

INSTRUCTOR'S MANUAL

to accompany

*Linear Algebra:
4th Edition*

Stephen H. Friedberg
Arnold J. Insel
Lawrence E. Spence

Illinois State University



PEARSON EDUCATION, UpperSaddle River, NJ 07458

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Vector Spaces

1.1 INTRODUCTION

2. (b) $x = (2, 4, 0) + t(-5, -10, 0)$ (d) $x = (-2, -1, 5) + t(5, 10, 2)$
 3. (b) $x = (3, -6, 7) + s(-5, 6, -11) + t(2, -3, -9)$
 (d) $x = (1, 1, 1) + s(4, 4, 4) + t(-7, 3, 1)$
 4. (0, 0)

1.2 VECTOR SPACES

2. $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
 4. (b) $\begin{pmatrix} 1 & -1 \\ 3 & -5 \\ 3 & 8 \end{pmatrix}$ (d) $\begin{pmatrix} 30 & -20 \\ -15 & 10 \\ -5 & -40 \end{pmatrix}$
 (f) $-x^3 + 7x^2 + 4$ (h) $3x^5 - 6x^3 + 12x + 6$
 5. $\begin{pmatrix} 8 & 3 & 1 \\ 3 & 0 & 0 \\ 3 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 9 & 1 & 4 \\ 3 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 17 & 4 & 5 \\ 6 & 0 & 0 \\ 4 & 1 & 0 \end{pmatrix}$
 16. Yes 18. No, (VS 1) fails. 19. No, (VS 8) fails.

1.3 SUBSPACES

2. (b) $\begin{pmatrix} 0 & 3 \\ 8 & 4 \\ -6 & 7 \end{pmatrix}$ (d) $\begin{pmatrix} 10 & 2 & -5 \\ 0 & -4 & 7 \\ -8 & 3 & 6 \end{pmatrix}$ (f) $\begin{pmatrix} -2 & 7 \\ 5 & 0 \\ 1 & 1 \\ 4 & -6 \end{pmatrix}$
 The trace is 12.
 (h) $\begin{pmatrix} -4 & 0 & 6 \\ 0 & 1 & -3 \\ 6 & -3 & 5 \end{pmatrix}$
 The trace is 2.
 8. (b) No (d) Yes (f) No
 9. $W_1 \cap W_3 = \{0\}$, $W_1 \cap W_4 = W_1$,
 $W_3 \cap W_4 = \{(a_1, a_2, a_3) \in \mathbb{R}^3 : a_1 = -11a_3 \text{ and } a_2 = -3a_3\}$

1.4 LINEAR COMBINATIONS AND SYSTEMS OF LINEAR EQUATIONS

2. (b) $(-2, -4, -3)$
 (d) $\{x_3(-8, 3, 1, 0) + (-16, 9, 0, 2) : x_3 \in R\}$
 (f) $(3, 4, -2)$
3. (a) $(-2, 0, 3) = 4(1, 3, 0) - 3(2, 4, -1)$
 (b) $(1, 2, -3) = 5(-3, 2, 1) + 8(2, -1, -1)$
 (d) No
 (f) $(-2, 2, 2) = 4(1, 2, -1) + 2(-3, -3, 3)$
4. (a) $x^3 - 3x + 5 = 3(x^3 + 2x^2 - x + 1) - 2(x^3 + 3x^2 - 1)$
 (b) No
 (c) $-2x^3 - 11x^2 + 3x + 2 = 4(x^3 - 2x^2 + 3x - 1) - 3(2x^3 + x^2 + 3x - 2)$
 (d) $x^3 + x^2 + 2x + 13 = -2(2x^3 - 3x^2 + 4x + 1) + 5(x^3 - x^2 + 2x + 3)$
 (f) No
5. (b) No (d) Yes (f) No (h) No
11. The span of $\{x\}$ is $\{0\}$ if $x = 0$ and is the line through the origin of R^3 in the direction of x if $x \neq 0$.
17. if W is finite

1.5 LINEAR DEPENDENCE AND LINEAR INDEPENDENCE

2. (b) Linearly independent (d) Linearly dependent
 (f) Linearly independent (h) Linearly independent
 (j) Linearly dependent
10. $(1, 0, 0), (0, 1, 0), (1, 1, 0)$

1.6 BASES AND DIMENSION

2. (b) Not a basis (d) Basis
3. (b) Basis (d) Basis
4. No, $\dim(P_3(R)) = 4$. 5. No, $\dim(R^3) = 3$.
8. $\{u_1, u_3, u_5, u_7\}$
10. (b) $12 - 3x$ (d) $-x^3 + 2x^2 + 4x - 5$
14. $\{(0, 1, 0, 0, 0), (0, 0, 0, 0, 1), (1, 0, 1, 0, 0), (1, 0, 0, 1, 0)\}$ and
 $\{(-1, 0, 0, 0, 1), (0, 1, 1, 1, 0)\}$; $\dim(W_1) = 4$ and $\dim(W_2) = 2$.
16. $\dim(W) = \frac{1}{2}n(n + 1)$

