HINTS FOR TEACHING FINITE MATHEMATICS AND CALCULUS WITH APPLICATIONS

Algebra Reference

This chapter is not as important for finite mathematics as it is for calculus. The instructor may cover Chapter R before Chapter 1 (the beginning of the finite mathematics part), before Chapter 10 (the beginning of the calculus part), as needed throughout any part of the course, or not at all. Use whichever method works best for your students. We refer to the chapter as a "Reference" rather than a "Review," and the regular page numbers don't begin until Chapter 1. We hope this will make your students less anxious if you don't cover this material.

Section 1.1

This section and the next may seem fairly basic to students who covered linear functions in high school. Some students have difficulty finding the equation of a line from two points. Emphasize that there is no point-point form.

Perpendicular lines are not used in future chapters and could be skipped if you are in a hurry.

Section 1.2

Linear functions are the only functions students learn about in this section, giving them a gentle introduction to functions. Review graphing lines using intercepts, especially horizontal lines, vertical lines, and lines through the origin.

Supply and demand provides the students' first experience with a mathematical model. Spend time developing both the economics and the mathematics involved.

Stress that for cost, revenue, and profit functions, x represents the number of units. For supply and demand functions, we use the economists' notation of q to represent the number of units.

Emphasize the difference between the profit earned on 100 units sold as opposed to the number of units that must be sold to produce a profit of \$100.

Section 1.3

The statistical functions on a calculator can greatly simplify these calculations, allowing more time for discussion and further examples. In this edition, we use "parallel presentation" to allow instructor choice on the extent technology is used. This section may be skipped if you are in a hurry, but your students can benefit from the realistic model and the additional work with equations of lines.

Chapter 2

The echelon method and the Gauss-Jordan method presented in the text are improved variations of the traditional methods. The "leading ones" are postponed until the last step, so as to avoid fractions and decimals. You may want to practice a few examples before presenting this method in class. We also present the traditional Gauss-Jordan method using a graphing calculator, for which keeping track of the fractions presents no difficulty.

Section 2.1

We have found it useful to spend less time on the echelon method and save the larger examples for the Gauss-Jordan method in the next section. Consequently, most of the exercises in this section involve only two equations. Use this section to introduce the concept of solving a system of linear equations and to show how a system can have one solution, no solutions, or an infinite number of solutions. Also use this section to show students how to solve applied exercises. Notice the application exercises in which a dependent system only has a finite number of solutions because the solution is restricted to nonnegative integers.

Emphasize the row notation as a way to keep track of and to check the problem solving process.

Stress the guidelines, found before Example 5, for solving an application problem.

Section 2.2

Shown on the next page is a comparison between the improved version and the traditional Gauss-Jordan method. Note the absence of tedious fractions in the improved version.

Solve:

$$\begin{aligned} 4x - 2y - 3z &= -23 \\ -4x + 3y + z &= 11 \\ 8x - 5y + 4z &= 6 \end{aligned}$$

New Method

$$\left[\begin{array}{rrr|r} 4 & -2 & -3 & -23 \\ -4 & 3 & 1 & 11 \\ 8 & -5 & 4 & 6 \end{array}\right]$$

$$\begin{array}{r} \\ R_1 + R_2 \to R_2 \\ -2R_1 + R_3 \to R_3 \end{array} \quad \left[\begin{array}{rrr|r} 4 & -2 & -3 & -23 \\ 0 & 1 & -2 & -12 \\ 0 & -1 & 10 & 52 \end{array}\right]$$

$$\begin{array}{r} R_1 + 2R_2 \to R_1 \\ \\ R_2 + R_3 \to R_3 \end{array} \quad \left[\begin{array}{rrr|r} 4 & 0 & -7 & -47 \\ 0 & 1 & -2 & -12 \\ 0 & 0 & 8 & 40 \end{array}\right]$$

$$\begin{array}{r} 7R_3 + 8R_1 \to R_1 \\ R_3 + 4R_2 \to R_2 \\ \\ \end{array} \quad \left[\begin{array}{rrr|r} 32 & 0 & 0 & -96 \\ 0 & 4 & 0 & -8 \\ 0 & 0 & 8 & 40 \end{array}\right]$$

$$\begin{array}{r} \frac{1}{32}R_1 \to R_1 \\ \frac{1}{4}R_2 \to R_2 \\ \frac{1}{8}R_3 \to R_3 \end{array} \quad \left[\begin{array}{rrr|r} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$(-3, -2, 5)$ is the solution.

Traditional Method

$$\left[\begin{array}{rrr|r} 4 & -2 & -3 & -23 \\ -4 & 3 & 1 & 11 \\ 8 & -5 & 4 & 6 \end{array}\right]$$

$$\begin{array}{r} \frac{1}{4}R_1 \to R_1 \\ \\ \\ \end{array} \quad \left[\begin{array}{rrr|r} 1 & -\frac{1}{2} & -\frac{3}{4} & -\frac{23}{4} \\ -4 & 3 & 1 & 11 \\ 8 & -5 & 4 & 6 \end{array}\right]$$

$$\begin{array}{r} \\ 4R_1 + R_2 \to R_2 \\ -8R_1 + R_3 \to R_3 \end{array} \quad \left[\begin{array}{rrr|r} 1 & -\frac{1}{2} & -\frac{3}{4} & -\frac{23}{4} \\ 0 & 1 & -2 & -12 \\ 0 & -1 & 10 & 52 \end{array}\right]$$

$$\begin{array}{r} \frac{1}{2}R_2 + R_1 \to R_1 \\ \\ R_2 + R_3 \to R_3 \end{array} \quad \left[\begin{array}{rrr|r} 1 & 0 & -\frac{7}{4} & -\frac{47}{4} \\ 0 & 1 & -2 & -12 \\ 0 & 0 & 8 & 40 \end{array}\right]$$

$$\begin{array}{r} \\ \\ \frac{1}{8}R_3 \to R_3 \end{array} \quad \left[\begin{array}{rrr|r} 1 & 0 & -\frac{7}{4} & -\frac{47}{4} \\ 0 & 1 & -2 & -12 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$$\begin{array}{r} \frac{7}{4}R_3 + R_1 \to R_1 \\ 2R_3 + R_2 \to R_2 \\ \\ \end{array} \quad \left[\begin{array}{rrr|r} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$(-3, -2, 5)$ is the solution.

By reworking a problem from Section 2.1 using the Gauss-Jordan method, students will see how closely this method parallels the echelon method given there.

Remind students to operate on the entire row. A common error is to forget the entry to the right of the vertical bar.

Section 2.3

Mention that, as in algebra, only like things can be added or subtracted. In this case, the like things are matrices having the same dimensions.

Section 2.4

Using the visual approach to matrix multiplication given after Example 2, students will have no trouble multiplying matrices. Most will eventually no longer need this tool.

Section 2.5

Explain that the technique used in finding the multiplicative inverse of a matrix is still the Gauss-Jordan method, now with more than one entry per row to the right of the vertical bar.

Students may be resistant to learning another method for solving a system of equations. Stress the advantage of using the inverse method to solve systems having the same matrix of coefficients. See Example 4. Point out that these systems can be found in many different fields of application. See Exercise 60.

Section 2.6

Discuss how the entries of A, the input-output matrix, could be determined. Stress the economic significance of the matrices A, D, X, and AX.

Section 3.1

Emphasize that the test point can be *any* point *not* on the boundary. Choose several points on either side of the boundary and on the boundary itself to illustrate this concept.

Students may fall into the habit of always choosing $(0, 0)$ as the test point. Do a couple of problems where $(0, 0)$ is not available for use as a test point.

Using a different color to shade each half plane for a system of inequalities will make their overlap easier to recognize.

Section 3.2

Use diagrams like Figures 11 and 12 to convince students of the believability of the corner point theorem. Emphasize that a corner point *must* be a point in the feasible region. Also, stress that not all corner points can be found by inspection. Some require solving a system of two linear equations. Have students note the equation of the boundary line next to its graph, so they will know which equations to solve as a system.

Section 3.3

Review the guidelines for setting up an applied problem (Section 2.1) to determine the objective function and all necessary constraints.

Students find those constraints comparing two unknown quantities the most difficult. See Exercise 12 for an example of this type of constraint.

Section 4.1

The simplex method in this chapter is modified from the traditional method along the lines of the Gauss-Jordan method in Chapter 2, eliminating tedious fractions until the last step. The notation of s instead of x for slack and surplus variables will help students remember which variables are the originals and which are slack or surplus variables.

Note the horizontal line in the simplex tableau to separate the constraints from the objective function.

Students may need several examples to be able to pick out the basic variables and to find the basic feasible solution from a matrix.

Section 4.2

Vocabulary is extremely important in this section. An understanding of the terms basic variables, basic feasible solution, indicators, and pivots is a necessity.

Remind students that the simplex method stated before Example 1 works only for problems in standard maximum form.

Example 1 is extremely important because it connects the two methods of solving a linear programming problem. You may want to do a similar example in class. Emphasize the advantages of the simplex method, especially for larger problems.

Section 4.3

If you are in a hurry, either Section 4.3 or 4.4 can be skipped. Section 4.3 is needed if you wish to cover Section 11.3 on game theory and linear programming. If you choose to cover this section and skip Section 4.4, your students will only know how to solve standard minimization and maximization problems, but they at least they will see the profound and amazing theorem of duality. Notice in Exercises 18 and 19 that for maximization problems, shadow costs become shadow profits.

Provide numerous examples for reading the optimal solution from the last row of the final tableau of the dual problem.

Section 4.4

The usual method for solving nonstandard problems (those with mixed constraints) is the two-phase method, which is somewhat complicated for students at this level. We use a modification of this method which students should find simpler.

Stress that slack variables are used for $\leq$ constraints, while surplus variables are used for $\geq$ constraints. Artificial variables only need to be covered if you want to solve constraints with an $=$. Even then, they can be avoided by replacing each $=$ constraint with two inequalities, one with $\leq$ and one with $\geq$.

Emphasize that to use the simplex method to find the optimal feasible solution, one must start with a feasible solution.

In Step 5 of the box "Solving a Nonstandard Problem," our choice of the positive entry that is farthest to the left is arbitrary. If your students choose a different column, they may still come up with the correct answer, and it might even require fewer steps.

Remind students to convert from z to w as the last step in solving a minimization problem.

Chapter 5

The chapter on mathematics of finance does not depend on earlier chapters and may be covered at any time.

Students may feel overwhelmed by the number of formulas presented in Chapter 5. Guidelines for choosing the appropriate formula can be found at the end of the chapter. This summary may be referred to throughout Chapter 5.

Chapter 5 requires numerous financial calculations. Make sure students are familiar with their calculators. The financial features of the TI-83/84 Plus make calculations easy.

Section 5.1

Interest is the key concept in Chapter 5. It is important that students understand that interest is the cost of borrowing money (or the reward for lending money). Both simple and compound interest are covered in the first section.

Point out that as the frequency of compounding increases, so does the amount of interest earned. Also note, however, that this increase in interest gets smaller and smaller as the interest is compounded more frequently. See Exercises 63 and 64.

The effective rate of interest is a topic that students find most useful and interesting. Bring in advertisements for loans that hide the effective rate (the APR) in the fine print.

Chapter 5 is full of symbols and formulas. It is imperative that students become familiar with the notation and know which formula is appropriate for a given problem. A summarization of the formulas in Section 5.1 is found at the end of the section.

Section 5.2

This section starts with an introduction to geometric sequences, which lays the groundwork for developing the future value formula as the sum of a geometric sequence.

Section 5.3

Make sure students understand that the present value formula presented here is for an ordinary annuity only.

Many students have had experience with amortization. Illustrate this topic using examples with present day interest rates. Students may bring in personal examples that may be used in class.

Chapter 6

This chapter leads students toward the construction of proofs in Section 5, with quantifiers briefly introduced in Section 6. Proofs are more difficult than truth tables, but they are also far more important. In this edition we use meaningful variables, such as d for "Django is a good dog," rather than generic variables such as p and q. We find (and our students do too) that this makes it easier to keep track of which variable stands for each statement.

This is a nice chapter to cover before sets (Chapter 7), because many of the same ideas appear in both contexts. You should point out the parallels to the students whenever possible.

Section 6.1

The chapter starts with fairly easy material, but the statement "Neither p nor q" can cause trouble. Notice that we introduce the basic truth tables in this section, and the material on the quantifiers "For all" and "There exists" appear in Section 6.

Section 6.2

Students usually find truth tables fun. The alternative method presented in Example 5 helps alleviate any tedium.

Sections 6.3 and 6.4

The conditional is probably the most challenging logical operator, perhaps because its usage in mathematical language is just different enough from that of common language to cause confusion. We have found that even after students have studied logic, they still give erroneous answers to tests of reason such as those in Exercises 52 and 53 of Section 6.4. The common translations of $p \to q$ given before Example 1 of Section 6.4 are particularly troublesome. Don't assume your students have mastered this material until you have firm evidence.

Section 6.5

This section is the culmination of the first four sections. The two most important skills are showing an argument is invalid by counterexample, and showing an argument is valid by proof. Look carefully at Examples 5 through 8. This is the most difficult material in the chapter, so students need a lot of practice to master these ideas. The payoff is worth it when students learn to create a proof. The puzzles by Lewis Carroll in Example 6 and Exercises 38–43 are fun.

Section 6.6

Some students won't believe Euler is pronounced "oiler." The material on quantifiers appears here. This is just an introduction; we do not try to teach proofs using quantifiers except by Euler diagrams.

Chapter 7

The material on probability is arranged so that if you are in a hurry to get to Markov chains, you can skip Section 7.6 and all of Chapters 8 and 9.

Section 7.1

Manipulatives are quite useful in this chapter, especially a deck of cards. Further, set brackets may be modeled as a box and the elements of the set as objects inside the box.

Stress the key word for each set operation: "not" for complement; "and" for intersection; "or" for union.

Section 7.2

Mention the order of set operations: If parentheses are present, simplify within them in the following order:

1) Take all complements.
2) Take the unions or intersections in the order they occur from left to right.

If no parentheses are present, start with 1).

To solve a survey problem, students must first be able to identify what type of object belongs to a certain region before they can determine how many objects belong to that region.

Have students explore the union rule for sets by determining the number of cards that are red or a king in their decks. Compare this problem with the problem of determining how many cards are fives or sevens (disjoint sets).

Section 7.3

Students need to be able to identify the experiment, the number of trials, the sample space, and the event in each probability problem.

Illustrate the basic probability principle using numerous examples with manipulatives.

Section 7.4

Redo the examples used to explore the union rule for sets to explore the related union rule for probability.

The complement rule is most useful for problems that contain statements of the form greater than, less than, etc.

Section 7.5

Sometimes independent events can be thought of as events that have the same sample space. For example, when two cards are drawn one at a time with replacement, both draws have a sample space consisting of all 52 cards. If these cards are drawn, instead, without replacement, the sample space for the second card has been reduced to 51 cards. Emphasize that the notation $P(A|B)$ reminds us how the sample space was reduced.

Section 7.6

Point out that trying to calculate $P(F|E)$ directly is sometimes impossible, too expensive, or too inconvenient. Thus, there is a need for Bayes' theorem which allows for the indirect calculation of $P(F|E)$ using $P(E|F)$. If a tree diagram is employed, then Bayes' theorem can be stated as

$$P(F|E) = \frac{\text{the probability of the branch through } F \text{ and } E}{\text{the sum of the probabilities of all branches ending in } E}.$$

Point out that the branch in the numerator will also be one of the branches in the denominator.

Section 8.1

To use the multiplication principle, break down the problem (the task) into parts. Draw a blank for each part. Fill in each blank with the number of ways that part of the task can be completed. Finally, multiply these numbers to obtain the solution.

Permutations are a special case of the multiplication principle that does not allow for repetition.

Section 8.2

An additional way to determine whether to use combinations or permutations is as follows:

1) Give a label to each of the n objects.
2) Pick r objects from the n objects.
3) Rearrange the r objects.
4) If this rearrangement can be considered the same as the original arrangement, use combinations. If it is different, use permutations.

Section 8.3

This section combines the counting techniques of the previous two sections with the basic probability principle.

Section 8.4

Students often have difficulty dealing with the phrases "at least," "at most," "no more than," etc. Have the students work numerous examples that include these phrases.

Section 8.5

In this section, we complete the discussion of binomial probability from the previous section by giving the formula $E(x) = np$ for the expected value in binomial probability. Having students work out expected value in a binomial probability exercise by the definition and by this formula will increase their confidence in both.

Section 9.1

Warn students that the term "average" is ambiguous. Illustrate this concept using the average salary example following Example 6. Have students find the modal salary. Discuss the problems this ambiguity may cause.

Section 9.2

The square of the standard deviation is the variance, while the square root of the variance is the standard deviation. Students often get these confused.

Section 9.3

Note that the standard normal table used in this text is different from the table that is found in many statistics books. Call this to the attention of students. Some may be familiar with the other table.

Students may find it helpful to draw the nonstandard normal curve with x-values first, then convert to z-scores and draw the standard normal curve.

If your students have graphing calculators that give normal probability, they will have no need for the standard normal table in the back of the book. Their answers to exercises and examples, however, may be slightly different from ours, which were found using the table.

Section 9.4

When using the normal approximation to the binomial, students often have difficulty choosing the appropriate x-value(s) on the normal curve. Provide numerous examples to practice this technique.

Note: The chapters on Markov Chains and Game Theory are no longer included in this book. This is because few users of the book covered these chapters, but many wanted the new chapter on logic. These chapters are still included in *Finite Mathematics.* If you wish to cover these topics, you may prefer to have your students purchase the individual books *Finite Mathematics* and *Calculus with Applications* rather than use the combined book.

Chapter 10

Instructors sometimes go to either of two extremes in this chapter. Some feel that their students have already covered enough precalculus in high school or in previous courses, and consequently begin with Chapter 11. Unfortunately, if they are wrong, their students may do poorly. Other instructors spend so much time on Chapters 1, 10 and the algebra reference chapter that they have little time for calculus. Such a course should not be labeled as calculus. We recommend trying to strike a balance, which may still not make all your students happy. A few may complain that the review of algebra, functions, and graphs is too quick; such students should be sent to a more basic course. Those students who are familiar with this material may become lazy and develop habits that will hurt them later in the course. You may wish to assign a few challenging exercises to keep these students on their toes.

Section 10.1

After learning about linear functions in Chapter 1, students now learn about functions in general. This concept is critical for success in calculus. Unless sufficient time is devoted to this section, the results will become apparent later when students don't understand the derivative. One device that helps students distinguish $f(x+h)$ from $f(x)+h$ is to use a box in place of the letter x, as we do in this section after Example 4.

Section 10.2

This section combines the topics of quadratic functions and translation and reflection, with a minimal amount of material on completing the square. Our experience is that students graph quadratics most easily by first finding the y-intercept, then finding the x-intercepts when they exist (using factoring or the quadratic formula), and finding the vertex last by locating the point midway between the x-intercepts or, if the quadratic formula was used, by letting $x = -b/(2a)$.

Quadratics are among a small group of functions that can be analyzed completely with ease, so they are used throughout the text. On the other hand, the advent of graphing calculators has made ease of graphing less important, so we rely on quadratics less than in previous editions.

Some instructors pressed for time may choose to skip translations and reflections. But we have found that students who understand that the graph of $f(x) = 5 - \sqrt{4 - x}$ is essentially the same as the graph of $f(x) = \sqrt{x}$, just shifted and reflected, will have an easier time when using the derivative to graph functions. Since students are familiar with very few classes of functions at this point, it helps to work with functions defined solely by their graphs, such as Exercises 25–28.

Exercises 33–40 cover stretching and shrinking of graphs in the vertical and horizontal directions. Covering these exercises carefully will not only give students a better grasp of functions, but will help them later to interpret the chain rule.

Section 10.3

Graphing calculators have made point plotting of functions less important than before. Plotting points by hand should not be entirely neglected, however, because a small amount is helpful when using the derivative to graph functions.

The two main goals of this section are to have an understanding of what an n-th degree polynomial looks like, and to be able to find the asymptotes of a rational function. Students who master these ideas will be better prepared for the chapter on curve sketching.

Exercise 59 is the first of several in this chapter asking students to find what type of function best fits a set of data. (See also Section 2.4, Exercises 49 and 50, and the Review Exercises 92 and 105.) The class can easily get bogged down in these exercises, particularly if you decide to explore the regression features in a calculator such as the TI-83/84 Plus. But there is a powerful payoff in terms of mastery of functions for the student who succeeds at these exercises.

Section 10.4

Some instructors may prefer at this point to continue with Chapter 11 and to postpone discussion of the exponential and logarithmic functions until later. The overwhelming preference of instructors we surveyed, however, was to cover exponential and logarithmic functions early and then to use these functions throughout the rest of the course. Instructors who wish to postpone this material will also need to omit for now those examples and exercises in Sections 11.1–12.3 that refer to exponential and logarithmic functions.

Students typically have no problem with $f(x) = 2^x$, but the number e often remains a mystery. Like π, the number e is a transcendental number, but students have had years of schooling to get used to π. Have your students approximate e with a calculator, as the textbook does before the definition of e. Notice how we use compound interest to help students get a handle on this number.

Section 10.5

Logarithms are a very difficult topic for many students. It's easy to say that a logarithm is just an exponent, but the fact that it is the exponent to which one must raise the base to get the number whose logarithm we are calculating is a rather obtuse concept. Therefore, spend lots of time going over examples that can be done without a calculator, such as $\log_2 8$. Students will also tend to come up with many incorrect pseudoproperties of logarithms, similar in form to the properties of logarithms given in this section. Take as much time and patience as necessary in gently correcting the many errors students inevitably will make at first.

Even after receiving a thorough treatment of logarithms, some students will still be stumped when solving a problem such as Example 7. Some of these students can get the correct answer using trial-and-error. The instructor should take consolation in the fact that at least such a student understands exponentials better than the one who uses logarithms incorrectly to solve Example 7 and comes up with the nonsensical answer $t = -7.51$ without questioning whether this makes sense. Be sure to teach your students to question the reasonableness of their answers; this will help them catch their errors.

Section 10.6

This section gives students much needed practice with exponentials and logarithms, and the applications keep students interested and motivated. Instructors should keep this in mind and not worry about having students memorize formulas. We have removed the formulas for present value in this edition, having decided that it's better for students to just solve the compound amount formula for P. This reduces by two the number of formulas that students need to remember.

There is a summary of graphs of basic functions in the end-of-chapter review.

Section 11.1

This is the first section on calculus, and perhaps the most important, since limits are what really distinguish calculus from algebra. Students will have the best understanding of limits if they have studied them graphically (as in Exercises 5–12), numerically (as in Exercises 15–20), and analytically (as in Exercises 31–52). The graphing calculator is a powerful tool for studying limits. Notice in Example 9 (c) and (d) that we have modified the method of finding limits at infinity by dividing by the highest power of x in the denominator, which avoid the problem of division by 0.

Section 11.2

The section on continuity should be straightforward if students have mastered limits from the previous section.

Section 11.3

This section introduces the derivative, even though that term doesn't appear until the next section. In a class full of business and social science majors, an instructor may wish to place less emphasis on velocity, an approach more suited to physics majors. But we have found velocity to be the manifestation of the derivative that is most intuitive to all students, regardless of their major.

Instructors in a hurry can skip the material on estimating the instantaneous rate of change from a set of data, but it helps solidify students' understanding of the derivative by giving them one more point of view.

Section 11.4

Students who have learned differentiation formulas in high school usually want (and deserve) some explanation of why they need to learn to take derivatives using the definition. You might try explaining to your students that getting the right formula is not the only goal; graphing calculators can give derivatives numerically. The most important thing for students to learn is the concept of the derivative, which they don't learn if they only memorize differentiation formulas.

Zooming in on a function with a graphing calculator until the graph appears to be a straight line gives students a very concrete image of what the derivative means.

After students have learned the differentiation formulas, they may forget about the definition of derivative. We have found that if we want them to use the definition on a test, it is important to say so clearly and emphatically, or they will simply use the shortcut formulas.

Section 11.5

One way to get students to focus on the concept of the derivative, rather than the mechanics, is to emphasize graphical differentiation. We have therefore devoted an entire section to this topic. Graphical differentiation is difficult for many students because there are no formulas to rely on. One must thoroughly understand what's going on to do anything. On the other hand, we have seen students who are weak in algebra but who possess a good intuitive grasp of geometry find this topic quite simple.

Section 12.1

Students tend to learn these differentiation formulas fairly quickly. These and the formulas in the next few sections are included in a summary at the end of the chapter.

Section 12.2

The product and quotient rules are more difficult for students to keep straight than those of the previous section. People seem to remember these rules better if they use an incantation such as "The first times the derivative of the second, plus the second times the derivative of the first." Some instructors have argued that this formulation of the product rule doesn't generalize well to products of three or more functions, but that's not important at this level.

Some instructors allow their students to bring cards with formulas to the tests. This does not eliminate the need for students to understand the use of the formulas, but it does eliminate the anxiety students may have about forgetting a key formula under the pressure of an exam.

Section 12.3

No matter how many times an instructor cries out to his or her students, "Remember the chain rule!", many will still forget this rule at some time later in the course. But if a few more students remember the rule because the instructor reminds them so often, such reminders are worthwhile.

Section 12.4 and 12.5

In going through these sections, you may need to frequently refer to the rules of differentiation in the previous sections. You may also need to review the last three sections of Chapter 10.

Section 13.1 and 13.2

If students have understood Chapter 11, then the connection between the derivative, increasing and decreasing functions, and relative extrema should be obvious, and these sections should go quickly and smoothly.

Section 13.3

Students often confuse concave downward and upward with increasing and decreasing; carefully go over Figure 31 or the equivalent with your class.

Section 13.4

Graphing calculators have made curve sketching techniques less essential, but curve sketching is still one of the best ways to unify the various concepts introduced in this and the previous two chapters. Students should use graphing calculators to check their work.

Because this section is the culmination of many ideas, students often find it difficult and start to forget things they previously knew. For example, a student might state that a function is increasing on an interval and then draw it decreasing. The best solution seems to be lots of practice with immediate help and feedback from the instructor.

Students sometimes stumble over this topic because they use the rules for differentiation incorrectly, or because they make mistakes in algebra when simplifying. Exercises such as 35–39 are excellent for testing whether students really grasp the concepts.

Section 14.1

This section should not be conceptually difficult, but students need constant reminders to check the endpoints of an interval when finding the absolute maxima and minima.

Section 14.2

This section is one of the high points of the course. Some of the best applications of calculus involve maxima and minima. Notice that some exercises have a maximum or minimum at the endpoint of an interval, so students cannot ignore checking endpoints.

Almost everyone finds this material difficult because most people are not skilled at word problems. Remind your students that if they ever wonder whether mathematics is of any use, this section will show them.

Why are word problems so difficult? One theory is that word problems require the use of two different modes of thinking, which students are using simultaneously for the first time. People use words in daily life without difficulty, but when they study mathematics, they often turn off that part of their brain and begin thinking in a very formal, mechanical way. In word problems, both modes of thinking must be active. If and when the NCTM Standards become widely accepted in the schools, children will get more practice at an early age in such ways of thinking. Meanwhile, the steps for solving applied problems given in this book might make the process a little more straightforward, and hence achievable by the average student.

Section 14.3

This section continues the ideas of the previous one. The point of studying economic lot size should not be to apply Equation (3), but to learn how to apply calculus to solve such problems. We therefore urge you to cover Exercises 10–13, in which we vary the assumptions, so Equation (3) does not necessarily apply. In this edition, we have changed the presentation to be consistent with that of business texts.

The material on elasticity can be skipped, but it is an important application that should interest students who have studied even a little economics.

Section 14.4

There are two main reasons for covering implicit differentiation. First, it reinforces the chain rule. Second, it is needed for doing related rate problems. If you skip related rates due to lack of time, it is not essential to cover implicit differentiation, either.

Section 14.5

Related rate applications are less important than applied extrema problems, but they use some of the same skills in setting up word problems, and for that reason are worth covering. The best application exercises are under the heading "Physical Sciences," because those are the exercises in which no formula is given to the student; the student must construct a formula from the words. The geometrical formulas needed are kept to a minimum: the Pythagorean theorem, the area of a circle, the volume of a sphere, the volume of a cone, and the volume of a cylinder with a triangular cross section. Some instructors allow their students to use a card with such formulas on the exam. These formulas are summarized in a table in the back of the book.

Section 14.6

Differentials may be skipped by instructors in a hurry; you need not fear that this omission will hamper your students in the chapter on integration. The differentials used there are not the same as those used here, and the required techniques are easily picked up when integration by substitution is covered.

As in the previous edition, our presentation of differentials emphasizes linear approximation, a topic of considerable importance in mathematics.

Section 15.1

Students sometimes start to get differentiation and antidifferentiation confused when they reach this section. Some believe the antiderivative of x^{-2} is $(-1/3)x^{-3}$; after all, if n is -2, isn't $n + 1 = -3$? Carefully clarify this point.

Section 15.2

The main difficulty here is teaching students what to choose as u. The advice given before Example 3 should be helpful.

Section 15.3

Some instructors who are pressed for time go lightly over the topic of the area as the limit of the sum of rectangular areas. This is possible, but care should be taken that students don't lose track of what the definite integral represents. Also, a light treatment here lessens the excitement of the Fundamental Theorem of Calculus.

We have continued the trend of the previous edition in covering three ways of approximating a definite integral with rectangles: the right endpoint, the left endpoint, and the midpoint. The trapezoidal rule is briefly introduced here as the average of the left sum and the right sum.

Section 15.4

The Fundamental Theorem of Calculus should be one of the high points of the course. Make a big deal about how the theorem unifies these two separate topics of area as a limit of sums and the antiderivative.

When using substitution on a definite integral, the text recommends changing the limits and the variable of integration. (See Example 4 and the Caution which follows the example.) Some instructors prefer instead to have their students solve an indefinite integral, and then to evaluate the integral using the limits on x. One advantage of this method is that students don't have to remember to change the limits. This method also has two disadvantages. The first is that it takes slightly longer, since it requires changing the integral to u and then back to x. Second, it prevents students at this stage from solving problems such as $\int_0^{1/2} x\sqrt{1-16x^4}\,dx$, which can be solved using the substitution $u = 4x^2$ and the fact that the integral $\int_0^1 \sqrt{1-u^2}\,du$ represents the area of a quarter circle. This is one section in which we deliberately did not use more than one method of presentation, because this would inevitably lead to confusion in the minds of some students.

Section 15.5

This section gives more motivation to the topic of integration. Consumers' and producers' surplus are important, realistic applications. We have downplayed sketching the curves that bound the area under consideration. Such sketches take time and are not necessary in solving these problems. But they clarify what is happening and make it possible to avoid memorizing formulas. Using a graphing calculator to sketch the curves can be helpful.

Section 15.6

The ubiquity of computers and graphing calculators has made numerical integration more important. Rather than computing a definite integral by an integration technique, one can just as easily enter the function into a calculator and press the integration key. Point out to students that this is valuable when the function to be integrated is complicated. On the other hand, using the antiderivative makes it easier to see the effect of changing the upper limit, say, from 2 to 3, or for working with the definite integral when one or both limits are parameters, such as a and b, rather than numbers.

Simpson's rule is the most accurate of the simple integration formulas. To achieve greater accuracy, a more complicated method must be used. This is why, unlike the trapezoidal rule, Simpson's rule is actually used by mathematicians and engineers.

You may wish to give your students the programs for the trapezoidal rule and Simpson's rule in *The Graphing Calculator Manual* that is available with the text.

Section 16.1

Students usually find column integration simpler than traditional integration by parts. We show both methods to give instructors a choice, and also to emphasize that the two methods are equivalent. Column integration makes organizing the details simpler, but is no more mechanical than the traditional method, as some have mistakenly claimed. At Hofstra University, students even use this method when neither the instructor nor the book discuss it. They find out about it from other students, and so it has become an underground method. Some instructors feel that students will lose any theoretical understanding of what they are doing if they use this method. Our experience is that almost no students at this level have a theoretical understanding of what integration by parts is about, but the better students can at least master the mechanics. With column integration, almost all of the students master the mechanics.

Section 16.2 and 16.3

These two sections give more applications of integration. Coverage of either section is optional.

Section 16.4

Improper integration is not really an application of integration, but it makes further applications of integration possible.

Many mathematicians use shorthand notation such as the following:
$\int_0^\infty e^{-x}\,dx = -e^{-x}\big|_0^\infty = 0-(-1) = 1$. For students at this level, it may be best to avoid the shorthand notation.

Section 16.5

Differential equations of the form $dy/dx = f(x)$ are treated lightly in this section because they were already covered in the chapter on integration, although the terminology and notation were different then. Remind students that solving such differential equations is the same as antidifferentiation. The rest of the section is on separable differential equations. Students sometimes have trouble with this section because they have forgotten how to find an antiderivative, particularly one requiring substitution.

Section 17.1

The major difficulty students have with this section, and indeed with this entire chapter, is that they cannot visualize surfaces in 3-dimensional space, even though they live there. Fortunately, such visualization is not really necessary for doing the exercises in this chapter. A student who wants to explore what various surfaces look like can use any of the commercial or public domain computer programs available.

Section 17.2

Students who have mastered the differentiation techniques should have no difficulty with this section.

Section 17.3

This section corresponds to the section on applied extrema problems in the chapter on applications of the derivative, but with less emphasis on word problems. In Exercise 30, we revisit the topic of the least squares line, first covered in Section 1.3.

Section 17.4

Lagrange multipliers are an important application of calculus to economics. At some colleges, the business school is very insistent that the mathematics department cover this material.

Section 17.5

This section corresponds to the section on differentials in the chapter on applications of the derivative.

Section 17.6

Students who have trouble visualizing surfaces in 3-dimensional space are sometimes bothered by double integrals over variable regions. Instructors should assure such students that all they need to do is draw a good sketch of the region in the xy-plane, and not try to draw the volume in three dimensions.

Chapter 18

Probability is one of the best applications of calculus around. In fact, statistics instructors sometimes feel the temptation to start discussing the definite integral even when their students know no calculus. This chapter is just a brief introduction, but it covers some of the most important concepts, such as mean, variance, standard deviation, expected value, and probability as the area under the curve. The third section covers three of the most important continuous probability distributions: uniform, exponential, and normal.