

## TEST QUESTIONS - CHAPTER #2

### Short Answer Questions

1. State Pascal's law.

*Ans. A pressure applied at any point in a liquid at rest is transmitted equally and undiminished in all directions to every other point in the liquid.*

2. (T or F) The difference in pressure between any two points in still water is always equal to the product of the density of water and the difference in elevation between the two points.

*Ans. False – specific weight, not density.*

3. Gage pressure is defined as

- a) the pressure measured above atmospheric pressure.
- b) the pressure measured plus atmospheric pressure.
- c) the difference in pressure between two points.
- d) pressure expressed in terms of the height of a water column.

*Ans. (a) is true*

4. Some species of seals dive to depths of 400 m. Determine the pressure at that depth in  $N/m^2$  assuming sea water has a specific gravity of 1.03.

*Ans.  $P = \gamma \cdot h = (1.03)(9790 N/m^3)(400 m) = 4.03 \cdot 10^6 N/m^2$*

5. Pressure below the surface in still water (or hydrostatic pressure)

- a) is linearly related to depth.
- b) acts normal (perpendicular) to any solid surface.
- c) is related to the temperature of the fluid.
- d) at a given depth, will act equally in any direction.
- e) all of the above.
- f) (a) and (b) only.

*Ans. (e)*

6. (T or F) A single-reading manometer makes use of a reservoir of manometry fluid with a large cross sectional area so that pressure calculations are only based on one reading.

*Ans. True.*

7. What is an open manometer?

*Ans. A manometer is a pressure measurement device that utilizes fluids of known specific gravity and differences in fluid elevations. An open manometer has one end open to the air.*

8. (T or F) The total hydrostatic pressure force on any submerged plane surface is equal to the product of the surface area and the pressure acting at the center of pressure of the surface.

*Ans. False. The total hydrostatic pressure force on any submerged plane surface is equal to the product of the surface area and the pressure acting at the centroid of the plane surface.*

9. A surface of equal pressure requires all of the following except:
- points of equal pressure must be at the same elevation.
  - points of equal pressure must be in the same fluid.
  - points of equal pressure must be interconnected.
  - points of equal pressure must be at the interface of immiscible fluids.
- Ans. (d); points of equal pressure do not need to be at and interface of fluids.*

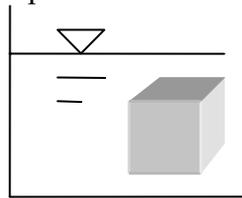
10. The center of pressure on inclined plane surfaces is:
- at the centroid.
  - is always above the centroid.
  - is always below the centroid.
  - is not related to the centroid.
- Ans. (c); points of equal pressure do not need to be at and interface of fluids.*

11. (T or F) The location of the centroid of a submerged plane area and the location where the resultant pressure force acts on that area are identical.
- Ans. False. The resultant force acts at the center of pressure.*

12. The equation for the determination of a hydrostatic force on a plane surface and its location are derived using all of the following concepts except
- integration of the pressure equation
  - moment of inertia concept
  - principle of moments
  - Newton's 2<sup>nd</sup> Law
- Ans. Since this deals with hydrostatics (i.e., no acceleration), (d) is the answer.*

13. The equation for the righting moment on a submerged body is  $M = W \cdot GM \cdot \sin \theta$ , where  $GM = MB - GB$  or  $MB + GB$ . Under what conditions is the sum used instead of the difference?
- Ans. Use the sum when the center of gravity is below the center of buoyancy.*

14. Given the submerged cube with area (A) on each face, derive the buoyant force on the cube if the depth (below the surface of the water) to the top of the cube is x and the depth to the bottom of the cube is y. Show all steps.



*Ans.  $F_{bottom} = P_{avg} \cdot A = \gamma \cdot y \cdot A$ ;  $F_{top} = P_{avg} \cdot A = \gamma \cdot x \cdot A$ ;  $F_{bottom} - F_{top} = \gamma \cdot (y - x) \cdot A = \gamma \cdot Vol$*

15. A 3 ft x 3 ft x 3 ft wooden cube (specific weight of 37 lb/ft<sup>3</sup>) floats in a tank of water. How much of the cube extends above the water surface? If the tank were pressurized to 2 atm (29.4 psi), how much of the cube would extend above the water surface? Explain.
- Ans.  $\sum F_y = 0$ ;  $W = B$ ;  $(37 \text{ lb/ft}^3)(3 \text{ ft})^3 = (62.3 \text{ lb/ft}^3)(3 \text{ ft})^2(y)$ ;  $y = 1.78 \text{ ft}$  Note: The draft does not change with pressure. That is, the added pressure on the top of the cube would be compensated by the increased pressure in the water under the cube.*

16. The derivation of the flotation stability equation utilizes which principles? Note: More than one answer is possible.

- a) moment of a force couple
- b) moment of inertia
- c) Newton's 2<sup>nd</sup> Law
- d) buoyancy

*Ans. It utilizes (a), (b), and (d).*

17. Rotational stability is a major concern in naval engineering. Draw the cross section of the hull of a ship and label the three important points (i.e., centers) which affect rotational stability.

*Ans. See Figure 2.16.*

18. A 4 m (length) by 3 m (width) by 2 m (height) homogeneous box floats with a draft 1.4 m. What is the distance between the center of buoyancy and the center of gravity?

*Ans. G is 1 m up from bottom and B is 0.7 m up from bottom. Thus, GB = 0.3 m.*

19. Determine the waterline moment of inertia about the width of a barge (i.e., used to assess stability from side to side about its width) if it is 30 m long, 12 m wide, and 8 m high?

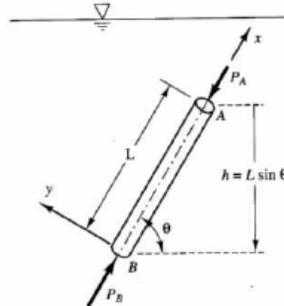
*Ans.  $I_o = (30m)(12m)^3/12 = 4320 m^4$*

20. (T or F) Flotation stability is dependent on the relative positions of the center of gravity and the center of buoyancy.

*Ans. True.*

## Problems

- Given the submerged, inclined rod with a top and bottom area ( $dA$ ), a length of  $L$ , and an angle of incline of  $\theta$ , derive an expression that relates the pressure on the top of the rod to the pressure on the bottom. Show all steps and define all variables.



*Ans.* Because the prism is at rest, all forces acting upon it must be in equilibrium in all directions. For the force components in the inclined direction, we may write

$$\sum F_x = P_A dA - P_B dA + \gamma L dA \sin \theta = 0$$

Note that  $L \cdot \sin \theta = h$  is the vertical elevation difference between the two points. The above equation reduces to

$$P_B - P_A = \gamma h$$

- A certain saltwater (S.G. = 1.03) fish does not survive well at an absolute pressure greater than 5 standard atmospheric pressures. How deep (in meters and feet) can the fish go before it experiences stress? (One atmospheric pressure is  $1.014 \times 10^5 \text{ N/m}^2$ .)

*Ans.* The absolute pressure includes atmospheric pressure. Therefore,

$$P_{\text{abs}} = P_{\text{atm}} + (\gamma_{\text{water}})(h) \leq 5(P_{\text{atm}}); \text{ thus}$$

$$h = 4(P_{\text{atm}})/\gamma_{\text{water}} = 4(1.014 \times 10^5 \text{ N/m}^2)/(1.03)(9790 \text{ N/m}^3)$$

$$\mathbf{h = 40.2 \text{ m (132 ft)}}$$

- A weight of 5,400 lbs is to be raised by a hydraulic jack. If the large piston has an area of 120 in.<sup>2</sup> and the small piston has an area of 2 in.<sup>2</sup>, what force must be applied through a lever having a mechanical advantage of 6 to 1?

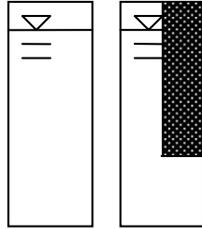
*Ans.* From Pascal's law, the pressure on the small piston is equal to the pressure on the large.

$$F_{\text{small}}/A_{\text{small}} = F_{\text{large}}/A_{\text{large}}$$

$$F_{\text{small}} = [(F_{\text{large}})(A_{\text{small}})]/(A_{\text{large}}) = [(5400 \text{ lb})(2 \text{ in}^2)]/(120 \text{ in}^2) = 90 \text{ lb}$$

$\therefore$  The **applied force** =  $90 \text{ lb}/6 = \mathbf{15 \text{ lb}}$  based on the mechanical advantage of the lever.

4. The two containers of water shown below have the same bottom areas (2 m by 2 m), the same depth of water (10 m), and are both open to the atmosphere. However, the L-shaped container on the right holds less fluid. Determine hydrostatic force (in kN), not the pressure, on the bottom of each container.



*Ans.* The pressure on the bottom of each container is identical, based on

$$P = (\gamma_{\text{water}})(h) = (9790 \text{ N/m}^3)(10 \text{ m}) = \mathbf{97.9 \text{ kN/m}^2}$$

The force on the bottom of each is identical as well, based on

$$F = P \cdot A = (97.9 \text{ kN/m}^2)(2 \text{ m})(2 \text{ m}) = \mathbf{391 \text{ kN}}$$

5. The gage pressure at the bottom of a water tank reads 30 mm of mercury (S.G. = 13.6). The tank is open to the atmosphere. Determine the water depth (in cm) above the gage. Find the equivalency in  $\text{N/m}^2$  of absolute pressure at  $20^\circ\text{C}$ .

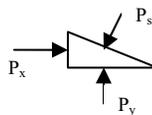
*Ans.* Since mercury has a specific gravity of 13.6, the water height can be found from

$$h_{\text{water}} = (h_{\text{Hg}})(SG_{\text{Hg}}) = (30 \text{ mm})(13.6) = 408 \text{ mm} = \mathbf{40.8 \text{ cm of water}}$$

$$P_{\text{abs}} = P_{\text{gage}} + P_{\text{atm}} = [(40.8 \text{ cm})\{(1 \text{ m})/(100 \text{ cm})\} + 10.3 \text{ m}] = 10.7 \text{ m of water}$$

$$P_{\text{abs}} = (10.7 \text{ m})(9790 \text{ N/m}^3) = \mathbf{1.05 \times 10^5 \text{ N/m}^2}$$

6. A triangle is submerged beneath the surface of a fluid. Three pressures ( $P_x$ ,  $P_y$ , and  $P_s$ ) act on the three tiny surfaces of length ( $\Delta y$ ,  $\Delta x$ , and  $\Delta s$ ). Prove that  $P_x = P_s$  and  $P_y = P_s$  (i.e., pressure is omni-directional) using principles of statics. (Note that  $P_x$  acts on  $\Delta y$ ,  $P_y$  acts on  $\Delta x$ , and  $P_s$  acts on  $\Delta s$ . Also, the angle between the horizontal leg of the triangle and the hypotenuse is  $\theta$ .)



$$\sum F_x = 0; \quad (P_x)(\Delta y) - (P_s \sin \theta)(\Delta s) = 0; \quad \text{since } (\Delta s \cdot \sin \theta) = \Delta y, \quad \mathbf{P_s = P_x}$$

$$\sum F_y = 0; \quad (P_y)(\Delta x) - (P_s \cos \theta)(\Delta s) - (\Delta x \cdot \Delta y / 2)(\gamma) = 0;$$

$$\text{since } (\Delta s \cdot \cos \theta) = \Delta x \quad \text{and } (\Delta y)(\Delta x) \rightarrow 0; \quad \mathbf{P_s = P_y}$$

7. A significant amount of mercury is poured into a U-tube with both ends open to the atmosphere. Then water is poured into one leg of the U-tube until the water column is 1 meter above the mercury-water meniscus. Finally, oil (S.G. = 0.79) is poured into the other leg to a height of 60 cm. What is the elevation difference between the mercury surfaces?

*Ans.* The mercury-water meniscus will be lower than the mercury-oil meniscus based on the relative amounts of each poured in and their specific gravity. Also, a surface of equal pressure can be drawn at the mercury-water meniscus. Therefore,

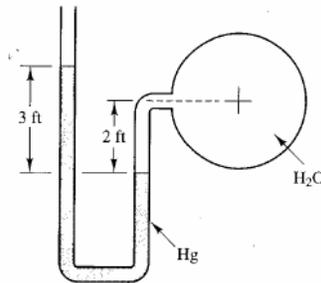
$$(1 \text{ m})(\gamma_{\text{water}}) = (h)(\gamma_{\text{Hg}}) + (0.6 \text{ m})(\gamma_{\text{oil}})$$

$$(1 \text{ m})(\gamma_{\text{water}}) = (h)(\text{SG}_{\text{Hg}})(\gamma_{\text{water}}) + (0.6 \text{ m})(\text{SG}_{\text{oil}})(\gamma_{\text{water}}); \text{ therefore}$$

$$h = [1 \text{ m} - (0.6 \text{ m})(\text{SG}_{\text{oil}})]/(\text{SG}_{\text{Hg}}) = [1 \text{ m} - (0.6 \text{ m})(0.79)]/(13.6)$$

$$\mathbf{h = 0.0387 \text{ m} = 3.87 \text{ cm}}$$

8. In the figure below, water is flowing in the pipe, and mercury (S.G = 13.6) is the manometer fluid. Determine the pressure in the pipe in psi and in inches of mercury.



*Ans.* A surface of equal pressure can be drawn at the mercury-water meniscus. Therefore,

$$(3 \text{ ft})(\gamma_{\text{Hg}}) = P + (2 \text{ ft})(\gamma) \quad \text{where } \gamma_{\text{Hg}} = (\text{SG}_{\text{Hg}})(\gamma)$$

$$(3 \text{ ft})(13.6)(62.3 \text{ lb/ft}^3) = P + (2 \text{ ft})(62.3 \text{ lb/ft}^3);$$

$$\mathbf{P = 2,420 \text{ lb/ft}^2 = 16.8 \text{ psi}}$$

Since  $p = \gamma \cdot h$ ; pressure can be expressed as the height of any fluid. For mercury,

$$h = P/\gamma_{\text{Hg}} = (2420 \text{ lb/ft}^2)/[(13.6)(62.3 \text{ lb/ft}^2)]$$

$$\mathbf{h = 2.86 \text{ ft of Hg (or 34.3 inches)}}$$

9. Manometer computations for the figure above would yield a pressure of 16.8 lb/in.<sup>2</sup> (psi). If the fluid in the pipe was oil (S.G. = 0.80) under the same pressure, would the manometer measurements (2 ft and 3 ft) still be the same? If not, what would the new measurements be?

*Ans.* The measurements will not be the same since oil is now in the manometer instead of water. A surface of equal pressure can be drawn at the mercury-oil interface.

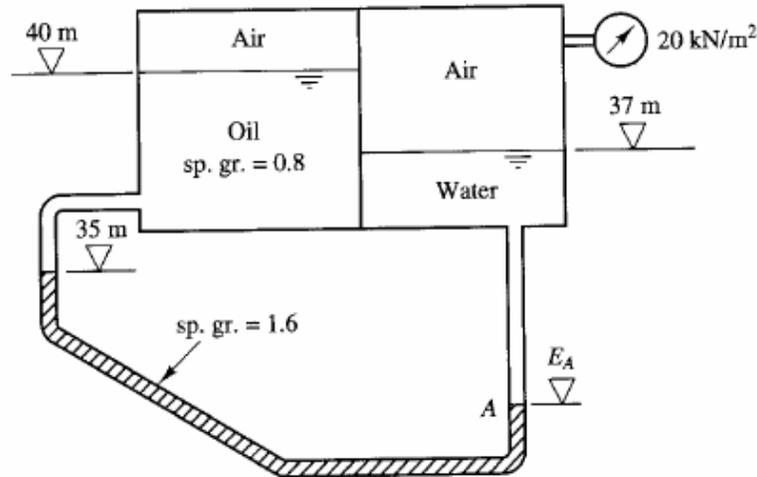
$$P_{\text{pipe}} + (2 \text{ ft} + \Delta h)(\gamma_{\text{oil}}) = (3 \text{ ft} + 2\Delta h)(\gamma_{\text{Hg}})$$

This is based on volume conservation. If the mercury-oil meniscus goes down  $\Delta h$  on the right, it must climb up  $\Delta h$  on the left making the total difference  $2\Delta h$ . Now

$$(2.42 \times 10^3 \text{ lb/ft}^2) + (2 \text{ ft} + \Delta h)(0.80)(62.3 \text{ lb/ft}^3) = (3 \text{ ft} + 2\Delta h)(13.6)(62.3 \text{ lb/ft}^3)$$

$$\Delta h = -0.0135 \text{ ft}$$

10. Determine the elevation at point A ( $E_A$ ) in the figure below if the air pressure in the sealed left tank is  $-29.0 \text{ kPa}$  ( $\text{kN/m}^2$ ).



*Ans.* Using the “swim through” technique, start at the right tank where pressure is known and “swim through” the tanks and pipes, adding pressure when “swimming” down and subtracting when “swimming” up until you reach the known pressure in the left tank. Solve for the variable elevation ( $E_A$ ) in the resulting equation.

$$20 \text{ kN/m}^2 + (37 \text{ m} - E_A)(9.79 \text{ kN/m}^3) - (35 \text{ m} - E_A)(1.6)(9.79 \text{ kN/m}^3) - (5 \text{ m})(0.8)(9.79 \text{ kN/m}^3) = -29.0 \text{ kN/m}^2$$

$$E_A = 30 \text{ m}$$

11. A 1-m-diameter viewing window is mounted into the inclined side ( $45^\circ$ ) of a dolphin pool. The center of the flat window is 10 m below the water’s surface measured along the incline. Determine the magnitude and location of the hydrostatic force acting on the window.

Ans. The hydrostatic force and its locations are:

$$F = \gamma \cdot \bar{h} \cdot A = (9790 \text{ N/m}^3)(10 \text{ m})(\sin 45^\circ)(\pi)(0.5 \text{ m})^2 = 5.44 \times 10^4 \text{ N} = \mathbf{54.4 \text{ kN}}$$

$$y_P - \bar{y} = \frac{I_0}{A\bar{y}} + \bar{y} = \left[ \frac{\pi(1\text{m})^4 / 64}{\pi(1\text{m})^2 / 4} \right] (10\text{m}) + 10\text{m} = \mathbf{10.01 \text{ m}}$$

12. A square gate 3m x 3m lies in a vertical plane. Determine the total pressure force on the gate and the distance between the center of pressure and the centroid when the upper edge of the gate is at the water surface. Compare these values to those that would occur if the upper edge is 15 m below the water surface.

Ans. The hydrostatic force and its locations are:

$$F = \gamma \cdot \bar{h} \cdot A = (9790 \text{ N/m}^3)(1.5 \text{ m})(9 \text{ m}^2) = 1.32 \times 10^5 \text{ N} = 132 \text{ kN}$$

$$y_P - \bar{y} = \frac{I_0}{A\bar{y}} = \frac{[(3\text{m})(3\text{m})^3 / 12]}{[9\text{m}^2](1.5\text{m})} = 0.500 \text{ m}$$

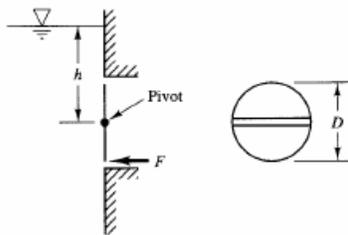
If the square gate was submerged by 15 m (to the top of the gate):

$$F = \gamma \cdot \bar{h} \cdot A = (9790 \text{ N/m}^3)(16.5 \text{ m})(9 \text{ m}^2) = 1.45 \times 10^6 \text{ N} = 1,450 \text{ kN}$$

$$y_P - \bar{y} = \frac{I_0}{A\bar{y}} = \frac{[(3\text{m})(3\text{m})^3 / 12]}{[9\text{m}^2](16.5\text{m})} = \mathbf{0.0455 \text{ m}}; \text{ Note that the force increases tremendously with}$$

depth; the distance between the centroid and the center of pressure becomes negligible.

13. A circular gate is installed on a vertical wall as shown in the figure below. Determine the horizontal force,  $F$ , necessary to hold the gate closed (in terms of diameter,  $D$ , and height,  $h$ ). Neglect friction at the pivot.



Ans. The hydrostatic force and its locations are:

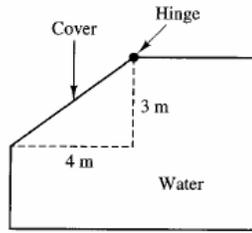
$$P = \gamma \cdot \bar{h} \cdot A = (\gamma)(h)[\pi(D)^2/4]$$

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \left[ \frac{\pi(D)^4 / 64}{\pi(D)^2 / 4} \right] (h) + h; \quad y_P = D^2/(16h) + h \text{ (depth to the center of pressure)}$$

Thus, summing moments:  $\sum M_{\text{hinge}} = 0$ ;  $F(D/2) - P(y_P - h) = 0$

$$F(D/2) - \{(\gamma)(h)[\pi(D)^2/4][D^2/(16h)]\} = 0; \quad \mathbf{F = (1/32)(\gamma)(\pi)(D)^3}$$

14. Calculate the minimum weight of the cover necessary to keep it closed. The cover dimensions are 5 meters by 10-meters.



*Ans.* The hydrostatic force on the cover and its locations are:

$$F = \gamma \cdot \bar{h} \cdot A = (9790 \text{ N/m}^3)(1.5 \text{ m})[(10\text{m})(5\text{m})] = 7.34 \times 10^5 \text{ kN} = 734 \text{ kN}$$

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[(10\text{m})(5\text{m})^3 / 12]}{[(10\text{m})(5\text{m})](2.5\text{m})} + 2.5\text{m} = 3.33 \text{ m (inclined distance to center of pressure)}$$

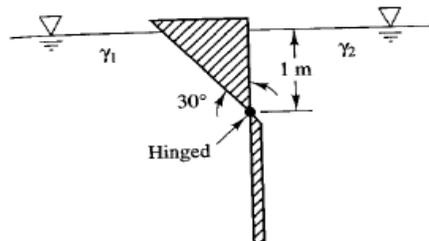
$$\sum M_{\text{hinge}} = 0; (734 \text{ kN})(3.33 \text{ m}) - W(2 \text{ m}) = 0; \quad \mathbf{W = 1,220 \text{ kN}}$$

15. A vertical, rectangular gate 3 m high and 2 m wide is located on the side of a water tank. The tank is filled with water to a depth 5 m above the upper edge of the gate. Locate a horizontal line that divides the gate area into two parts so that (a) the forces on the upper and lower parts are the same and (b) the moments of the forces about the line are the same.

*Ans.* The center of pressure represents the solution to both parts of the question. Thus,

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[(2\text{m})(3\text{m})^3 / 12]}{[(2\text{m})(3\text{m})](6.5\text{m})} + 6.5\text{m} = \mathbf{6.62 \text{ m}}$$

16. Determine the relationship between  $\gamma_1$  and  $\gamma_2$  in the figure below if the weightless triangular gate is in equilibrium in the position shown. (Hint: Use a unit length for the gate.)



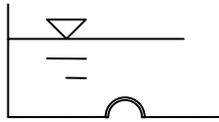
*Ans.*  $F_{\text{incline}} = \gamma \cdot \bar{h} \cdot A = (\gamma_1)(0.5 \text{ m})[(1\text{m}/\cos 30^\circ)(1\text{m})] = 0.577 \cdot \gamma_1 \text{ N}$ ; Since  $(1\text{m}/\cos 30^\circ) = 1.15 \text{ m}$

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[(1\text{m})(1.15\text{m})^3 / 12]}{[(1\text{m})(1.15\text{m})](1.15\text{m}/2)} + (1.15\text{m}/2) = 0.767 \text{ m (inclined distance to center of pressure)}$$

$$F_{\text{right}} = \gamma \cdot \bar{h} \cdot A = (\gamma_2)(0.5 \text{ m})[(1\text{m})(1\text{m})] = 0.500 \cdot \gamma_2 \text{ N}; \text{ and } y_P = 0.667 \text{ m}$$

$$\sum M_{\text{hinge}} = 0; (0.577 \cdot \gamma_1 \text{ N})(1.15\text{m} - 0.767 \text{ m}) - (0.500 \cdot \gamma_2 \text{ N})(1 \text{ m} - 0.667 \text{ m}) = 0; \quad \mathbf{\gamma_2 = 1.33 \cdot \gamma_1 \text{ N}}$$

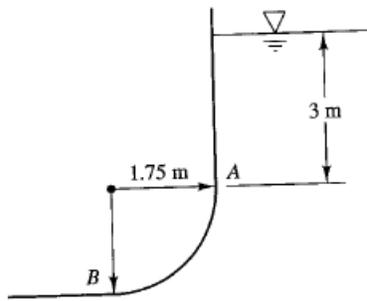
17. A hemispherical viewing port (under the bottom of a coral reef tank in a marine museum) has a 1-m outside radius. The top of the viewing port is 5 m below the surface of the water. Determine the total resultant (horizontal and vertical) components of the force on the viewing port (but not their locations). Salt water has an S.G. = 1.03.



*Ans.* The resultant force in the horizontal direction is zero ( $F_H = 0$ ) since equal pressures surround the viewing port in a complete circle. For the vertical direction;

$$F_V = \gamma \cdot Vol = (1.03)(9790 \text{ N/m}^3)[\pi(1 \text{ m})^2(6 \text{ m}) - (1/2)(4/3)\pi(1 \text{ m})^3] = \mathbf{169 \text{ kN}}$$

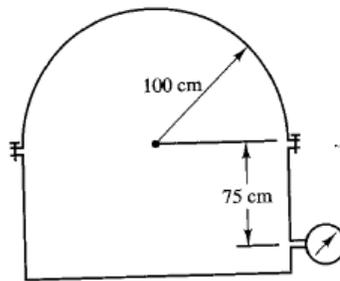
18. The corner plate of a large hull (depicted in the figure below) is curved with the radius of 1.75 m. When the barge is submerged in sea water (sp. gr. = 1.03), determine whether or not the vertical force component is greater than the horizontal component on plate  $AB$ .



$$\text{Ans. } F_H = \gamma \cdot \bar{h} \cdot A = (1.03)(9790 \text{ N/m}^3)(3.875 \text{ m})[(1.75 \text{ m})(1 \text{ m})] = 68.4 \text{ kN (per unit length of hull)}$$

$$F_V = \gamma \cdot Vol = (1.03)(9790 \text{ N/m}^3)[(3 \text{ m})(1.75 \text{ m})(1 \text{ m}) + \pi/4(1.75 \text{ m})^2](1 \text{ m}) = \mathbf{77.2 \text{ kN; larger}}$$

19. The cylindrical dome in the figure below is 8 m long and is secured to the top of an oil tank by bolts. If the oil has a specific gravity of 0.90 and the pressure gage reads  $2.75 \times 10^5 \text{ N/m}^2$ , determine the total tension force in the bolts. Neglect the weight of the cover.



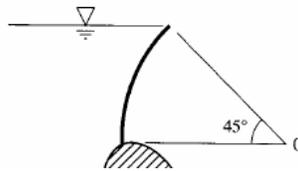
*Ans.*  $h = P/\gamma = [2.75 \times 10^5 \text{ N/m}^2]/[(0.9)(9790 \text{ N/m}^3)] = 31.2 \text{ m of oil}$

The total upward force is the weight of an oil column 31.2 m high minus the column of oil that is resident above the gage already.

$$F_V = \gamma \cdot Vol = (0.9)(9790 \text{ N/m}^3)[(31.2 \text{ m} - 0.75 \text{ m}) - (1/2)\pi(1.0 \text{ m})^2] (2.0 \text{ m})(8.0 \text{ m})$$

$F_V = 4.07 \times 10^6 \text{ N}$ ; which is the also the total tension force in the bolts holding the top on.

20. The tainter gate section shown in the figure below has a cylindrical surface with a 40-ft radius and is supported by a structural frame hinged at *O*. The gate is 33 ft long (in the direction perpendicular to the page). Determine the magnitude and location of the total hydrostatic force on the gate.



*Ans.* The height of the vertical projection is  $(R)(\sin 45^\circ) = 28.3 \text{ ft}$ . Thus,

$$F_H = \gamma \cdot \bar{h} \cdot A = (62.3 \text{ lb/ft}^3)(28.3 \text{ ft}/2)[(33 \text{ ft})(28.3 \text{ ft})] = 8.23 \times 10^5 \text{ lb}$$

Obtain the vertical component of the total pressure force by determining the weight of the water column above the curved gate. The volume of water above the gate is:

$$Vol = (A_{\text{rectangle}} - A_{\text{triangle}} - A_{\text{arc}})(\text{length})$$

$$Vol = [(40 \text{ ft})(28.3 \text{ ft}) - (1/2)(28.3 \text{ ft})(28.3 \text{ ft}) - (\pi/8)(40 \text{ ft})^2](33 \text{ ft}) = 3,410 \text{ ft}^3$$

$$F_V = \gamma \cdot Vol = (62.3 \text{ lb/ft}^3)(3410 \text{ ft}^3) = 2.12 \times 10^5 \text{ lb}; \quad \text{The total force is}$$

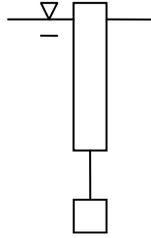
$$F = [(8.23 \times 10^5 \text{ lb})^2 + (2.12 \times 10^5 \text{ lb})^2]^{1/2} = 8.50 \times 10^5 \text{ lb}; \quad \theta = \tan^{-1} (F_V/F_H) = 14.4^\circ \rightarrow$$

Since all hydrostatic pressures pass through point *O* (i.e., they are all normal to the surface upon which they act), then the resultant must also pass through point *O*.

21. A 1-m length of a certain standard steel pipe weighs 248 N and has an outside diameter of 158 mm. Will the pipe sink in glycerin (S.G. = 1.26) if its ends are sealed?

*Ans.*  $B = \gamma \cdot Vol = (1.26)(9790 \text{ N/m}^3)[\pi(0.079 \text{ m})^2(1 \text{ m})] = 242 \text{ N} < 248 \text{ N}$ , thus it will sink.

22. A concrete block that has a total volume of 12 ft<sup>3</sup> and a specific gravity of 2.67 is tied to one end of a long cylindrical buoy as depicted in the figure below. The buoy is 10 ft long and is 2 ft in diameter. Unfortunately, it is floating away with 1 ft sticking above the water surface. Determine the specific gravity of the buoy. The fluid is brackish bay water (S.G. = 1.02).



*Ans.*  $W = 2.67\gamma(12 \text{ ft}^3) + (\text{SG})\gamma\cdot\pi(1 \text{ ft})^2(10 \text{ ft}); \quad B = 1.02\gamma(12 \text{ ft}^3) + 1.02\gamma\cdot\pi(1 \text{ ft})^2(9 \text{ ft})$   
 $\sum F_y = 0; \text{ or } W = B; \quad 32.0 + 31.4(\text{SG}) = 12.2 + 28.8; \quad \text{SG} = \mathbf{0.287}$

23. A 40-ft long, 30-ft diameter cylindrical caisson floats upright in the ocean (S.G. = 1.03) with 10 feet of the caisson above the water. The center of gravity measure 6 ft from the bottom of the caisson. Determine the metacentric height and the righting moment when the caisson is tipped through an angle of  $10^\circ$ .

*Ans.* The center of buoyancy (B) is 15 feet from the bottom since 30 feet is in the water.

$$GB = 9.0 \text{ ft, and } \overline{GM} = \overline{MB} \pm \overline{GB} = \frac{I_0}{\text{Vol}} \pm \overline{GB}; \text{ where } I_0 \text{ is the waterline moment of inertial}$$

about the tilting axis. The waterline area is a circle with a 30 ft diameter. Thus,

$$\overline{GM} = \frac{I_0}{\text{Vol}} \pm \overline{GB} = \frac{[\pi(30 \text{ ft})^4 / 64]}{(\pi / 4)(30 \text{ ft})^2(30 \text{ ft})} + 9.0 \text{ ft} = 10.9 \text{ ft}; \quad \text{Note: Vol is the submerged}$$

volume and a positive sign is used since G is located below the center of buoyancy.

$$M = W \cdot \overline{GM} \cdot \sin \theta = [(1.03)(62.3 \text{ lb/ft}^3)(\pi/4)(30 \text{ ft})^2(30 \text{ ft})](10.9 \text{ ft})(\sin 10^\circ)$$

$$\mathbf{M = 2.57 \times 10^6 \text{ ft-lb}} \text{ (for a heel angle of } 10^\circ)$$