Rightarrow -10x - 4 + 4 \geq -1 + 4 \Rightarrow -10x \geq 3 \Rightarrow$ $\frac{-10x}{-10} \leq \frac{3}{-10} \Rightarrow x \leq -\frac{3}{10} \Rightarrow \left\{x \,\middle|\, x \leq -\frac{3}{10}\right\}$

74. $3(2 - x) - 5 > -4(5 - x) \Rightarrow 6 - 3x - 5 > -20 + 4x \Rightarrow 1 - 3x > 4x - 20 \Rightarrow$ $1 - 3x - 4x > 4x - 4x - 20 \Rightarrow 1 - 7x > -20 \Rightarrow 1 - 1 - 7x > -20 - 1 \Rightarrow -7x > -21 \Rightarrow$ $\frac{-7x}{-7} < \frac{-21}{-7} \Rightarrow x < 3 \Rightarrow \{x \mid x < 3\}$

75. $1.6x + 0.4 \leq 0.4x \Rightarrow 1.6x - 0.4x + 0.4 \leq 0.4x - 0.4x \Rightarrow 1.2x + 0.4 \leq 0 \Rightarrow$ $1.2x + 0.4 - 0.4 \leq 0 - 0.4 \Rightarrow 1.2x \leq -0.4 \Rightarrow \frac{1.2x}{1.2} \leq \frac{-0.4}{1.2} \Rightarrow x \leq -\frac{1}{3} \Rightarrow \left\{x \,\middle|\, x \leq -\frac{1}{3}\right\}$

76. $-5.1x + 1.1 < 0.1 - 0.1x \Rightarrow -5.1x + 0.1x + 1.1 < 0.1 - 0.1x + 0.1x \Rightarrow -5x + 1.1 < 0.1 \Rightarrow$
$-5x + 1.1 - 1.1 < 0.1 - 1.1 \Rightarrow -5x < -1 \Rightarrow \frac{-5x}{-5} > \frac{-1}{-5} \Rightarrow x > \frac{1}{5} \Rightarrow \left\{x \,\middle|\, x > \frac{1}{5}\right\}$

77. $0.8x - 0.5 < x + 1 - 0.5x \Rightarrow 0.8x - 0.5 < 0.5x + 1 \Rightarrow 0.8x - 0.5x - 0.5 < 0.5x - 0.5x + 1 \Rightarrow$
$0.3x - 0.5 < 1 \Rightarrow 0.3x - 0.5 + 0.5 < 1 + 0.5 \Rightarrow 0.3x < 1.5 \Rightarrow \frac{0.3x}{0.3} < \frac{1.5}{0.3} \Rightarrow x < 5 \Rightarrow \{x \mid x < 5\}$

78. $0.1(x + 1) - 0.1 \le 0.2x - 0.5 \Rightarrow 0.1x + 0.1 - 0.1 \le 0.2x - 0.5 \Rightarrow 0.1x \le 0.2x - 0.5 \Rightarrow$
$0.1x - 0.2x \le 0.2x - 0.2x - 0.5 \Rightarrow -0.1x \le -0.5 \Rightarrow \frac{-0.1x}{-0.1} \ge \frac{-0.5}{-0.1} \Rightarrow x \ge 5 \Rightarrow \{x \mid x \ge 5\}$

79. $-\frac{1}{2}\left(\frac{2}{3}x + 4\right) \ge x \Rightarrow -\frac{1}{3}x - 2 \ge x \Rightarrow -\frac{1}{3}x - x - 2 \ge x - x \Rightarrow -\frac{4}{3}x - 2 \ge 0 \Rightarrow$
$-\frac{4}{3}x - 2 + 2 \ge 0 + 2 \Rightarrow -\frac{4}{3}x \ge 2 \Rightarrow -\frac{3}{4}\left(-\frac{4}{3}x\right) \le 2\left(-\frac{3}{4}\right) \Rightarrow x \le -\frac{3}{2} \Rightarrow \left\{x \,\middle|\, x \le -\frac{3}{2}\right\}$

80. $-5x > \frac{4}{5}\left(\frac{10}{3}x + 10\right) \Rightarrow -5x > \frac{8}{3}x + 8 \Rightarrow -5x - \frac{8}{3}x > \frac{8}{3}x - \frac{8}{3}x + 8 \Rightarrow -\frac{15}{3}x - \frac{8}{3}x > 8 \Rightarrow$
$-\frac{23}{3}x > 8 \Rightarrow -\frac{3}{23}\left(-\frac{23}{3}x\right) < 8\left(-\frac{3}{23}\right) \Rightarrow x < -\frac{24}{23} \Rightarrow \left\{x \,\middle|\, x < -\frac{24}{23}\right\}$

81. $\frac{3}{7}x + \frac{2}{7} > -\frac{1}{7}x - \frac{5}{14} \Rightarrow \frac{3}{7}x - \frac{1}{7}x + \frac{2}{7} > -\frac{1}{7}x + \frac{1}{7}x - \frac{5}{14} \Rightarrow \frac{4}{7}x + \frac{2}{7} > -\frac{5}{14} \Rightarrow$
$\frac{4}{7}x + \frac{2}{7} - \frac{2}{7} > -\frac{5}{14} - \frac{2}{7} \Rightarrow \frac{4}{7}x > -\frac{5}{14} - \frac{4}{14} \Rightarrow \frac{4}{7}x > -\frac{9}{14} \Rightarrow \frac{7}{4}\left(\frac{4}{7}x\right) > -\frac{9}{14}\left(\frac{7}{4}\right) \Rightarrow x > -\frac{63}{56} \Rightarrow$
$x > -\frac{9}{8} \Rightarrow \left\{x \,\middle|\, x > -\frac{9}{8}\right\}$

82. $\frac{5}{6} - \frac{1}{3}x \ge -\frac{1}{3}\left(\frac{5}{6}x - 1\right) \Rightarrow \frac{5}{6} - \frac{1}{3}x \ge -\frac{5}{18}x + \frac{1}{3} \Rightarrow \frac{5}{6} - \frac{1}{3}x + \frac{5}{18}x \ge -\frac{5}{18}x + \frac{5}{18}x + \frac{1}{3} \Rightarrow$
$\frac{5}{6} - \frac{6}{18}x + \frac{5}{18}x \ge \frac{1}{3} \Rightarrow \frac{5}{6} - \frac{1}{18}x \ge \frac{1}{3} \Rightarrow \frac{5}{6} - \frac{5}{6} - \frac{1}{18}x \ge \frac{1}{3} - \frac{5}{6} \Rightarrow -\frac{1}{18}x \ge \frac{2}{6} - \frac{5}{6} \Rightarrow -\frac{1}{18}x \ge -\frac{1}{2} \Rightarrow$
$-\frac{18}{1}\left(-\frac{1}{18}x\right) \le -\frac{1}{2}\left(-\frac{18}{1}\right) \Rightarrow x \le 9 \Rightarrow \{x \mid x \le 9\}$

83. $\frac{x}{3} + \frac{5x}{6} \le \frac{2}{3} \Rightarrow \frac{2x}{6} + \frac{5x}{6} \le \frac{2}{3} \Rightarrow \frac{7x}{6} \le \frac{2}{3} \Rightarrow 6\left(\frac{7x}{6}\right) \le \frac{2}{3}(6) \Rightarrow 7x \le 4 \Rightarrow \frac{7x}{7} \le \frac{4}{7} \Rightarrow x \le \frac{4}{7} \Rightarrow$
$\left\{x \,\middle|\, x \le \frac{4}{7}\right\}$

84. $\frac{3x}{4} - \frac{x}{2} < 1 \Rightarrow \frac{3x}{4} - \frac{2x}{4} < 1 \Rightarrow \frac{x}{4} < 1 \Rightarrow 4\left(\frac{x}{4}\right) < 1(4) \Rightarrow x < 4 \Rightarrow \{x \mid x < 4\}$

85. $\frac{6x}{7} < \frac{1}{3}x + 1 \Rightarrow \frac{6x}{7} - \frac{1}{3}x < \frac{1}{3}x - \frac{1}{3}x + 1 \Rightarrow \frac{6x}{7} - \frac{x}{3} < 1 \Rightarrow \frac{18x}{21} - \frac{7x}{21} < 1 \Rightarrow \frac{11x}{21} < 1 \Rightarrow$
$21\left(\frac{11x}{21}\right) < 1(21) \Rightarrow 11x < 21 \Rightarrow \frac{11x}{11} < \frac{21}{11} \Rightarrow x < \frac{21}{11} \Rightarrow \left\{x \,\middle|\, x < \frac{21}{11}\right\}$

86. $\frac{5x}{8} - \frac{3x}{4} \le 8 \Rightarrow \frac{5x}{8} - \frac{6x}{8} \le 8 \Rightarrow \frac{-x}{8} \le 8 \Rightarrow 8\left(\frac{-x}{8}\right) \le 8(8) \Rightarrow -x \le 64 \Rightarrow -1(-x) \ge 64(-1) \Rightarrow$
$x \ge -64 \Rightarrow (x \mid x \ge -64\}$

Translating Inequalities

87. $x > 60$

88. $x \le 60$

89. $x \ge 21$

90. $x < 21$

91. $x > 40{,}000$

92. $x \le 40{,}000$

93. $x \le 70$

94. $x \ge 70$

Applications

95. $2(x + 5) + 2x < 50 \Rightarrow 2x + 10 + 2x < 50 \Rightarrow 4x + 10 < 50 \Rightarrow 4x + 10 - 10 < 50 - 10 \Rightarrow$ $4x < 40 \Rightarrow \frac{4x}{4} < \frac{40}{4} \Rightarrow x <$ 10 feet.

96. Let l represent length and w represent width. Then, $l = 2w$. Then, $l = 2w \Rightarrow 2(2w) + 2w \ge 36 \Rightarrow$ $4w + 2w \ge 36 \Rightarrow 6w \ge 36 \Rightarrow \frac{6w}{6} \ge \frac{36}{6} \Rightarrow w \ge 6$. Thus, the width must be 6 inches or more.

97. The area of a triangle is $\frac{1}{2}bh$. Substitute 12 for h and solve. $\frac{1}{2}bh < 120 \Rightarrow \frac{1}{2}b(12) < 120 \Rightarrow$ $6b < 120 \Rightarrow \frac{6b}{6} < \frac{120}{6} \Rightarrow b < 20$. Thus, the base of the triangle must be less than 20 inches.

98. Substitute h with 6 and solve. $\frac{1}{2}h(a + b) \le 120 \Rightarrow \frac{1}{2}(6)(a + b) \le 120 \Rightarrow 3(a + b) \le 120 \Rightarrow$ $\frac{3(a + b)}{3} \le \frac{120}{3} \Rightarrow a + b \le 40$. Thus, the sum of the bases must be 40 inches or less.

99. Let x represent the unknown test score. Then, $\frac{74 + x}{2} \ge 80 \Rightarrow 2\left(\frac{74 + x}{2}\right) \ge 80(2) \Rightarrow 74 + x \ge 160 \Rightarrow$ $74 - 74 + x \ge 160 - 74 \Rightarrow x \ge 86$. Thus, the student needs a score of 86 or more to maintain an average of at least 80.

100. Let x represent the unknown test score. Then, $\frac{65 + 82 + x}{3} \ge 70 \Rightarrow 3\left(\frac{65 + 82 + x}{3}\right) \ge 70(3) \Rightarrow$ $65 + 82 + x \ge 210 \Rightarrow 147 + x \ge 210 \Rightarrow 147 - 147 + x \ge 210 - 147 \Rightarrow x \ge 63$. Thus, the student needs a score of 63 or more to maintain an average of at least 70.

101. Let x represent the number of hours. We see that there is a \$2.00 cost for the first half hour and \$1.25 cost for each hour after that. Therefore, $2 + 1.25x \le 8 \Rightarrow 2 - 2 + 1.25x \le 8 - 2 \Rightarrow 1.25x \le 6 \Rightarrow$ $\frac{1.25x}{1.25} \le \frac{6}{1.25} \Rightarrow x \le 4.8$. This result would indicate that the student can park for as long as 4.8 hours for \$8.00. However, because a partial hour of parking is charged as a full hour, the longest amount of time that the student could park for \$8.00 is 4.5 hours.

102. Let x represent the number of parking hours in the student lot and y represent the number of parking hours in the nearby lot. Examine the first question and compare the results for x and y. Therefore, $2.50 + 1x \leq 5 \Rightarrow 2.50 - 2.50 + x \leq 5 - 2.50 \Rightarrow x \leq 2.50$. Because a partial hour is charged as a full hour, the student could park in the lot for 3 hours. Then, $1.25y \leq 5 \Rightarrow \frac{1.25y}{1.25} \leq \frac{5}{1.25} \Rightarrow y \leq 4$. Therefore, the student could park in this lot for 4 hours. Thus, for \$5.00, the student could park for a longer time in the nearby lot.
Now, examine the second question, $2.50 + 1x \leq 11 \Rightarrow 2.50 - 2.50 + x \leq 11 - 2.50 \Rightarrow x \leq 8.50$. Because a partial hour is charged as a full hour, the student could park in the lot for 9 hours. Then, $1.25y \leq 11 \Rightarrow \frac{1.25y}{1.25} \leq \frac{11}{1.25} \Rightarrow y \leq 8.8$. Because a partial hour is charged as a full hour, the student could park in this lot for 9 hours. Thus, for \$11.00, each lot would allow 9 hours of parking.

103. Let x represent the number of days. Then, $25x + 0.20(90)x \leq 200 \Rightarrow 25x + 18x \leq 200 \Rightarrow 43x \leq 200 \Rightarrow \frac{43x}{43} \leq \frac{200}{43} \Rightarrow x \leq 4.65$. Because the car can not be rented for a partial day, the person can rent the car for 4 days.

104. Let x represent the number of miles. Then, $37 < 20 + 0.25x \Rightarrow 37 - 20 < 20 - 20 + 0.25x \Rightarrow 17 < 0.25x \Rightarrow \frac{17}{0.25} < \frac{0.25x}{0.25} \Rightarrow 68 < x \Rightarrow x > 68$. Therefore, for more than 68 miles per day, the second rental agency is a better deal.

105. (a) $C = 1.5x + 2000$

(b) $R = 12x$

(c) $P = 12x - (1.5x + 2000) \Rightarrow P = 10.5x - 2000$

(d) To yield a positive profit, revenue must be greater than cost. Then, $12x > 1.5x + 2000 \Rightarrow 12x - 1.5x > 1.5x - 1.5x + 2000 \Rightarrow 10.5x > 2000 \Rightarrow \frac{10.5x}{10.5} > \frac{2000}{10.5} \Rightarrow x > 190.476$. Thus, 191 or more compact discs must be sold to yield a profit.

106. (a) $C = 100{,}000 + 890x$

(b) $R = 1520x$

(c) $P = 1520x - (100{,}000 + 890x) \Rightarrow P = 630x - 100{,}000$

(d) To yield a positive profit, revenue must exceed cost. Then, $1520x > 100{,}000 + 890x \Rightarrow 1520x - 890x > 100{,}000 + 890x - 890x \Rightarrow 630x > 100{,}000 \Rightarrow \frac{630x}{630} > \frac{100{,}000}{630} \Rightarrow x > 158.73$.
Thus, 159 or more laptop computers must be sold to yield a positive profit.

107. (a) Set the distances equal and then solve for x. Then, $\frac{1}{6}x = \frac{1}{8}x + 2 \Rightarrow 4x = 3x + 48 \Rightarrow x = 48$.
Thus, at 48 minutes the athletes are the same distance from the parking lot.

(b) $\frac{1}{6}x > \frac{1}{8}x + 2 \Rightarrow 4x > 3x + 48 \Rightarrow x > 48$. Thus, after more than 48 minutes, the first athlete is farther from the parking lot than the second athlete.

108. (a) Set the cassette tape equation to be greater than the CD equation and solve. Then,

$5.96(x - 1987) + 11.5 < -4.68(x - 1987) + 62.5 \Rightarrow$

$5.96x - 11{,}842.52 + 11.5 < -4.68x + 9299.16 + 62.5 \Rightarrow$

$5.96x - 11{,}831.02 < -4.68x + 9361.66 \Rightarrow$

$5.96x + 4.68x - 11{,}831.02 < -4.68x + 4.68x + 9361.66 \Rightarrow$

$10.64x - 11{,}831.02 < 9361.66 \Rightarrow 10.64x - 11{,}831.02 + 11{,}831.02 < 9361.66 + 11{,}831.02 \Rightarrow$

$10.64x < 21{,}192.68 \Rightarrow \frac{10.64x}{10.64} < \frac{21{,}192.68}{10.64} \Rightarrow x < 1991.79$. Thus, in the years before about 1992, sales of cassette tapes were greater than sales of CD's.

(b) By substituting various years after 1995 for x in each equation, we can see that CD sales increased and cassette tape sales declined.

109. Because $T = 80 - 29x$, set an inequality statement with T equal to 7.5 and solve. Then,

$80 - 29x < 7.5 \Rightarrow 80 - 80 - 29x < 7.5 - 80 \Rightarrow -29x < -72.5 \Rightarrow \frac{-29x}{-29} > \frac{-72.5}{-29} \Rightarrow x > 2.5.$

Thus, at altitudes more than 2.5 miles, the air temperature is less than 7.5ºF.

110. $65 - 5.8x < 36 \Rightarrow 65 - 65 - 5.8x < 36 - 65 \Rightarrow -5.8x < -29 \Rightarrow \frac{-5.8x}{-5.8} > \frac{-29}{-5.8} \Rightarrow x > 5$. Thus, at altitudes more than 5 miles, the dew point is less than 36ºF.

111. $0.1(x - 1970) + 13.1 \geq 15.5 \Rightarrow 0.1x - 197 + 13.1 \geq 15.5 \Rightarrow 0.1x - 183.9 \geq 15.5 \Rightarrow$

$0.1x - 183.9 + 183.9 \geq 15.5 + 183.9 \Rightarrow 0.1x \geq 199.4 \Rightarrow \frac{0.1x}{0.1} \geq \frac{199.4}{0.1} \Rightarrow x \geq 1994$. Thus, in year 1994 and later, a 65-year-old man could expect to live an additional 15.5 years or more.

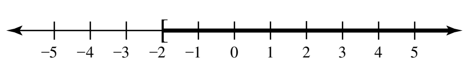
112. (a) $W = 0.96x - 14.4 \Rightarrow 7.2 = 0.96x - 14.4 \Rightarrow 7.2 + 14.4 = 0.96x - 14.4 + 14.4 \Rightarrow 21.6 = 0.96x \Rightarrow$

$\frac{21.6}{0.96} = \frac{0.96x}{0.96} \Rightarrow 22.5 = x \Rightarrow x = 22.5$. Thus, the length of bass is likely to be 22.5 inches.

(b) $0.96x - 14.4 < 7.2 \Rightarrow 0.96x - 14.4 + 14.4 < 7.2 + 14.4 \Rightarrow 0.96x < 21.6 \Rightarrow \frac{0.96x}{0.96} < \frac{21.6}{0.96} \Rightarrow$

$x < 22.5$. Thus, bass less than 22.5 inches in length are likely to weigh less than 7.2 pounds.

Checking Basic Concepts for Section 2.5

1. 

2. $x < 1$

3. When $x = -2$, then $5 - 2(-2) = 5 + 4 = 9$; When $x = -1$, then $5 - 2(-1) = 5 + 2 = 7$; When $x = 0$, then $5 - 2(0) = 5 - 0 = 5$; When $x = 1$, then $5 - 2(1) = 5 - 2 = 3$. Therefore, the numbers that complete the table are 9, 7, 5 and 3. See Figure 3.

x	-2	-1	0	1	2
$5 - 2x$	9	7	5	3	1

Figure 3

From the table, we see that $5 - 2x \le 7$ whenever $x \ge -1$. Thus, the solution to the inequality is $x \ge -1$.

4. (a) $x + 5 > 8 \Rightarrow x + 5 - 5 > 8 - 5 \Rightarrow x > 3 \Rightarrow \{x \mid x > 3\}$

(b) $-\frac{5}{7}x \le 25 \Rightarrow -\frac{7}{5}\left(-\frac{5}{7}x\right) \ge 25\left(-\frac{7}{5}\right) \Rightarrow x \ge -\frac{175}{5} \Rightarrow x \ge -35 \Rightarrow \{x \mid x \ge -35\}$

(c) $3x \ge -2(1 - 2x) + 3 \Rightarrow 3x \ge -2 + 4x + 3 \Rightarrow 3x \ge 1 + 4x \Rightarrow 3x - 4x \ge 1 + 4x - 4x \Rightarrow$ $-x \ge 1 \Rightarrow -1(-x) \le -1(1) \Rightarrow x \le -1 \Rightarrow \{x \mid x \le -1\}$

5. Let l represent length and w represent width. Then, $l = 2w + 5$. Therefore, $2(2w + 5) + 2w > 88 \Rightarrow$ $4w + 10 + 2w > 88 \Rightarrow 6w + 10 > 88 \Rightarrow 6w + 10 - 10 > 88 - 10 \Rightarrow 6w > 78 \Rightarrow$ $\frac{6w}{6} > \frac{78}{6} \Rightarrow w > 13$. Thus, the possible widths must be more than 13 inches.

Chapter 2 Review Exercises

Section 2.1

1. $x + 9 = 3 \Rightarrow x + 9 - 9 = 3 - 9 \Rightarrow x = -6$
2. $x - 4 = -2 \Rightarrow x - 4 + 4 = -2 + 4 \Rightarrow x = 2$
3. $x - \frac{3}{4} = \frac{3}{2} \Rightarrow x - \frac{3}{4} + \frac{3}{4} = \frac{3}{2} + \frac{3}{4} \Rightarrow x = \frac{6}{4} + \frac{3}{4} \Rightarrow x = \frac{9}{4}$
4. $x + 0.5 = 0 \Rightarrow x + 0.5 - 0.5 = 0 - 0.5 \Rightarrow x = -0.5 \Rightarrow x = -\frac{1}{2}$
5. $4x = 12 \Rightarrow \frac{4x}{4} = \frac{12}{4} \Rightarrow x = 3$
6. $3x = -7 \Rightarrow \frac{3x}{3} = \frac{-7}{3} \Rightarrow x = -\frac{7}{3}$
7. $-0.5x = 1.25 \Rightarrow \frac{-0.5x}{-0.5} = \frac{1.25}{-0.5} \Rightarrow x = -2.5$
8. $-\frac{1}{3}x = \frac{7}{6} \Rightarrow -\frac{3}{1}\left(-\frac{1}{3}x\right) = \frac{7}{6}\left(-\frac{3}{1}\right) \Rightarrow x = -\frac{21}{6} \Rightarrow x = -\frac{7}{2}$

Section 2.2

9. The equation $5x - 3 = 0$ is linear; $a = 5, b = -3$
10. The equation $-4x + 3 = 2$ is linear.

$-4x + 3 = 2 \Rightarrow -4x + 3 - 2 = 2 - 2 \Rightarrow -4x + 1 = 0$; $a = -4, b = 1$

11. The equation $0.55x = 0.05$ is linear.

$0.55x = 0.05 \Rightarrow 0.55x - 0.05 = 0.05 - 0.05 \Rightarrow 0.55x - 0.05 = 0;\ a = 0.55, b = -0.05$

12. The equation $\frac{3}{8}x^2 - x = \frac{1}{4}$ is not a linear equation because it cannot be written in the form $ax + b = 0$.

13. $4x - 5 = 3 \Rightarrow 4x - 5 + 5 = 3 + 5 \Rightarrow 4x = 8 \Rightarrow \frac{4x}{4} = \frac{8}{4} \Rightarrow x = 2$. To check the solution, substitute 2 for x in the original equation: $4x - 5 = 3 \Rightarrow 4(2) - 5 = 3 \Rightarrow 8 - 5 = 3$. Because this statement is true, the solution checks.

14. $7 - \frac{1}{2}x = -4 \Rightarrow 7 - 7 - \frac{1}{2}x = -4 - 7 \Rightarrow -\frac{1}{2}x = -11 \Rightarrow -\frac{2}{1}\left(-\frac{1}{2}x\right) = -\frac{11}{1}\left(-\frac{2}{1}\right) \Rightarrow x = 22$. To check the solution, substitute 22 for x in the original equation: $7 - \frac{1}{2}x = -4 \Rightarrow 7 - \frac{1}{2}(22) = -4 \Rightarrow 7 - 11 = -4$. Because this statement is true, the solution checks.

15. $5(x - 3) = 12 \Rightarrow 5x - 15 = 12 \Rightarrow 5x - 15 + 15 = 12 + 15 \Rightarrow 5x = 27 \Rightarrow \frac{5x}{5} = \frac{27}{5} \Rightarrow x = \frac{27}{5}$. To check the solution, substitute $\frac{27}{5}$ for x in the original equation: $5\left(\frac{27}{5} - 3\right) = 12 \Rightarrow 5\left(\frac{27}{5} - \frac{15}{5}\right) = 12 \Rightarrow$ $5\left(\frac{12}{5}\right) = 12 \Rightarrow \frac{60}{5} = 12$. Because this statement is true, the solution checks.

16. $1 - (x - 3) = 6 + 2x \Rightarrow 1 - x + 3 = 6 + 2x \Rightarrow 4 - x = 6 + 2x \Rightarrow 4 - x + x = 6 + 2x + x \Rightarrow$ $4 = 6 + 3x \Rightarrow 4 - 6 = 6 - 6 + 3x \Rightarrow -2 = 3x \Rightarrow \frac{-2}{3} = \frac{3x}{3} \Rightarrow -\frac{2}{3} = x \Rightarrow x = -\frac{2}{3}$. To check the solution, substitute $-\frac{2}{3}$ for x in the original equation: $1 - \left(-\frac{2}{3} - 3\right) = 6 + 2\left(-\frac{2}{3}\right) \Rightarrow$ $1 + \frac{2}{3} + 3 = 6 - \frac{4}{3} \Rightarrow 4\frac{2}{3} = 4\frac{2}{3}$. Because this statement is true, the solution checks.

17. $3.4x - 4 = 5 - 0.6x \Rightarrow 3.4x - 4 + 4 = 5 + 4 - 0.6x \Rightarrow 3.4x = 9 - 0.6x \Rightarrow$ $3.4x + 0.6x = 9 - 0.6x + 0.6x \Rightarrow 4x = 9 \Rightarrow \frac{4x}{4} = \frac{9}{4} \Rightarrow x = \frac{9}{4}$. To check the solution, substitute $\frac{9}{4}$ for x in the original equation: $3.4\left(\frac{9}{4}\right) - 4 = 5 - 0.6\left(\frac{9}{4}\right) \Rightarrow 3.4(2.25) - 4 = 5 - 0.6(2.25) \Rightarrow$ $7.65 - 4 = 5 - 1.35 \Rightarrow 3.65 = 3.65$. Because this statement is true, the solution checks.

18. $-\frac{1}{3}(3 - 6x) = -(x + 2) + 1 \Rightarrow -1 + 2x = -x - 2 + 1 \Rightarrow 2x - 1 = -x - 1 \Rightarrow$ $2x - 1 + 1 = -x - 1 + 1 \Rightarrow 2x = -x \Rightarrow 2x + x = -x + x \Rightarrow 3x = 0 \Rightarrow \frac{3x}{3} = \frac{0}{3} \Rightarrow x = 0$. To check the solution, substitute 0 for x in the original equation: $-\frac{1}{3}(3 - 6(0)) = -(0 + 2) + 1 \Rightarrow$ $-\frac{1}{3}(3) = -2 + 1 \Rightarrow -1 = -1$. Because this statement is true, the solution checks.

19. $\frac{2}{3}x - \frac{1}{6} = \frac{5}{12} \Rightarrow \frac{2}{3}x - \frac{1}{6} + \frac{1}{6} = \frac{5}{12} + \frac{1}{6} \Rightarrow \frac{2}{3}x = \frac{5}{12} + \frac{2}{12} \Rightarrow \frac{2}{3}x = \frac{7}{12} \Rightarrow \frac{3}{2}\left(\frac{2}{3}x\right) = \frac{7}{12}\left(\frac{3}{2}\right) \Rightarrow$ $x = \frac{21}{24} \Rightarrow x = \frac{7}{8}$. To check the solution, substitute $\frac{7}{8}$ for x in the original equation: $\frac{2}{3}\left(\frac{7}{8}\right) - \frac{1}{6} = \frac{5}{12} \Rightarrow$ $\frac{14}{24} - \frac{4}{24} = \frac{10}{24} \Rightarrow \frac{10}{24} = \frac{10}{24}$. Because this statement is true, the solution checks.

20. $2y - 3(2 - y) = 5 + y \Rightarrow 2y - 6 + 3y = 5 + y \Rightarrow 5y - 6 = 5 + y \Rightarrow 5y - y - 6 = 5 + y - y \Rightarrow$ $4y - 6 = 5 \Rightarrow 4y - 6 + 6 = 5 + 6 \Rightarrow 4y = 11 \Rightarrow \frac{4y}{4} = \frac{11}{4} \Rightarrow y = \frac{11}{4}$. To check the solution, substitute $\frac{11}{4}$ for y in the original equation: $2\left(\frac{11}{4}\right) - 3\left(2 - \frac{11}{4}\right) = 5 + \frac{11}{4} \Rightarrow \frac{22}{4} - 6 + \frac{33}{4} = \frac{20}{4} + \frac{11}{4} \Rightarrow$ $\frac{22}{4} - \frac{24}{4} + \frac{33}{4} = \frac{20}{4} + \frac{11}{4} \Rightarrow \frac{31}{4} = \frac{31}{4}$. Because this statement is true, the solution checks.

21. First, solve for x: $4(3x - 2) = 2(6x + 5) \Rightarrow 12x - 8 = 12x + 10 \Rightarrow$ $12x - 12x - 8 = 12x - 12x + 10 \Rightarrow -8 = 10$. Because this statement is not true, the equation has zero solutions.

22. First, solve for x: $5(3x - 1) = 15x - 5 \Rightarrow 15x - 5 = 15x - 3$. Because this statement is true for any value of x, the equation has infinitely many solutions.

23. First, solve for x: $8x = 5x + 3x \Rightarrow 8x = 8x$. Because this statement is true for any value of x, the equation has infinitely many solutions.

24. First solve for x: $9x - 2 = 8x - 2 \Rightarrow 9x - 8x - 2 = 8x - 8x - 2 \Rightarrow x - 2 = -2 \Rightarrow$ $x - 2 + 2 = -2 + 2 \Rightarrow x = 0$. Thus, there is one solution to the equation.

25. When $x = 1.0$, then $-2(1.0) + 3 = -2 + 3 = 1$; When $x = 1.5$, then $-2(1.5) + 3 = -3 + 3 = 0$; When $x = 2.0$, then $-2(2.0) + 3 = -4 + 3 = -1$; When $x = 2.5$, then $-2(2.5) + 3 = -5 + 3 = -2$; Thus, the missing values in the table are 1, 0, −1 and −2. See Figure 25. From the table we see that when $x = 1.5$, the value of $-2x + 3$ is 0.

x	0.5	1.0	1.5	2.0	2.5
$-2x + 3$	2	1	0	−1	−2

Figure 25

x	−2	−1	0	1	2
$-(x + 1) + 3$	4	3	2	1	0

Figure 26

26. When $x = -2$, then $-(-2 + 1) + 3 = 2 - 1 + 3 = 4$; When $x = -1$, then $-(-1 + 1) + 3 = 1 - 1 + 3 = 3$; When $x = 0$, then $-(0 + 1) + 3 = 0 - 1 + 3 = 2$; When $x = 1$, then $-(1 + 1) + 3 = -2 + 3 = 1$; Thus, the missing values in the table are 4, 3, 2 and 1. See Figure 26. From the table we see that when $x = 0$, the value of $-(x + 1) + 3$ is 2.

Section 2.3

27. $6x = 72 \Rightarrow \frac{6x}{6} = \frac{72}{6} \Rightarrow x = 12$

28. $x + 18 = -23 \Rightarrow x + 18 - 18 = -23 - 18 \Rightarrow x = -41$

29. $2x - 5 = x + 4 \Rightarrow 2x - 5 + 5 = x + 4 + 5 \Rightarrow 2x = x + 9 \Rightarrow 2x - x = x - x + 9 \Rightarrow x = 9$

30. $x + 4 = 3x \Rightarrow x - 3x + 4 = 3x - 3x \Rightarrow -2x + 4 = 0 \Rightarrow -2x + 4 - 4 = 0 - 4 \Rightarrow -2x = -4 \Rightarrow$ $\frac{-2x}{-2} = \frac{-4}{-2} \Rightarrow x = 2$

31. $x + (x + 1) + (x + 2) + (x + 3) = 70 \Rightarrow 4x + 6 = 70 \Rightarrow 4x + 6 - 6 = 70 - 6 \Rightarrow 4x = 64 \Rightarrow$ $\frac{4x}{4} = \frac{64}{4} \Rightarrow x = 16$. The numbers are 16, 17, 18 and 19.

32. $x + (x + 1) + (x + 2) = -153 \Rightarrow 3x + 3 = -153 \Rightarrow 3x + 3 - 3 = -153 - 3 \Rightarrow 3x = -156 \Rightarrow$
$\frac{3x}{3} = \frac{-156}{3} \Rightarrow x = -52$. The numbers are -52, -51 and -50.

33. $85\% = \frac{85}{100} = \frac{17}{20}$; $85\% = 0.85$

34. $5.6\% = \frac{56}{1000} = \frac{7}{125}$; $5.6\% = 0.056$

35. $0.03\% = \frac{.03}{100} = \frac{3}{10{,}000}$; $0.03\% = 0.0003$

36. $342\% = \frac{342}{100} = \frac{171}{50}$; $342\% = 3.42$

37. $0.89 = 89\%$

38. $0.005 = 0.5\%$

39. $2.3 = 230\%$

40. $1 = 100\%$

Section 2.4

41. $d = rt \Rightarrow d = 8(3) \Rightarrow d = 24$ miles.

42. $d = rt \Rightarrow d = 70(55) \Rightarrow d = 3850$ feet.

43. $d = rt \Rightarrow 500 = r(20) \Rightarrow \frac{500}{20} = r\frac{20}{20} \Rightarrow \frac{500}{20} = r \Rightarrow r = 25$ yd/sec.

44. $d = rt \Rightarrow 125 = 15t \Rightarrow \frac{125}{15} = \frac{15t}{15} \Rightarrow \frac{125}{15} = t \Rightarrow t = \frac{25}{3}$ hours.

45. The area of a triangle is given as $\frac{1}{2}$(base)(height). Thus, $A = \frac{1}{2}(b)(h) = \frac{1}{2}(5)(3) = 7.5 \text{ m}^2$.

46. The area of a circle is given as πr^2 where r represents radius. Thus, $A = \pi r^2 = \pi(6^2) = 36\pi \approx 113.1 \text{ ft}^2$.

47. The area of a rectangle is given as length (l) times width (w). Thus, $A = lw = (36)(24) = 864 \text{ in}^2$ or 6 ft^2.

48. $A = \frac{1}{2}bh$, where A represents area, b represents the length of the base and h represents the height. Thus,
$A = \frac{1}{2}bh = \frac{1}{2}(13)(7) = \frac{1}{2}(91) = 45\frac{1}{2} = 45.5 \text{ in}^2$.

49. The circumference of a circle is given as $2\pi r$, where r represents radius. Thus, $r = \frac{1}{2}$(diameter) $= \frac{1}{2}(18) = 9$.
$C = 2\pi r = 2\pi(9) = 18\pi \approx 56.5$ feet.

50. $A = \pi r^2$, where A represents area and r represents radius. Thus, $A = \pi r^2 = \pi(5^2) = 25\pi \approx 78.5 \text{ in}^2$.

51. The angles in a triangle must add up to 180º. Let x represent the unknown angle. Thus,
$90 + 40 + x = 180 \Rightarrow 130 + x = 180 \Rightarrow 130 - 130 + x = 180 - 130 \Rightarrow x = 50^\circ$.

52. The angles in a triangle must add up to 180º. Thus, $x + 3x + 4x = 180 \Rightarrow 8x = 180 \Rightarrow \frac{8x}{8} = \frac{180}{8} \Rightarrow$
$x = 22.5^\circ$.

53. $V = \pi r^2 h = \pi(5^2)(25) = \pi(25)(25) = 625\pi \approx 1963.5 \text{ in}^3$.

54. First, convert height (h) and base (b) to inches. $h = 5$ feet $= 5(12) = 60$ inches and

$b = 3$ feet $= 3(12) = 36$ inches. Then, $A = \frac{1}{2}(a + b)h = \frac{1}{2}(36 + 18)60 = \frac{1}{2}(54)60 = (27)60 = 1620 \text{ in}^2$.

Or, convert the base in inches to feet. $b = 18$ inches $= \frac{18}{12} = 1.5$ feet. Then,

$A = \frac{1}{2}(a + b)h = \frac{1}{2}(3 + 1.5)5 = \frac{1}{2}(4.5)5 = (2.25)5 = 11.25 \text{ ft}^2$.

55. $a = x + y \Rightarrow a - y = x + y - y \Rightarrow a - y = x \Rightarrow x = a - y$

56. $P = 2x + 2y \Rightarrow P - 2y = 2x + 2y - 2y \Rightarrow P - 2y = 2x \Rightarrow \frac{P - 2y}{2} = \frac{2x}{2} \Rightarrow \frac{P - 2y}{2} = x \Rightarrow$

$x = \frac{P - 2y}{2}$

57. $z = 2xy \Rightarrow \frac{z}{2x} = \frac{2xy}{2x} \Rightarrow \frac{z}{2x} = y \Rightarrow y = \frac{z}{2x}$

58. $S = \frac{a + b + c}{3} \Rightarrow 3S = \frac{a + b + c}{3} \cdot 3 \Rightarrow 3S = a + b + c \Rightarrow 3S - a - c = a + b + c - a - c \Rightarrow$

$3S - a - c = b \Rightarrow b = 3S - a - c$

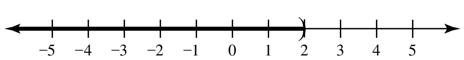
59. $T = \frac{a}{3} + \frac{b}{4} \Rightarrow \frac{12T}{12} = \frac{4a}{12} + \frac{3b}{12} \Rightarrow 12\left(\frac{12T}{12}\right) = 12\left(\frac{4a}{12}\right) + 12\left(\frac{3b}{12}\right) \Rightarrow 12T = 4a + 3b \Rightarrow$

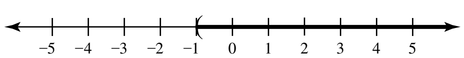
$12T - 4a = 4a - 4a + 3b \Rightarrow 12T - 4a = 3b \Rightarrow \frac{12T - 4a}{3} = \frac{3b}{3} \Rightarrow \frac{12T - 4a}{3} = b \Rightarrow b = \frac{12T - 4a}{3}$

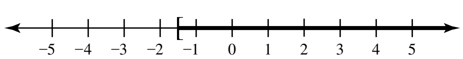
60. $cd = ab + bc \Rightarrow cd - bc = ab + bc - bc \Rightarrow cd - bc = ab \Rightarrow c(d - b) = ab \Rightarrow$

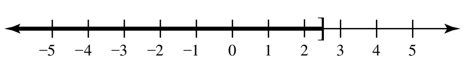
$\frac{c(d - b)}{(d - b)} = \frac{ab}{(d - b)} \Rightarrow c = \frac{ab}{d - b}$

Section 2.5

61. 

62. 

63. 

64. 

65. $x < 3$

66. $x \geq -1$

67. Substitute -3 for x and check for accuracy: $2x + 1 \leq 5 \Rightarrow 2(-3) + 1 \leq 5 \Rightarrow -6 + 1 \leq 5 \Rightarrow -5 \leq 5$.

Because this statement is true, $x = -3$ is a solution to the inequality.

68. Substitute 4 for x and check for accuracy: $5 - \frac{1}{2}(x) \geq -1 \Rightarrow 5 - \frac{1}{2}(4) \geq -1 \Rightarrow 5 - 2 \geq -1 \Rightarrow 3 \geq -1$.

Because this statement is true, $x = 4$ is a solution to the inequality.

69. Substitute −2 for x and check for accuracy: $1 - (x + 3) \geq x \Rightarrow 1 - (-2 + 3) \geq -2 \Rightarrow 1 - 1 \geq -2 \Rightarrow$ $0 \geq -2$. Because this statement is true, $x = -2$ is a solution to the inequality.

70. Substitute −1 for x and check for accuracy: $4(x + 1) < -(5 - x) \Rightarrow 4(-1 + 1) < -(5 - (-1)) \Rightarrow$ $4(0) < -(6) \Rightarrow 0 < -6$. Because this statement is not true, $x = -1$ is not a solution to the inequality.

71. When $x = 1$, then $5 - x = 5 - 1 = 4$; When $x = 2$, then $5 - x = 5 - 2 = 3$;

When $x = 3$, then $5 - x = 5 - 3 = 2$; When $x = 4$, then $5 - x = 5 - 4 = 1$;

Thus, the missing values in the table are 4, 3, 2 and 1. See Figure 71. From the table we see that $5 - x > 3$, when $x < 2$.

x	0	1	2	3	4
$5 - x$	5	4	3	2	1

Figure 71

x	1	1.5	2	2.5	3
$2x - 5$	−3	−2	−1	0	1

Figure 72

72. When $x = 1.5$, then $2x - 5 = 2(1.5) - 5 = 3 - 5 = -2$;

When $x = 2$, then $2x - 5 = 2(2) - 5 = 4 - 5 = -1$;

When $x = 2.5$, then $2x - 5 = 2(2.5) - 5 = 5 - 5 = 0$;

When $x = 3$, then $2x - 5 = 2(3) - 5 = 6 - 5 = 1$;

Thus, the missing values in the table are −2, −1, 0 and 1. See Figure 72. From the table we see that $2x - 5 \leq 0$ when $x \leq 2.5$.

73. $x - 3 > 0 \Rightarrow x - 3 + 3 > 0 + 3 \Rightarrow x > 3 \Rightarrow \{x \mid x > 3\}$

74. $-2x \leq 10 \Rightarrow \dfrac{-2x}{-2} \geq \dfrac{10}{-2} \Rightarrow x \geq -5 \Rightarrow \{x \mid x \geq -5\}$

75. $5 - 2x \geq 7 \Rightarrow 5 - 5 - 2x \geq 7 - 5 \Rightarrow -2x \geq 2 \Rightarrow \dfrac{-2x}{-2} \leq \dfrac{2}{-2} \Rightarrow x \leq -1 \Rightarrow \{x \mid x \leq -1\}$

76. $3(x - 1) < 20 \Rightarrow 3x - 3 < 20 \Rightarrow 3x - 3 + 3 < 20 + 3 \Rightarrow 3x < 23 \Rightarrow \dfrac{3x}{3} < \dfrac{23}{3} \Rightarrow x < \dfrac{23}{3} \Rightarrow$ $\left\{x \,\middle|\, x < \dfrac{23}{3}\right\}$

77. $5x \leq 3 - (4x + 2) \Rightarrow 5x \leq 3 - 4x - 2 \Rightarrow 5x + 4x \leq 3 - 4x + 4x - 2 \Rightarrow 9x \leq 1 \Rightarrow \dfrac{9x}{9} \leq \dfrac{1}{9} \Rightarrow$ $x \leq \dfrac{1}{9} \Rightarrow \left\{x \,\middle|\, x \leq \dfrac{1}{9}\right\}$

78. $3x - 2(4 - x) \geq x + 1 \Rightarrow 3x - 8 + 2x \geq x + 1 \Rightarrow 5x - 8 + 8 \geq x + 1 + 8 \Rightarrow 5x \geq x + 9 \Rightarrow$ $5x - x \geq x - x + 9 \Rightarrow 4x \geq 9 \Rightarrow \dfrac{4x}{4} \geq \dfrac{9}{4} \Rightarrow x \geq \dfrac{9}{4} \Rightarrow \left\{x \,\middle|\, x \geq \dfrac{9}{4}\right\}$

79. $x < 50$

80. $x \leq 45{,}000$

81. $x \geq 16$

82. $x < 1995$

Applications

83. (a) See Figure 83.

(b) $R = 2 + \frac{3}{4}x$

(c) At 5 PM, $x = 5$; $R = 2 + \frac{3}{4}(5) = 2 + \frac{15}{4} = \frac{23}{4} = 5\frac{3}{4}$ inches. This value does agree with the table.

(d) At 3:45 PM, $x = 3.75$; $R = 2 + \frac{3}{4}\left(3\frac{3}{4}\right) = 2 + \frac{3}{4}\left(\frac{15}{4}\right) = 2 + \frac{45}{16} = \frac{32}{16} + \frac{45}{16} = \frac{77}{16} = 4\frac{13}{16}$ inches.

Time	12:00	1:00	2:00	3:00	4:00	5:00
Rainfall (R)	2	2.75	3.5	4.25	5	5.75

Figure 83

Hours (x)	1	2	3	4	5
Distance (D)	40	30	20	10	0

Figure 85

84. Let x represent the cost of the laptop. $0.05x = 106.25 \Rightarrow \frac{0.05x}{0.05} = \frac{106.25}{0.05} \Rightarrow x = 2125$. Thus, the cost of the laptop is \$2125.

85. (a) See Figure 85.

(b) $D = 50 - 10x$

(c) $D = 50 - 10x = 50 - 10(3) = 50 - 30 = 20$ miles. This value does agree with the table.

(d) $D \geq 20$. Thus, $50 - 10x \geq 20 \Rightarrow 50 - 50 - 10x \geq 20 - 50 \Rightarrow -10x \geq -30 \Rightarrow \frac{-10x}{-10} \leq \frac{-30}{-10} \Rightarrow$ $x \leq 3$. Thus, the bicyclist was at least 20 miles from home when he had traveled for 3 or fewer hours, or from noon to 3 PM.

86. $N = \frac{1}{15}x - 130.4$. Substitute 2.8 for N and solve for x: $2.8 = \frac{1}{15}x - 130.4 \Rightarrow$ $2.8 + 130.4 = \frac{1}{15}x - 130.4 + 130.4 \Rightarrow 133.2 = \frac{1}{15}x \Rightarrow 133.2\left(\frac{15}{1}\right) = \frac{15}{1}\left(\frac{1}{15}x\right) \Rightarrow 1998 = x \Rightarrow$ $x = 1998$. Thus, in the year 1998, the number reached 2.8 million.

87. First, subtract the smaller number from the larger to obtain the difference between them: $419{,}401 - 230{,}500 = 188{,}901$. Then, determine what percentage 188,901 is of 230,500. Do this by dividing the smaller number by the larger: $\frac{188{,}901}{230{,}500} \approx 0.82$. Thus, there was about an 82% change in master's degrees received between 1971 and 1997.

88. Use the distance (d) = rate (r) $\times$ time (t) formula. Determine how long it takes the faster car to be 2 miles ahead of the slower car, let $(r + 12)$ be the rate of the faster car and r be the rate of the slower car. Thus, $d = rt \Rightarrow 2 = (r + 12 - r)t \Rightarrow 2 = 12t \Rightarrow \frac{2}{12} = \frac{12t}{12} \Rightarrow \frac{1}{6} = t \Rightarrow t = \frac{1}{6}$ hour, or 10 minutes.

89. Let x represent the amount of water. The amount of salt on one side of the equation must equal the amount of salt on the other side. Thus, $100(0.03) + x(0.00) = (100 + x)(0.02) \Rightarrow 3 + 0 = 2 + 0.02x \Rightarrow$ $3 - 2 = 2 - 2 + 0.02x \Rightarrow 1 = 0.02x \Rightarrow \frac{1}{0.02} = \frac{0.02x}{0.02} \Rightarrow 50 = x \Rightarrow x = 50$. Thus, 50 ml of water must be added.

90. Let x represent the higher interest rate. Then, $800(x) + 500(x - 0.02) = 55 \Rightarrow 800x + 500x - 10 = 55 \Rightarrow$ $1300x - 10 + 10 = 55 + 10 \Rightarrow 1300x = 65 \Rightarrow \frac{1300x}{1300} = \frac{65}{1300} \Rightarrow x = 0.05$. Thus, the interest rate on the \$800 loan is 5% and the interest rate on the \$500 loan is 3%.

91. Perimeter $(P) = 2 \times$ width $(w) =$ length (l). Then, $w = l - 10 \Rightarrow 2(l - 10) + 2l = 112 \Rightarrow$ $2l - 20 + 2l = 112 \Rightarrow 2l - 20 + 20 + 2l = 112 + 20 \Rightarrow 4l = 132 \Rightarrow \frac{4l}{4} = \frac{132}{4} \Rightarrow l = 33$. Because the length is 33, the width is $(l - 10) = 23$. Thus, the dimensions are 33 by 23 inches.

92. Area (A) of a triangle is $\frac{1}{2} \times$ base $(b) \times$ height (h). Thus, $A = \frac{1}{2}bh \Rightarrow \frac{1}{2}bh \leq 100 \Rightarrow \frac{1}{2}b(8) \leq 100 \Rightarrow$ $4b \leq 100 \Rightarrow \frac{4b}{4} \leq \frac{100}{4} \Rightarrow b \leq 25$. Therefore, the base must be 25 inches or less.

93. Let x represent the unknown test score. Then, $\frac{75 + 91 + x}{3} = 80 \Rightarrow \frac{166 + x}{3} = 80 \Rightarrow$ $3\left(\frac{166 + x}{3}\right) = 3(80) \Rightarrow 166 + x = 240 \Rightarrow 166 - 166 + x = 240 - 166 \Rightarrow x = 74$. Thus, the student must score 74 or more.

94. Let x represent the unknown number of hours. Then, $2.25 + 1.25x = 9 \Rightarrow$ $2.25 - 2.25 + 1.25x = 9 - 2.25 \Rightarrow 1.25x = 6.75 \Rightarrow \frac{1.25x}{1.25} = \frac{6.75}{1.25} \Rightarrow x = 5.4$. Because each partial hour is charged as a full hour, the person can park for 6 hours.

95. (a) $C = 150{,}000 + 85x$

(b) $R = 225x$

(c) $P = 225x - (150{,}000 + 85x) \Rightarrow P = 140x - 150{,}000$

(d) $140x - 150{,}000 < 0 \Rightarrow 140x - 150{,}000 + 150{,}000 < 0 + 150{,}000 \Rightarrow 140x < 150{,}000 \Rightarrow$ $\frac{140x}{140} < \frac{150{,}000}{140} \Rightarrow x < 1071.43$. Therefore, if 1071 or fewer DVD players are sold, there will be a loss.

Chapter 2 Test

1. $9 = 3 - x \Rightarrow 9 - 3 = 3 - 3 - x \Rightarrow 6 = -x \Rightarrow 6(-1) = (-x)(-1) \Rightarrow -6 = x \Rightarrow x = -6$

To check the solution: $9 - 3 - (-6) \Rightarrow 9 = 9$. The solution checks.

2. $4x - 3 = 7 \Rightarrow 4x - 3 + 3 = 7 + 3 \Rightarrow 4x = 10 \Rightarrow \frac{4x}{4} = \frac{10}{4} \Rightarrow x = \frac{5}{2}$

To check the solution: $4\left(\frac{5}{2}\right) - 3 = 7 \Rightarrow \frac{20}{2} - \frac{6}{2} = 7 \Rightarrow \frac{14}{2} = 7 \Rightarrow 7 = 7$. The solution checks.

3. $4x - (2 - x) = -3(2x + 6) \Rightarrow 4x - 2 + x = -6x - 18 \Rightarrow 5x - 2 = -6x - 18 \Rightarrow$ $5x - 2 + 2 = -6x - 18 + 2 \Rightarrow 5x = -6x - 16 \Rightarrow 5x + 6x = -6x + 6x - 16 \Rightarrow 11x = -16 \Rightarrow$ $\frac{11x}{11} = \frac{-16}{11} \Rightarrow x = -\frac{16}{11}$. To check the solution: $4\left(-\frac{16}{11}\right) - \left(2 - \left(-\frac{16}{11}\right)\right) = -3\left(2\left(-\frac{16}{11}\right) + 6\right) \Rightarrow$ $-\frac{64}{11} - 2 - \frac{16}{11} = -6\left(-\frac{16}{11}\right) - 18 \Rightarrow -\frac{64}{11} - \frac{22}{11} - \frac{16}{11} = \frac{96}{11} - \frac{198}{11} \Rightarrow -\frac{102}{11} = -\frac{102}{11}$. The solution checks.

4. $\frac{1}{12}x - \frac{2}{3} = \frac{1}{2}\left(\frac{3}{4} - \frac{1}{3}x\right) \Rightarrow \frac{1}{12}x - \frac{2}{3} = \frac{3}{8} - \frac{1}{6}x \Rightarrow \frac{1}{12}x + \frac{1}{6}x - \frac{2}{3} = \frac{3}{8} - \frac{1}{6}x + \frac{1}{6}x \Rightarrow$

$\frac{3}{12}x - \frac{2}{3} + \frac{2}{3} = \frac{3}{8} + \frac{2}{3} \Rightarrow \frac{3}{12}x = \frac{9}{24} + \frac{16}{24} \Rightarrow \frac{3}{12}x = \frac{25}{24} \Rightarrow \frac{12}{3}\left(\frac{3}{12}x\right) = \frac{12}{3}\left(\frac{25}{24}\right) \Rightarrow x = \frac{300}{72} = \frac{25}{6}.$

To check the solution: $\frac{1}{12}\left(\frac{25}{6}\right) - \frac{2}{3} = \frac{1}{2}\left(\frac{3}{4} - \frac{1}{3}\left(\frac{25}{6}\right)\right) \Rightarrow \frac{25}{72} - \frac{48}{72} = \frac{1}{2}\left(\frac{27}{36} - \frac{50}{36}\right) \Rightarrow$

$-\frac{23}{72} = \frac{1}{2}\left(-\frac{23}{36}\right) \Rightarrow -\frac{23}{72} = -\frac{23}{72}$. The solution checks.

5. First, solve for x: $6(2x - 1) = -4(3 - 3x) \Rightarrow 12x - 6 = -12 + 12x \Rightarrow$

$12x - 12x - 6 = -12 + 12x - 12x \Rightarrow -6 = -12$. Because this statement is not true, there are no solutions.

6. When $x = 1$, then $6 - 2x = 6 - 2(1) = 4$; When $x = 2$, then $6 - 2x = 6 - 2(2) = 2$;

When $x = 3$, then $6 - 2x = 6 - 2(3) = 0$; When $x = 4$, then $6 - 2x = 6 - 2(4) = -2$;

Thus, the missing values in the table are 4, 2, 0 and -2. See Figure 6. From the table we see that

$6 - 2x = 0$, when $x = 3$.

x	0	1	2	3	4
$6 - 2x$	6	4	2	0	−2

Figure 6

7. $x + (-7) = 6 \Rightarrow x - 7 = 6 \Rightarrow x - 7 + 7 = 6 + 7 \Rightarrow x = 13$

8. $2x + 6 = x - 7 \Rightarrow 2x - x + 6 = x - x - 7 \Rightarrow x + 6 = -7 \Rightarrow x + 6 - 6 = -7 - 6 \Rightarrow x = -13$

9. $x + (x + 1) + (x + 2) = 336 \Rightarrow 3x + 3 = 336 \Rightarrow 3x + 3 - 3 = 336 - 3 \Rightarrow 3x = 333 \Rightarrow$

$\frac{3x}{3} = \frac{333}{3} \Rightarrow x = 111$. Thus, the three numbers are 111, 112 and 113.

10. $5.6\% = 0.056$; $5.6\% = \frac{56}{1000} = \frac{14}{250} = \frac{7}{125}$

11. $0.345 = 34.5\%$

12. Let x represent the unknown number. To find 7.5% of \$500, multiply 500 by 0.075. Then, $500(0.075) = x \Rightarrow$

$37.5 = x \Rightarrow x = 37.5$. Thus, 7.5% of \$500 is \$37.50.

13. $\frac{5280}{5} = \frac{5280}{5} \div \frac{5}{5} = 1056$ ft/sec.

14. Area $(A) = \frac{1}{2} \times$ base $(b) \times$ height (h). Thus, $A = \frac{1}{2}bh = \frac{1}{2}(5)(3) = 7.5$ in^2.

15. Circumference of a circle is given as $C = 2\pi r$. Then, $C = 2\pi r = 2\pi\left(\frac{30}{2}\right) = 2\pi(15) = 30\pi \approx 94.2$ inches.

Area of a circle is given as $A = \pi r^2$. Then, $A = \pi r^2 = \pi(15)^2 = 225\pi \approx 706.9$ in^2.

16. The angles in a triangle must add up to 180º.

Then, $x + 2x + 3x = 180 \Rightarrow 6x = 180 \Rightarrow \frac{6x}{6} = \frac{180}{6} \Rightarrow x = 30$. Thus, the angles are 30º, 60º and 90º.

17. $z = y - 3xy \Rightarrow z - y = y - y - 3xy \Rightarrow z - y = -3xy \Rightarrow \frac{z - y}{-3y} = \frac{-3xy}{-3y} \Rightarrow \frac{z - y}{-3y} = x \Rightarrow x = \frac{y - z}{3y}$

18. $3(6 - 5x) < 20 - x \Rightarrow 18 - 15x < 20 - x \Rightarrow 18 - 18 - 15x < 20 - 18 - x \Rightarrow -15x < 2 - x \Rightarrow$ $-15x + x < 2 - x + x \Rightarrow -14x < 2 \Rightarrow \frac{-14x}{-14} > \frac{2}{-14} \Rightarrow x > -\frac{1}{7} \Rightarrow \left\{x \,\middle|\, x > -\frac{1}{7}\right\}$

19. (a) $S = 5 + 2x$, where x represents hours past noon.

    (b) $x = 8$. Thus, $S = 5 + 2(8) = 21$ inches.

    (c) $x = 6.25$. Thus, $S = 5 + 2(6.25) = 17.5$ inches.

20. The amount of acid on the left side of the equation must equal the amount of acid on the right side of the equation. Let x represent the unknown amount of water. Then, $1000(0.45) + x(0) = (1000 + x)(0.15) \Rightarrow$ $450 = 150 + 0.15x \Rightarrow 450 - 150 = 150 - 150 + 0.15x \Rightarrow 300 = 0.15x \Rightarrow \frac{300}{0.15} = \frac{0.15x}{0.15} \Rightarrow$ $2000 = x \Rightarrow x = 2000$. Thus, 2000 ml. of water must be added.

21. Subtract the lesser amount from the larger amount and then calculate the percentage difference as compared to the smaller amount. Then, $32 - 8 = 24$; $\frac{24}{8} = 3 = 300\%$. Therefore, there was a 300% increase in premiums from 1998 to 2003.

Chapter 2 Extended and Discovery Exercises

1. For the first hour, the distance traveled was $d = rt$ such that $d = (50)(1) = 50$ miles. For the second hour, the distance traveled was $d = rt$ such that $d = (70)(1) = 70$ miles. Thus, for the two hours $r = \frac{d}{t}$ such that $r = \frac{70 + 50}{1 + 1} = \frac{120}{2} = 60$. Thus, the average speed of the car was 60 mph.

2. Uphill, $d = \frac{d}{r}$ such that $t = \frac{1}{5} = \frac{1}{5}$ of an hour. Downhill, $t = \frac{d}{r}$ such that $t = \frac{1}{10} = \frac{1}{10}$ of an hour. Thus, the average speed $r = \frac{d}{t}$ is $r = \frac{1 + 1}{\frac{1}{5} + \frac{1}{10}} = \frac{2}{\frac{3}{10}} = \frac{20}{3} = 6.\overline{6}$ mph.

3. For the first two miles, $t = \frac{d}{r}$ such that $t = \frac{2}{8} = \frac{1}{4}$ of an hour. For the third mile, $t = \frac{d}{t}$ such that $t = \frac{1}{10} = \frac{1}{10}$ of an hour. Thus, the average speed of the athlete is $r = \frac{d}{t} = \frac{3}{\frac{1}{4} + \frac{1}{10}} = \frac{3}{\frac{5}{20} + \frac{2}{20}} = \frac{3}{\frac{7}{20}} = \frac{60}{7} \approx 8.6$ mph.

4. Choose a distance of 400 miles as the distance between the two cities (the distance is arbitrary because any distance gives the same average speed). Then, the pilot flew at 200 mph for 1 hour and at 100 mph for 2 hours. Then, $r = \frac{d}{t} = \frac{400}{1 + 2} = \frac{400}{3} = 133.\overline{3}$. Thus, the average speed is $133.\overline{3}$ mph.

5. The lighter coin can be found in two weighings as follows: Place two coins on each pan of the balance and set three coins off to the side. Case 1: The pans balance and the lighter coin is one of the three coins that were set off to the side. Case 2: The pans do not balance and the lighter coin is one of the two coins on the higher pan. To find the lighter coin in Case 1, work only with the three remaining coins. Place one coin on each side of the balance and set one coin off to the side. If the pans do not balance, the lighter coin is the one on the higher pan. To find the lighter coin in Case 2, work with only the two coins from the higher pan. Place one coin on each side of the balance. The lighter coin is on the higher pan.

6. (a) Surface area $(A) = 4\pi r^2 = 4\pi(3960)^2 \approx 197{,}060{,}797$ mi^2.

   (b) $.71(197{,}060{,}797) \approx 139{,}913{,}166$ mi^2.

   (c) $\dfrac{680{,}000}{139{,}913{,}166} \approx 0.00486$ miles. To convert 0.00486 miles to feet: $0.00486(5280) \approx 25.7$ feet.

   (d) They would be flooded.

   (e) Divide the volume of the Antarctic ice cap by the surface area of the oceans: $\dfrac{6{,}300{,}000}{139{,}913{,}166} \approx 0.045$ miles.

   To convert 0.0045 miles to feet: $0.045(5280) \approx 237.7$ feet.

Critical Thinking Solutions for Chapter 2

Section 2.1

- If an error is made, the resulting equation may not be equivalent to the given equation.

Section 2.2

- Solve for x: $bx - 2 = dx + 7 \Rightarrow bx - 2 + 2 = dx + 7 + 2 \Rightarrow bx = dx + 9 \Rightarrow$ $bx - dx = dx - dx + 9 \Rightarrow bx - dx = 9 \Rightarrow x(b - d) = 9 \Rightarrow \dfrac{x(b-d)}{b-d} = \dfrac{9}{b-d} \Rightarrow x = \dfrac{9}{b-d}$. Thus, if $b = d$, then $x = \dfrac{9}{0}$. Because dividing by 0 is not allowed, there are no solutions. If $b \neq d$, then there is one solution.

Section 2.3

- Let x represent the lower salary. Then, $x + 2x$ equals the increased salary amount. Thus, because $x + 2x = 3x$, the lower salary increased by a factor of 3.

Section 2.4

- $C = 2\pi r = \pi 2r = \pi d$; $A = \pi r^2 = \pi\left(\dfrac{1}{2}d\right)^2 = \dfrac{1}{4}\pi d^2$
- Yes. Multiply one expression by 1 in the form $\dfrac{-1}{-1}$ to transform it to the other.

Section 2.5

- $-5 - 3x > -2x + 7 \Rightarrow -5 - 3x + 3x > -2x + 3x + 7 \Rightarrow -5 > x + 7 \Rightarrow -5 - 7 > x + 7 - 7 \Rightarrow$ $-12 > x \Rightarrow x < -12$