CHAPTER 1 FUNCTIONS

The opening chapter of the text focuses on functions: their properties, their graphs, and their use in applications. It can also be viewed as an overview of the prerequisite knowledge from algebra and trigonometry that is necessary for success in a calculus course. Additional review material appears in Appendix A. Some departments skip this material and begin the calculus curriculum with Chapter 2; if that describes your situation, you may refer your students to this chapter and to Appendix A when algebra and trigonometry difficulties arise. For those instructors who provide a review of basic skills, a daunting task lies ahead of you. In one or two weeks, you must race through enough material to fill a term-long course. Plan accordingly, and recognize that you won't likely be able to cover everything. Rely on your students to fill in the details.

Section 1.1 Review of Functions

Overview

A function is defined and its properties are developed.

Lecture Support Notes

It's important to set the right tone at the beginning of a semester. Experience shows that when you set the bar high, your class will jump higher as a result. Assign a good deal of homework from these initial sections to ensure that your students get a thorough review of the prerequisites necessary for calculus, and so that they get used to devoting plenty of time to the study of calculus outside of class.

This chapter is also an ideal time to think about an early diagnostic quiz, assuming your department does not have an entrance exam for the class already in place. Providing immediate feedback for your students helps them to determine whether they are ready for the rigors of calculus, or whether they need to drop the course before your institution's deadline. It's an old story, but one that calculus teachers share often: The main reason for failure in calculus usually stems from difficulties in algebra and trigonometry. Try to develop a quiz—which could be given in the first week—in order to gather data for correlations between your students' scores on the quiz and their final grade in the course. Over time, you will be able to spot insurmountable deficiencies, or problem areas where your students need help, and you can address the challenges accordingly.

- Cover the definition of a function, its geometric interpretation (the vertical line test), and the concepts of domain and range (both the domain of definition and the domain in the context of an application).
- Review composition of functions. Composite functions are featured prominently in calculus, so cover all the bases (Examples 4–8).
- Focus on the idea of a difference quotient (examples 9 and 10), and be sure your students can simplify expressions such as $\frac{f(x+h)-f(x)}{h}$ and $\frac{f(x)-f(a)}{x-a}$.
- Explain the notion of symmetry in graphs, and give definitions of even and odd functions.

Interactive Figures

- Figures 1.4–1.5 display the domain and range of two functions.
- Figure 1.7 illustrates the distinction between the vertical trajectory of a stone thrown upward and the graph of its height as a function of time.
- Figure 1.13 illustrates the ideas of symmetry about the *y*-axis, *x*-axis, and origin.
- Figures 1.14–1.16 display symmetric and non-symmetric functions.

Connections

- Exercises 57–70 and Exercises 89–92 ask students to compute the difference quotients $\frac{f(x)-f(a)}{x-a}$ and $\frac{f(x+h)-f(x)}{h}$ for various functions; these exercises will prepare students for upcoming limit and derivative computations.
- The notion of symmetry is used when graphing functions in Cartesian coordinates (Section 4.3) and polar coordinates (Section 10.2), and when evaluating definite integrals (Section 5.4).

Additional Activities

Suggested Guided Projects: *Problem-solving skills* and *Constant rate problems*

- *Problem-solving skills* is a guided project that can be used in a variety of ways.
- *Constant rate problems* continues the theme of problem solving. Assign a few of the brain teasers found therein to develop critical thinking skills, and to introduce students to Pólya's four-step method to problem solving. The exercises in these guided projects could also be used as icebreakers in the initial days of class, as they are sure to generate discussion. Finally, you could help your students get to know one another by asking them to work on a handful of exercises in groups of 2–4 students.

Name: ________________________ Section: ________________ Date: ______________

Section 1.1 Quick Quiz

Answer the following multiple choice questions by circling the correct response.

1. The domain of the function $f(x) = x^3 - x$ is

(a) $\{x : x \geq 0\}$. (b) $\{x : x < 0\}$. (c) $\{x : -\infty < x < \infty\}$.

2. The range of $y = f(x) = x^3 - x$ is

(a) $\{y : y \geq 1\}$. (b) $\{y : -\infty < y < \infty\}$. (c) $\{y : y < 0\}$.

3. The domain of the function $f(x) = \sqrt{9 - x^2}$ is

(a) $\{x : |x| > 3\}$. (b) $\{x : |x| < 3\}$. (c) $\{x : |x| \leq 3\}$.

4. The graph of the function $f(x) = -3x + 8$ is

(a) a line with slope 8 and y-intercept (0, –3).
(b) a line with slope 3 and y-intercept (0, 8).
(c) a line with slope –3 and y-intercept (0, 8).

5. Suppose the height of a soccer ball that is kicked from the ground at time $t = 0$ is $h(t) = -5t^2 + 60t$ (in feet). An appropriate domain for this problem is

(a) $\{t : 0 \leq t \leq 12\}$. (b) $\{t : 0 \leq t \leq 6\}$. (c) $\{t : -\infty < t < \infty\}$.

6. If $f(x) = \sqrt{x}$ and $g(x) = 1/(x+1)$, then $f(g(x))$ is

(a) $\dfrac{1}{\sqrt{x+1}}$. (b) $\dfrac{1}{\sqrt{x}+1}$. (c) $\sqrt{x+1}$.

7. If $f(x) = x^3 - x$ and $g(x) = x^{-2}$, then $g(f(x))$ is

(a) $x^{-6} - x^{-2}$. (b) $(x^3 - x)^{-2}$. (c) $x^6 - x^2$.

8. With $f(x) = \sqrt{x}$ and $g(x) = 4 - x^2$, the function $f \circ g$ is defined for

(a) all real numbers. (b) $\{x : |x| \geq 2\}$. (c) $\{x : |x| \leq 2\}$.

9. Suppose $f(x) = 3x^2 - 2$. When simplified, the difference quotient $\dfrac{f(x+h) - f(x)}{h}$ is equal to

(a) $6x + 3h - 4$. (b) 1. (c) $6x + 3h$.

10. Suppose $f(x) = -\dfrac{3}{x}$. When simplified, the difference quotient $\dfrac{f(x) - f(a)}{x - a}$ is equal to

(a) 0. (b) $\dfrac{3}{ax}$. (c) –3.

11. The function $h(x) = x^4 - 3x + 1$

(a) is even. (b) is odd. (c) has no symmetry.

12. The curve described by the equation $x^2 - y^4 = 1$ is symmetric about the

(a) x-axis only. (b) y-axis only. (c) origin.

Section 1.2 Representing Functions

Overview

We introduce the full catalog of functions that will be encountered in calculus, and present four ways to represent a function: through formulas, graphs, tables, and words.

Lecture Support Notes

- Run through the gamut of standard functions and provide representative graphs for each family of functions. This section covers only the graphs of polynomial, algebraic, and rational functions—logarithmic, exponential, and trigonometric functions are featured in the following sections—and we keep things simple at this stage (e.g. power functions instead of polynomial functions).
- Cover piecewise functions, which are used repeatedly in the next chapter (limits). Include a piecewise definition of the absolute value function, another fact that will be used frequently in upcoming material.
- Let your students know where you stand regarding the question of technology and graphing: To what degree will they need to be able to produce graphs by hand? Whatever your policy is, it's probably best to ask your students to become familiar with at least the basic shapes of the standard functions.
- Reviewing transformations of graphs is important, as this topic gives students the tools needed to quickly visualize more complicated functions.
- Examples 6 and 7 provide a preview of the derivative and integral. Although the main goal of the chapter is to review algebra and trigonometry, these examples give an early introduction to slope functions and area functions, and they illustrate how the behavior of a function can be described with words.

Interactive Figures

- Figures 1.24–1.25 display $y = x^n$ for n even and n odd, respectively.
- Figures 1.26–1.27 display $y = x^{1/n}$ for n even and n odd, respectively.
- Figure 1.28 zooms in and out on a particular rational function.
- Figure 1.32 illustrates how the graph of g, which is a slope function for the function f, is related to the graph of f.
- Figures 1.35–1.36 illustrate how an area function A is generated by a given function f.
- Figures 1.37–1.42 display shifts and scalings in the x- and y-directions. Figures 1.43 and 1.44 display similar shifts and scalings in a parabola and absolute value function, respectively.

Connections

- Examples 6 and 7, and their related Exercises 35–42, are included as prelude to big ideas on the horizon. Exercises 80–81 and 85–87 are also designed with future chapters in mind.
- The theme of investigating the properties of the standard families of functions (polynomials, rational functions, algebraic functions, etc.) continues throughout the text. We first learn how to differentiate each family of functions (Chapter 3), and then later learn how to integrate each family (Section 4.9, Chapter 5, and Chapter 7). This program is repeated in multivariable calculus (in fact, several times over, as we learn how to differentiate and integrate vector-valued functions, followed by multivariable functions, and finally vector fields).

Additional Activities

Suggested Guided Projects: *Functions in action I, Functions in action II* and *Supply and demand*

- *Functions in action I and II* are guided projects that explore the behavior of functions in an applied setting. Each mini-project provides a detailed and compelling look into how functions are used to model various phenomena in a variety of disciplines (biology, meteorology, physics, economics, and general interest—see *Chasing a dog*—to name a few).
- *Supply and demand* is another guided project along these lines that is devoted to illustrating principles in economics with graphs.

Name: ______________________ Section: ______________ Date: ____________

Section 1.2 Quick Quiz

Answer the following multiple choice questions by circling the correct response.

1. The function $f(x)=\dfrac{x+1}{x-2}$ is a

(a) polynomial. (b) rational function. (c) transcendental function.

2. The function $f(x)=2x^{10}-3x^2$ is a

(a) polynomial. (b) trigonometric function. (c) transcendental function.

3. The function $f(x)=\sqrt{x-1}-3x^{-2}$ is

(a) a polynomial. (b) an algebraic function. (c) a rational function.

4. The graph of $f(x)=2x^5$

(a) lies in the first and second quadrants.

(b) lies in the second and fourth quadrants.

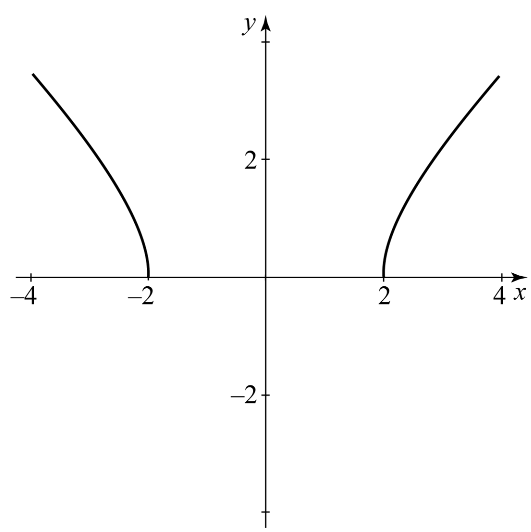
(c) has a point for every real number *x*.

5. The graph to the right best represents the function

(a) $f(x)=\sqrt{4-x^2}$.

(b) $f(x)=\dfrac{1}{x^2-4}$.

(c) $f(x)=\sqrt{x^2-4}$.



6. The data in the table below is best represented by the function

(a) $f(x)=\sqrt{x}+1$. (b) $f(x)=x^2-3$. (c) $f(x)=x^2-x$.

x	–2	–1	1	3	4	6
$f(x)$	1	–2	–2	6	13	33

7. Suppose your car gets exactly 30 miles per gallon of gasoline and you start driving with 15 gallons in the tank. The number of gallons of gasoline left in the tank after driving x miles is given by the function

(a) $g(x) = 30 - 15x$.   (b) $g(x) = 15 - 30x$.   (c) $g(x) = 15 - \frac{x}{30}$.

8. The value of $f(0)$ for the piecewise linear function $f(x) = \begin{cases} 2x+1 & \text{if } x \le 0 \\ -x & \text{if } x > 0 \end{cases}$ is

(a) 1.   (b) 0.   (c) undefined.

9. The function that gives the slope of $f(x) = \begin{cases} 2x+1 & \text{if } x \le 0 \\ -x & \text{if } x > 0 \end{cases}$ is

(a) $g(x) = \begin{cases} 2 & \text{if } x < 0 \\ 1 & \text{if } x > 0. \end{cases}$ (b) $g(x) = \begin{cases} 2 & \text{if } x < 0 \\ -1 & \text{if } x > 0. \end{cases}$ (c) $g(x) = \begin{cases} 1 & \text{if } x < 0 \\ 0 & \text{if } x > 0. \end{cases}$

10. The graph of $g(x) = (x-2)^2 + 3$ is obtained by shifting the graph of $f(x) = x^2$

(a) left 2 units and up 3 units.   (b) right 2 units and up 3 units.   (c) right 3 units and up 2 units.

11. The graph of $y = f(3x)$ is

(a) the graph of $y = f(x)$ compressed horizontally by a factor of 3.

(b) the graph of $y = f(x)$ stretched horizontally by a factor of 3.

(c) the graph of $y = f(x)$ shifted horizontally by 3 units.

12. The graph of $y = 3(x-2)^4$ is the graph of $y = x^4$

(a) stretched vertically by a factor of 3 and stretched horizontally by a factor of 2.

(b) stretched vertically by a factor of 3 and shifted horizontally 2 units to the right.

(c) stretched vertically by a factor of 2 and compressed horizontally by a factor of 2.

Section 1.3 Inverse, Exponential, and Logarithmic Functions

Overview

Exponential and logarithmic functions are explored, linked by a study of inverse functions.

Lecture Support Notes

- Introduce general exponential functions, followed by the natural exponential function, along with their graphs and properties. Note that at this point, we don't have the tools needed to buttress the claim that b^x is defined for all real numbers.
- Give the definition of an inverse function, state conditions under which a function is invertible (one-to-one; horizontal line test), and cover Theorem 1.1. If you want to present the exponential and logarithmic functions together, there's no harm done in beginning your lecture with a discussion of inverse functions.
- Show how to find the formula for the inverse of a function, and explain that the reversal of the roles played by x and y implies that the graph of a function and its inverse are reflections of one another over the line $y = x$.
- For functions that do not pass the horizontal line test, describe how the domain may be restricted to produce an invertible function (Example 3b, which is a continuation of Example 2).
- Introduce the general and natural logarithmic functions as the inverses of their exponential counterparts. Develop the properties of logarithmic functions based on the inverse relationship they have with exponential functions.
- Review the method of solution of exponential equations (Example 5).
- Present the change of base rules for both exponential and logarithmic expressions. The formula $b^x = e^{x \ln b}$ is used frequently in future work.

Interactive Figures

- Figure 1.45 displays $y = b^x$ for $b > 1$.
- Figure 1.46 displays $y = b^x$ for $0 < b < 1$.
- Figure 1.47 shows how the slope of the line tangent to $y = b^x$ at (0,1) varies with b.
- Figure 1.49 displays the horizontal line test for one-to-one functions.
- Figure 1.50 displays restricting the domain of a parabola, thereby making it one-to-one.
- Figure 1.52 illustrates the actions of a function f and its inverse f^{-1}.
- Figures 1.55–1.56 illustrate the graphical relationship between $y = x$, f, and f^{-1}.
- Figure 1.58 displays $y = b^x$ and $\log_b x$ for $b > 1$.
- Figure 1.59 plots $\log_b x$ for various values of b.

Connections

In this section, we make the claim that b^x is defined for all real numbers, and that $\log_b x$ is defined on $(0, \infty)$. The mathematical machinery necessary to back these claims won't arrive until Chapter 6. If you intend to cover Section 6.8, you might plant the seed of motivation for that material by noting it's a stretch (at this point in the term) to maintain that expressions such as $2^{\sqrt{3}}$ are well defined. We also note in the text that e^x, among all exponential functions, has the special property that the slope of its tangent at $x = 0$ is 1. This fact is presented again in Section 3.3 in order to find the derivative of e^x.

Additional Activities

Suggested Guided Project: *Acid, noise, and earthquakes* is a guided project that looks at logarithmic scales in the applied sciences.

Name: ______________________________ Section: __________________ Date: ______________

Section 1.3 Quick Quiz

Answer the following multiple choice questions by circling the correct response.

1. Simplify $\log_{10}(1000x^8)$.

(a) $80 \ \log_{10} x$ (b) $3 + \log_{10} x$ (c) $3 + 8\log_{10} x$

2. The solutions of $4^{x^2} = 8$ are

(a) $x = \pm 2$. (b) $x = \pm\sqrt{3/2}$. (c) $x = \pm\sqrt{2/3}$.

3. With any $b > 0$, the solution of $\log_b(x^2 - 2) = \log_b x$ is

(a) $x = 2$. (b) $x = 2$ and $x = -1$. (c) $x = -2$ and $x = 2$.

4. If $y = 3^x$, then

(a) $x = 3^y$. (b) $x = \log_3 y$. (c) $y = \log_3 x$.

5. If $p = \log_2 q$, then

(a) $q = \log_2 p$. (b) $p = 2^q$. (c) $q = 2^p$.

6. The domain of the function $f(x) = \log_8(x^2 - 1)$ is

(a) $\{x: |x| > 1\}$. (b) $\{x$: x is a real number$\}$. (c) $\{x: x > 0\}$.

7. The domain of the function $f(x) = 2^{x+1}$ is

(a) $\{x: x > 0\}$. (b) $\{x$: x is a real number$\}$. (c) $\{x: x > -1\}$.

8. The horizontal line $y = 3$ intersects the graph of $f(x) = x^2 + 1$

(a) never. (b) once. (c) twice.

9. On which of the following intervals does $f(x) = |x - 1|$ have an inverse?

(a) $x \geq 0$. (b) $x \geq -1$. (c) $x \geq 1$.

10. If f has a unique inverse and $f(2) = 4$, then

(a) $f(4) = 2$. (b) $f^{-1}(2) = 4$. (c) $f^{-1}(4) = 2$.

11. Suppose you graph the functions f and f^{-1} and you know that $y_0 = f(x_0)$. Then

(a) (x_0, y_0) is on the graph of $y = f^{-1}(x)$. (b) (y_0, x_0) is on the graph of $y = f^{-1}(x)$.

(c) (y_0, x_0) is on the graph of $y = f(x)$.

Section 1.4 Trigonometric Functions and Their Inverses

Overview

The trigonometric functions and their inverses are defined, and the graphs and properties of these functions are examined.

Lecture Support Notes

This section includes a lot of information, and some of it will be used only peripherally for the first several chapters of the text. Give solid coverage to the material you are certain to use early in the term, but be willing to breeze through the rest of it.

Take, for instance, the right-triangle relationship described in Example 4b. Students will need to understand this procedure when learning trigonometric substitutions (Section 7.4), but it won't appear until that time. We made the decision to collect the necessary prerequisite facts from trigonometry in this section so that students know where to look when confronted with difficulties in trigonometry; all of it will be used at some point in the book. However, you needn't feel obligated to cover every detail the first time around. Cherry-pick from the major ideas in the following bullet list to construct your lecture.

- Define the six trigonometric functions, using both the right-triangle definition and by treating them as circular functions (both are helpful interpretations in subsequent chapters).
- Ask students to memorize Figure 1.63—unless your teaching style does not require it—and explain how it is used to evaluate the trigonometric functions at standard angles.
- Cover the trigonometric identities assembled on page 41. These are the most frequently used identities in calculus. Additional important trigonometric identities are listed on the end papers of the text.
- Review the graphs of the trigonometric functions, and explain how a trigonometric graph can be translated using the methods of Section 1.3.
- Discuss the domain restrictions necessary for the definition of the inverse trigonometric functions. The three most important are $\sin^{-1} x$, $\tan^{-1} x$, and $\sec^{-1} x$, because these are the functions associated with trigonometric substitutions in integrals. Further, we focus on the derivatives of these three inverse functions in Section 3.10 (the derivatives of the other three are derived using trigonometric identities).
- If you have time, use inverse trigonometric functions to solve a trigonometric equation. Example 2 could be modified for this purpose, or you could present two solutions to part a), one where you appeal to the unit circle, and the other where the inverse sine function is employed.

Interactive Figures

- Figure 1.60 illustrates how changing the radius r and the angle θ impacts the arc length s.
- Figure 1.62 illustrates the relationship between the right triangle and circular definitions of the trigonometric functions.
- Figure 1.63 shows the coordinates of the points on the unit circle for all the standard angles.
- Figures 1.64–1.65 show how to evaluate trigonometric functions on angles that lie outside the interval $[0, 2\pi]$.
- Figures 1.66–1.67 display $\sin\theta$, $\csc\theta$, and $\cos\theta$, $\sec\theta$, and $\tan\theta$, $\cot\theta$.
- Figure 1.68 illustrates how the graph of $y = A\sin\left(B(\theta - C)\right) + D$ changes when the parameters A, B, C, and D are varied.
- Figure 1.69 illustrates how a daylight function varies over one year.
- Figure 1.71 displays how to restrict the domain of sine and cosine so that they are one-to-one.
- Figures 1.72–1.73 display the domain and range of $\sin x$, $\sin^{-1} x$, and $\cos x$, $\cos^{-1} x$, respectively.
- Figures 1.77–1.80 plot $\tan x$, $\cot x$, $\sec x$, $\csc x$, and their inverse functions.

Connections

- Use Figure 1.63 in later chapters anytime you want to refer to the unit circle for the purposes of evaluating a trigonometric function.
- As noted previously, all the material reviewed in this section (indeed, in the entire chapter) will appear in subsequent chapters of the book, so some level of coverage is essential.

Additional Activities

Suggested Guided Projects: *Phase and amplitude* and *Atmospheric* CO_2

- *Phase and amplitude* focuses on properties of the sine and cosine functions; in particular, it explains how the sum of two sine or cosine waves of different amplitude combine to produce a single sine (or cosine) wave, which can be expressed in phase-amplitude form.
- Data on the concentration of carbon dioxide in the atmosphere has been gathered at points around the globe since the mid 1950s; the most famous CO_2 time line comes from the observatory at Mauna Loa, Hawaii. *Atmospheric* CO_2 is a guided project that gives students the opportunity to create a mathematical model that fits the Mauna Loa CO_2 data set by superimposing the oscillations of a trigonometric function onto a linear or exponential function.

Name: ______________________ Section: ______________ Date: ____________

Section 1.4 Quick Quiz

Answer the following multiple choice questions by circling the correct response.

1. The radian measure of an angle corresponding to one-eighth of a circle is

(a) $\pi/8$. (b) $\pi/4$. (c) $\pi/6$.

2. The value of tan $(3\pi/4)$ is

(a) $-\sqrt{3}$. (b)1. (c) -1.

3. Among the zeros of the function $y = \cos 2\theta$ are

(a) 0, $\pm\pi/2$. (b) $\pm\pi$. (c) $\pm\pi/4$.

4. The function $y = \sec(x/2)$ is undefined at

(a) $x = \pm\pi$. (b) $x = 0, \pi/2$. (c) $x = 0, \pm 2\pi$.

5. Among the solutions of $\sin 2x = \cos 2x$ are

(a) $\pi/4$. (b) $\pi/8$. (c) $\pi/3$.

6. The function $f(x) = \cos(\pi x/4)$ has a period of

(a) 4. (b) 8. (c) 16.

7. The maximum value of the function $f(x) = 4\cos x + 1$ is

(a) 4. (b) 6. (c) 5.

8. The value of $\cos^{-1}(\cos(\pi/6))$ is

(a) 1/2. (b) $\pi/6$. (c) $\sqrt{3}/2$.

9. The value of $\tan^{-1}(\tan(9\pi/4))$ is

(a) $9\pi/4$. (b) 1. (c) $\pi/4$.

10. The value of $\cos(\sin^{-1} x)$ is

(a) $\sqrt{1-x^2}$. (b) $1/\sqrt{1-x^2}$. (c) x.

11. The range of $f(x) = \sin^{-1} x$ is

(a) $[-\pi/2, \pi/2]$. (b) all real numbers. (c) $[-1, 1]$.

12. The domain of $f(x) = \cos^{-1} x$ is

(a) $[-\pi/2, \pi/2]$. (b) all real numbers. (c) $[-1, 1]$.

Chapter 1 Key Terms and Concepts

Functions (p. 1)
Domain and range (p. 1)
Vertical line test (p. 2)
Composite functions (p. 3)
Secant lines and the difference quotient (p. 6)
Symmetry in graphs and functions (p. 8)
Representing functions (formulas, graphs, tables, words) (p. 12–17)
Linear and piecewise linear functions (p. 14–15)
Power and root functions (p. 15–16)
Rational functions (p. 16)
Transformations of functions and graphs (p. 19)
Exponential functions (p. 26)
Inverse functions (p. 28)
One-to-one functions and the horizontal line test (p. 29)
Finding inverse functions (p. 30)
Graphing inverse functions (p. 32)
Logarithmic and exponential functions to other bases (p. 32)
Change of base rules (p. 34)
Trigonometric functions (p. 40)
Properties and identities of trigonometric functions (p. 41)
Graphs of trigonometric functions (p. 42)
Amplitude and period (p. 43)
Inverse sine and cosine (p. 44)
Other inverse trigonometric functions (p. 46)

Chapter 1 Review Questions

1. Express the set $\{x : -2 < x \le 8\}$ using interval notation and draw it on the number line.

2. Describe the set $\{x : |x-3| < 4\}$ in words and draw it on the number line.

3. Describe the set $\{(x, y) : x^2 + (y-3)^2 < 4\}$ in words.

4. Use the terms *domain*, *range*, *independent variable*, and *dependent variable* to explain the concept of a function.

5. If $f(x) = x^3$ and $g(x) = \frac{1}{x}$, find $f(g(x))$, $g(f(x))$, and $g(g(x))$.

6. Give an example of a polynomial, a rational function, and an algebraic function.

7. What is the domain of the function $f(x) = \sqrt{4-x^2}$?

8. Describe the vertical line test. How does it detect functions?

9. What is a secant line? How is it related to a difference quotient?

10. What is the equation of the line that passes through $P(x_1, y_1)$ with slope m?

11. Explain what is meant by a piecewise linear function and explain how one might arise.

12. Why can't an entire circle be described by a single function?

13. What are the possible solution sets of the equation $x^2 + y^2 + Cx + Dy + E = 0$?

14. Draw a picture to show how a composite function takes a value of x and produces a value of $y = f(g(x))$.

15. Sketch an even function and a curve that is symmetric about the origin.

16. Explain in words how you would modify the graph of $y = f(x)$ to obtain the graph of $y = 3f(x-4) - 4$.

17. Explain how the six trigonometric functions are defined in terms of a point $P(x, y)$ on the unit circle.

18. How is the Pythagorean property for sine and cosine obtained from the Pythagorean theorem for right triangles?

19. What is the period and amplitude of the function $y = 3\sin(\theta / 3)$?

20. Explain the meaning of $\log_b x$.

21. Make a sketch of the graphs of $y = b^x$ and $y = \log_b x$ for $b > 1$.

22. Why must a function be one-to-one on an interval in order to have an inverse on that interval?

23. Sketch the graph of a function that is one-to-one on an interval $[a,b]$. Sketch the graph of its inverse on the same set of axes. Explain why the inverse function has the graph you sketched.

24. Explain the inverse relationship between the functions $f(x) = b^x$ and $f^{-1}(x) = \log_b x$.

25. How and why must the domain of the sine function be restricted in order to define the inverse sine function?

26. What are the domain and range of the inverse cosine function?

27. Use a right triangle to evaluate $\sin(\cos^{-1}(4/5))$.

Chapter 1 Test Bank Exercises

1–2. Algebra review Simplify or evaluate the following expressions without a calculator.

1. $(-27)^{4/3}$

2. $16^{-3/4}$

3–4. Solving equations

3. Solve $\frac{x^2-1}{x-1}=0$.

4. Solve $2x+3y=4$ and $4x-6y=20$ for x and y.

5–6. Composite functions and notation Let $f(x)=x^2-4$ and $F(x)=\frac{1}{x-3}$. Simplify or evaluate the following expressions.

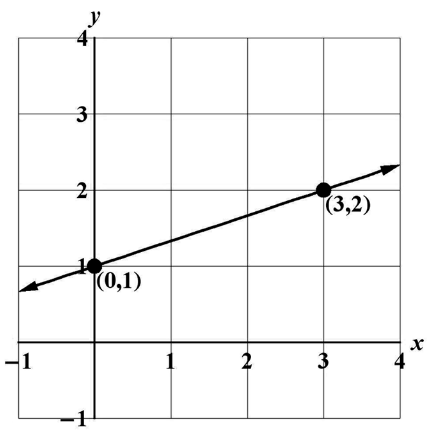
5. $\frac{F(x+h)-F(x)}{h}$

6. $F(f(x^2))$

7. More composite functions Let $g(x)=x^2-4$ and $F(x)=\sqrt{x}$. Determine $g \circ F$ and give its domain.

8–9. Graphs of functions Find the linear functions that correspond to the following graphs.

8.



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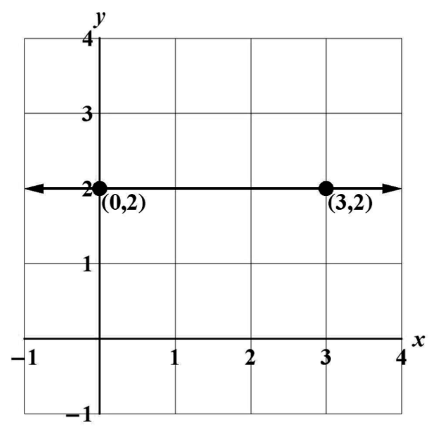
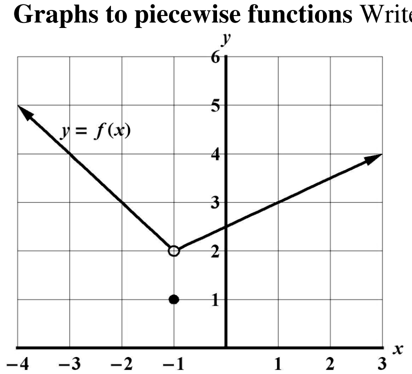


10. Graphs to piecewise functions Write a definition of the following function.



11. Intersection problem Find the point(s) of intersection between the parabola $y = x^2 + 3x - 4$ and the line $y + 4x + 16 = 0$.

12. Shifting and scaling Use analytical methods to graph $v(x) = 2x^2 - 4x + 6$. Then check your work with a graphing utility. Be sure to identify the original function on which the shifts and scalings are performed.

13–14. Finding inverse functions Determine whether the following functions are one-to-one on the given interval. If so, find the inverse function on that interval.

13. $f(x) = x^2 - 6x + 9; (-\infty, \infty)$

14. $f(x) = x^2 - 6x + 9;\ [3, \infty)$

15. Finding inverse functions

a. Find the inverse of $f(x) = \dfrac{x}{x+1}$ and write it in the form $y = f^{-1}(x)$.

b. Verify the relationships $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

16. Solving logarithmic equations Solve for x: $\log_3 x = -2$.

17–18. Properties of logarithms Assume $\log_b y = 0.56$ and $\log_b z = 0.83$ to evaluate the following expressions.

17. $\log_b \dfrac{1}{y}$

18. $\log_b \dfrac{z^3}{y^2}$

19–20. Solving Equations Without using a calculator, solve the following equations.

19. $2^{\cos x} = \dfrac{1}{2}$

20. $\log_{10}(\cos^2 x) = 0$

21. Changing bases Convert the following expression to the indicated base.

$\ln 6$ using $\log_6$

22–27. Evaluating trigonometric functions Evaluate the following expressions by drawing the unit circle and the appropriate right triangle. All angles are in radians.

22. $\sin \dfrac{7\pi}{4}$

23. $\cot \dfrac{7\pi}{3}$

24. $\sin \dfrac{\pi}{4}$

25. $\cot \dfrac{\pi}{2}$

26. $\sec \dfrac{\pi}{4}$

27. $\tan \dfrac{\pi}{3}$

28. Evaluating trigonometric functions Suppose $\sin\theta = -\dfrac{2}{3}$, where $\pi < \theta < \dfrac{3\pi}{2}$. Find $\cos\theta$ and $\tan\theta$. Be sure to sketch a picture of the angle with the terminal side in the proper quadrant.