

Chapter 2: Introduction to Algebra

CHAPTER 2

Exercise 1 • Algebraic Expressions

Mathematical Expressions

Which of the following mathematical expressions are also algebraic expressions? [See Example 1]

1. $x + 2y$: algebraic expression; contains only algebraic symbols and operations
2. $y - \log x$: not an algebraic expression; it is transcendental
3. $3 \sin x$: not an algebraic expression; it is transcendental
4. $x^2 - z^3$: algebraic expression; contains only algebraic symbols and operations

Algebraic Expressions

Which of the following algebraic expressions are literal expressions? [See Examples 4 and 5]

5. $5xy - 2x$: not a literal expression; letters at end of alphabet generally represent variables
6. $ax + by$: literal expression; letters at beginning of alphabet generally represent constants
7. $2az - 3bx$: literal expression; letters at beginning of alphabet generally represent constants
8. $4x^2 + 4y^2$: not a literal expression; letters at end of alphabet generally represent variables

Terms

How many terms are there in each expression? [See Examples 8 and 9]

9. $x^3 - 2x$: 2 terms
10. $(5y + y^2) - 5$: 2 terms; $(5y + y^2)$ is considered one term because of the symbol of grouping
11. $(z - 9)(z + 4)$: 1 term; terms (except within symbols of grouping) are separated by plus and/or minus signs
12. $5(1 + x) + 3(4 - 3x)$: 2 terms; $5(1 + x)$ is the first term, $3(4 - 3x)$ is the second term

Factors

Write the factors of each expression. [See Examples 10 and 11]

13. $3ax$: 3, a , x
14. $9xyz$: $3 (3 \cdot 3 = 9)$, x , y , z
15. $7x^2y^3$: 7, $x (x \cdot x = x^2)$, $y (y \cdot y \cdot y = y^3)$
16. $6a^2bx$: 2, 3 ($2 \cdot 3 = 6$), $a (a \cdot a = a^2)$, b , x

Coefficient

Write the coefficient of each term. [See Examples 12 and 13]

17. $6x^2$: 6 is the coefficient
18. x : 1 is the coefficient
19. $-x$: -1 is the coefficient
20. $3cx^3$: $3c$ (letters at the beginning of the alphabet are considered constants)
21. $\frac{ay^2}{2} = \frac{a}{2}(y^2)$: $\frac{a}{2}$ is the coefficient

22. $(1/3)(x + 1) = (b/3)(x + 1)$: $b/3$ is the coefficient

23. $(c - 2)3x^5$: $3(c - 2)$ is the coefficient

24. $2ax^5$: $2a$ is the coefficient

Degree

State the degree of each term. [See Examples 14 and 15]

25. $3x$: first degree (the exponent on x is 1)

26. $4y^2$: second degree

27. $3xy$: second degree (the degree of x is 1, the degree of y is 1; $1 + 1 = 2$)

28. $5x^2y^3$: fifth degree ($2 + 3 = 5$)

State the degree of each expression.

29. $3x + 4$: first degree

30. $5 - xy$: second

31. $3x^2 - 2x + 1$: second (the highest degree on x is 2)

32. $2xy^2 + xy - 4$: third degree

Exercise 2 • Adding and Subtracting Polynomials

Combining Like Terms

Combine as indicated and simplify. [See Examples 20, 21, 22, 23, 24 and 25]

1. $8y + 2y \Rightarrow (8 + 2)y = 10y$

2. $6x - 8x \Rightarrow (6 - 8)x = -2x$

3. $38.2a - 17.2a \Rightarrow 21.0a$

4. $2.94z + 5.37z \Rightarrow 8.31z$

5. $5z + 9z - 20z \Rightarrow (5 + 9 - 20)z = -6z$

6. $9xyz - 2xyz + 7xyz \Rightarrow 14xyz$

7. $7.39y - 6.62y + 1.94y \Rightarrow 2.71y$

8. $23.9ab + 54.9ab - 65.1ab \Rightarrow 13.7ab$

9. $5x + 2x - 8x - x \Rightarrow -2x$

10. $9a - 2a + 7a - 3a \Rightarrow 11a$

11. $2x^2 + 4x^2 - 3x^2 \Rightarrow 3x^2$

12. $5x^3 - x^3 - 2x^3 \Rightarrow 2x^3$

13. $7z^2 + 6z^2 - z^2 - 4z^2 \Rightarrow 8z^2$

14. $3.84z^3 - 1.27z^3 - 4.32z^3 - 7.52z^3 \Rightarrow -9.27z^3$

15. $88x + 23y - 17z + 68y - 36x + 39z \Rightarrow (88 - 36)x + (23 + 68)y - (17 - 39)z \Rightarrow 52x + 91y + 22z$

16. $a - 6b + 2c + a + 6b + n - 2c \Rightarrow 2a + n$

17. $1.95x - 4.38z + 2.83a - 5.21z - 9.27x \Rightarrow 2.83a - 7.32x - 9.59z$

18. $33.9ab - 82.4ac + 29.3ad - 84.2ac + 73.2ab \Rightarrow (33.9 + 73.2)ab - (82.4 + 84.2)ac + (29.3)ad$
 $\Rightarrow 107.1ab - 166.6ac + 29.3ad$

Combining More Than Two Expressions

Combine and simplify. Use either horizontal or vertical addition and subtraction. [See Examples 26, 27 and 28]

19. $(4ab + 6bc + 8cd) + (6ab - 3ab) + (4cd - 6bc) \Rightarrow (4ab + 6ab - 3ab) + (6bc - 6bc) + (8cd + 4cd) \Rightarrow 7ab + 12cd$

20. $(2a + 3b - 1) + (2c + d - b) - (3a - 4c + 5 - 6d) \Rightarrow (2a - 3a) + (3b - b) + (2c + 4c) + (d + 6d) + (-5 - 1)$
 $\Rightarrow -a + 2b + 6c + 7d - 6$

21. $(3b - 7p + 4r) + (3s - 11p - 19r) - (-3p + r - 10s) - (-5b + 2p - 2r + 4s)$
$$\begin{array}{r} 3b - 7p + 4r \\ -11p - 19r + 3s \\ 3p - r + 10s \\ \hline 5b - 2p + 2r - 4s \\ 8b - 17p - 14r + 9s \end{array}$$

22. $(2bx + 9x + 4a) - (4bx - a) + (3b + a) \Rightarrow 4a + a + a + 3b + 9x + 2bx - 4bx \Rightarrow 6a + 3b + 9x - 2bx$

23. $(-a + 5x) + (a^3 + 3a - 11x^2 + 2a^2 - 3) - (7a^2 + 3a^3 - 6x^3 + 12)$
 $\Rightarrow -a + 5x + a^3 + 3a - 11x^2 + 2a^2 - 3 - 7a^2 - 3a^3 + 6x^3 - 12 \Rightarrow -2a^3 - 5a^2 + 2a + 6x^3 - 11x^2 + 5x - 15$

24. $(2y - x + 3z - 13) - (4x + 2 - 5y) + (6z + 8) \Rightarrow 2y - x + 3z - 13 - 4x - 2 + 5y + 6z + 8 \Rightarrow -5x + 7y + 9z - 7$

Instructions Given Verbally [See Examples 29 and 30]

25. $(a - c + b) + (b + c - a) = (a - a) + (b + b) + (c - c) \Rightarrow 2b$

26. $(6bc + n^2 + 3p) + (-5x + 3n^2) \Rightarrow -5x + 6bc + n^2 + 3n^2 + 3p \Rightarrow -5x + 6bc + 4n^2 + 3p$

27. $(8b - 10c + 3a - d) - (5a + 7d - 4b + 6c) = 8b - 10c + 3a - d - 5a - 7d + 4b - 6c \Rightarrow -2a + 12b - 16c - 8d$

28. $(5x - 2xy + 8y) - (2xy + 4y - 3x) = 5x - 2xy + 8y - 2xy - 4y + 3x \Rightarrow 8x - 4xy + 4y$

29. $(24by^5 - 14bx^4) + (-72bx^5 + 2by^5 - 3bx^4) + (9bx^4 + 23by^4 - 21by^5)$
 $= 24by^5 + 2by^5 - 21by^5 - 14bx^4 - 3bx^4 + 9bx^4 + 23by^4 - 72bx^5 \Rightarrow 5by^5 - 8bx^4 - 72bx^5 + 23by^4$

30. $[3b(y^2 - z) - 6nx^2] + [2b(z + y^2) + 3nx^2] + [4b(-z - y^2) - 2nx^2]$
 $= 3by^2 - 3bz - 6nx^2 + 2bz + 2by^2 + 3nx^2 - 4bz - 4by^2 - 2nx^2$
 $\Rightarrow 3by^2 + 2by^2 - 4by^2 - 3bz + 2bz - 4bz - 6nx^2 + 3nx^2 - 2nx^2 \Rightarrow by^2 - 5bz - 5nx^2$

Applications

31. $2[2w^2 + 3w^2 + 6w^2] = 2[11w^2] \Rightarrow 22w^2$
32. $0.12x + 0.08(5000 - x) = 0.12x + 400 - 0.08x \Rightarrow 0.04x + 400$
33. $w + l + w + l \Rightarrow w + w + l + l = 2w + 2l$
34. $\pi r^2 + 2\pi rh + \pi r^2 = 2\pi r^2 + 2\pi rh \Rightarrow 2\pi r(r + h)$
35. $s_2 - s_1 = (9.76t^2 + 2.95t + 1.94) - (3.74t^2 + 5.83t + 4.22) \Rightarrow 6.02t^2 - 2.88t - 2.28$

Exercise 3 • Laws of Exponents**Definitions**

Evaluate each expression. [See Example 32]

1. $3^3 \Rightarrow 3 \cdot 3 \cdot 3 = 27$
2. $(3)^5 \Rightarrow 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$
3. $(-2)^4 \Rightarrow (-2)(-2)(-2)(-2) = 16$
4. $(-2)^5 \Rightarrow (-2)(-2)(-2)(-2)(-2) = -32$
5. $(0.001)^3 \Rightarrow (0.001)(0.001)(0.001) = 1 \times 10^{-9}$
6. $(-5)^3 \Rightarrow (-5)(-5)(-5) = -125$

Multiplying Powers

Multiply. [See Example 33 and 34]

7. $(x^4)(x^2) \Rightarrow [(x)(x)(x)(x)][(x)(x)] \Rightarrow [(x)(x)(x)(x)(x)(x)] = x^6$ or $(x^4)(x^2) \Rightarrow x^{4+2} = x^6$
8. $(y^b)(y^3) \Rightarrow y^{b+3}$
9. $(a^3)(a^6) \Rightarrow a^{3+6} = a^9$
10. $(10^5)(10^9) \Rightarrow 10^{5+9} = 10^{14}$
11. $(10^2)(10^6) \Rightarrow 10^{2+6} = 10^8$
12. $(z^{11})(z^2) \Rightarrow z^{11+2} = z^{13}$

Quotients

Divide. Write your answers without negative exponents. [See Example 35]

13. $\frac{a^6}{a^4} \Rightarrow \frac{a \cdot a \cdot a \cdot a \cdot a \cdot a}{a \cdot a \cdot a \cdot a} \Rightarrow \frac{\cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot a \cdot a}{\cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a}} = a^2$ or $\frac{a^6}{a^4} \Rightarrow a^{6-4} = a^2$
14. $\frac{2^4}{2^3} \Rightarrow 2^{4-3} = 2^1 \Rightarrow 2$
15. $\frac{y^{a+1}}{y^{a-2}} \Rightarrow y^{(a+1)-(a-2)} \Rightarrow y^{a+1-a+2} = y^3$

16. $\frac{10^6}{10^2} \Rightarrow 10^{6-2} = 10^4$

17. $\frac{10^{b-1}}{10^{b-3}} \Rightarrow 10^{(b-1)-(b-3)} \Rightarrow 10^{b-1-b+3} = 10^2$

18. $\frac{10^4}{10^3} \Rightarrow 10^{4-3} = 10^1 \Rightarrow 10$

19. $\frac{b^{-4}}{b^{-5}} \Rightarrow b^{-4-(-5)} = b^1 \Rightarrow b$

20. $\frac{x^{-6}}{y} \Rightarrow \frac{x^{-6}}{y} \cdot \frac{1}{1} \Rightarrow \frac{x^{-6}}{y} \div y \Rightarrow \frac{1}{x^6} \cdot \frac{1}{y} = \frac{1}{x^6 y}$

this line means "divided by"

Power Raised to a Power

Simplify. [See Example 36]

21. $(a^2)^4 \Rightarrow (a \cdot a)^4 \Rightarrow (a \cdot a)(a \cdot a)(a \cdot a)(a \cdot a) \Rightarrow a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a = a^8$ or $(a^2)^4 \Rightarrow a^{2 \cdot 4} = a^8$

22. $(2^5)^2 \Rightarrow 2^{5 \cdot 2} = 2^{10}$

23. $(z^c)^a \Rightarrow z^{c \cdot a} = z^{ac}$

24. $(y^{-1})^{-3} \Rightarrow y^{-1(-3)} = y^3$

25. $(a^{x-1})^3 \Rightarrow a^{3(x-1)} = a^{3x-3}$

Product Raised to a Power

Raise to the power indicated and remove parentheses. [See Example 37]

26. $(ac)^3 \Rightarrow (ac)(ac)(ac) \Rightarrow (aaacc) = a^3c^3$ or $(ac)^3 \Rightarrow a^{1 \cdot 3}c^{1 \cdot 3} = a^3c^3$

27. $(3a)^2 \Rightarrow 3^2a^2 = 9a^2$

28. $(4a^3c^2)^4 \Rightarrow 4^1 \cdot 4 \cdot a^3 \cdot 4 \cdot c^2 \cdot 4 \Rightarrow 4^4 a^{12} c^8 = 256 a^{12} c^8$

29. $(2xyz)^5 \Rightarrow 2^5 x^5 y^5 z^5 = 32 x^5 y^5 z^5$

Quotient Raised to a Power

Raise to the power indicated and remove parentheses. [See Example 38]

30. $\left(\frac{2}{3}\right)^3 \Rightarrow \frac{2^{1 \cdot 3}}{3^{1 \cdot 3}} \Rightarrow \frac{2^3}{3^3} \Rightarrow \frac{2 \cdot 2 \cdot 2}{3 \cdot 3 \cdot 3} = \frac{8}{27}$

31. $\left(-\frac{2}{5}\right)^3 \Rightarrow -\frac{2^3}{5^3} = -\frac{8}{125}$

32. $\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$

33. $\left(\frac{4x^2}{3y^2}\right)^2 \Rightarrow \frac{4^{1 \cdot 2} x^{2 \cdot 2}}{3^{1 \cdot 2} y^{2 \cdot 2}} \Rightarrow \frac{4^2 x^4}{3^2 y^4} = \frac{16x^4}{9y^4}$

$$34. \left(\frac{2ab^3}{3c^2d}\right)^3 \Rightarrow \frac{2^3 a^3 b^{3 \cdot 3}}{3^3 c^{2 \cdot 3} d^3} \Rightarrow \frac{2^3 a^3 b^9}{3^3 c^6 d^3} = \frac{8a^3 b^9}{27c^6 d^3}$$

Zero Exponent

Evaluate. [See Example 39]

$$35. (2x^2 - 8x + 32)^0 = 1$$

$$36. 108a^3c^0 \Rightarrow 108a^3(1) = 108a^3$$

$$37. \frac{82}{y^0} \Rightarrow \frac{82}{1} = 82$$

$$38. \frac{c}{y^0} \Rightarrow \frac{c}{1} = c$$

$$39. \frac{(z^{-n})(z^2)}{z^{2-n}} \Rightarrow \frac{z^{2-n}}{z^{2-n}} \Rightarrow z^{(2-n)-(2-n)} \Rightarrow z^{2-2-n+n} \Rightarrow z^0 = 1$$

$$40. 4\left(\frac{a^7}{y^4}\right)^0 \Rightarrow 4\left(\frac{a^{7 \cdot 0}}{y^{4 \cdot 0}}\right) \Rightarrow 4\left(\frac{a^0}{y^0}\right) \Rightarrow 4\left(\frac{1}{1}\right) \Rightarrow 4(1) = 4$$

Negative Exponent

Write each expression with positive exponents only. [See Example 40]

$$41. x^{-1} \Rightarrow \frac{1}{x^1} = \frac{1}{x}$$

$$42. (-b)^{-3} = -\frac{1}{b^3}$$

$$43. \left(\frac{2}{x}\right)^{-4} \Rightarrow \frac{2^{-4}}{x^{-4}} \Rightarrow \frac{\frac{1}{2^4}}{\frac{1}{x^4}} \Rightarrow \frac{1}{2^4} \div \frac{1}{x^4} \Rightarrow \frac{1}{2^4} \cdot \frac{x^4}{1} = \frac{x^4}{16}$$

$$44. ab^{-5}c^{-2} \Rightarrow a\left(\frac{1}{b^5}\right)\left(\frac{1}{c^2}\right) \Rightarrow \left(\frac{a}{1}\right)\left(\frac{1}{b^5}\right)\left(\frac{1}{c^2}\right) = \frac{a}{b^5c^2}$$

$$45. \left(\frac{4x^3}{3y^2}\right)^{-3} \Rightarrow \frac{4^{-3}x^{-9}}{3^{-3}y^{-6}} \Rightarrow \left(\frac{\frac{1}{64}}{\frac{1}{27}}\right)\left(\frac{\frac{1}{x^9}}{\frac{1}{y^6}}\right) \Rightarrow \frac{\frac{1}{64x^9}}{\frac{1}{27y^6}} \Rightarrow \frac{1}{64x^9} \div \frac{1}{27y^6} \Rightarrow \left(\frac{1}{64x^9}\right)\left(\frac{27y^6}{1}\right) = \frac{27y^6}{64x^9}$$

An alternative way to solve $\left(\frac{4x^3}{3y^2}\right)^{-3}$ based on the definition of the negative exponent

$$\left(\frac{4x^3}{3y^2}\right)^{-3} \Rightarrow \frac{1}{\left(\frac{4x^3}{3y^2}\right)^3} \Rightarrow \frac{1}{\frac{4^3x^9}{3^3y^6}} \Rightarrow \frac{1}{64x^9} \Rightarrow 1 \div \frac{64x^9}{27y^6} \Rightarrow \left(\frac{1}{1}\right)\left(\frac{27y^6}{64x^9}\right) = \frac{27y^6}{64x^9}$$

$$46. a^{-5}bc^{-2} \Rightarrow \left(\frac{1}{a^5}\right)\left(\frac{b}{1}\right)\left(\frac{1}{c^2}\right) = \frac{b}{a^5c^2}$$

$$47. 4w^{-4} - 3z^{-2} \Rightarrow 4\left(\frac{1}{w^4}\right) - 3\left(\frac{1}{z^2}\right) = \frac{4}{w^4} - \frac{3}{z^2}$$

$$48. \left(\frac{a}{b}\right)^{-4} \Rightarrow \frac{a^{-4}}{b^{-4}} \Rightarrow \frac{1}{\frac{a^4}{b^4}} \Rightarrow \left(\frac{1}{a^4}\right)\left(\frac{b^4}{1}\right) = \frac{b^4}{a^4}$$

Solving $\left(\frac{a}{b}\right)^{-4}$ by the alternative method

$$\left(\frac{a}{b}\right)^{-4} \Rightarrow \frac{1}{\left(\frac{a}{b}\right)^4} \Rightarrow \frac{1}{\frac{a^4}{b^4}} \Rightarrow 1 \div \frac{a^4}{b^4} \Rightarrow \left(\frac{1}{1}\right)\left(\frac{b^4}{a^4}\right) = \frac{b^4}{a^4}$$

Express without fractions, using negative exponents when needed [See Example 41]

$$49. \frac{1}{a} = a^{-1}$$

$$50. \frac{5}{x^3} = 5x^{-3}$$

$$51. \frac{c^2}{d^3} \Rightarrow c^2\left(\frac{1}{d^3}\right) \Rightarrow c^2d^{-3}$$

$$52. \frac{w^4z^{-2}}{x^{-3}} \Rightarrow \frac{w^4\left(\frac{1}{z^2}\right)}{\frac{1}{x^3}} \Rightarrow \frac{w^4}{\frac{z^2}{x^3}} \Rightarrow \left(\frac{w^4}{z^2}\right)\left(\frac{x^3}{1}\right) \Rightarrow \frac{w^4x^3}{z^2} \Rightarrow w^4x^3z^{-2}$$

$$53. \frac{y^2}{x^{-4}} \Rightarrow \frac{y^2}{\frac{1}{x^4}} \Rightarrow \left(\frac{y^2}{1}\right)\left(\frac{x^4}{1}\right) = x^4y^2$$

$$54. \frac{c^{-1}d^{-4}}{b^{-2}c^{-3}} \Rightarrow \frac{\left(\frac{1}{c}\right)\left(\frac{1}{d^4}\right)}{\left(\frac{1}{b^2}\right)\left(\frac{1}{c^3}\right)} \Rightarrow \frac{\frac{1}{cd^4}}{\frac{1}{b^2c^3}} \Rightarrow \left(\frac{1}{cd^4}\right)\left(\frac{b^2c^3}{1}\right) \Rightarrow \frac{b^2c^3}{cd^4} \Rightarrow b^2c^{3-1}d^{-4} = b^2c^2d^{-4}$$

Applications

$$55. (w)(2w)(3w) \Rightarrow (2 \cdot 3)(w \cdot w \cdot w) = 6w^3$$

$$56. 16.1(2t)^2 \Rightarrow 16.1(4t^2) = 64.4t^2 \text{ ft}$$

$$57. \left(\frac{i}{3}\right)^2 R \Rightarrow \left(\frac{i^2}{3^2}\right)R \Rightarrow \left(\frac{i^2}{9}\right)R \Rightarrow \left(\frac{i^2}{9}\right)\left(\frac{R}{1}\right) = \frac{i^2R}{9}$$

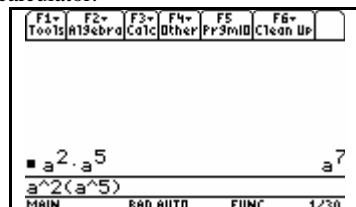
$$58. \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R^{-1} = R_1^{-1} + R_2^{-1}$$

Exercise 4 • Product of Two Monomials

Multiply the following monomials, and simplify. [See Examples 43, 44, 45, 46, 47, 48, 49 and 50]

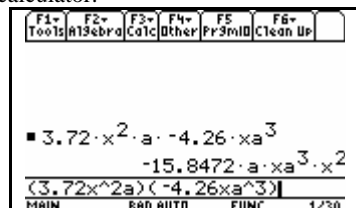
- $(x)(-y) \Rightarrow -xy$
- $(-w)(-z) \Rightarrow wz$
- $(a)(-b)(c) \Rightarrow -abc$
- $(-x)(-y)(z) \Rightarrow xyz$
- $(2.82x)(3.26y) \Rightarrow (2.82 \cdot 3.26)(xy) \Rightarrow 9.1932xy = 9.19xy$
- $(33.5a)(3.72ab) \Rightarrow (33.5 \cdot 3.72)(a^{1+1}b) \Rightarrow 124.62a^2b = 125a^2b$ (round to 3 significant digits)
- $(-8xy)(-2x) \Rightarrow [(-8)(-2)](x^{1+1}y) \Rightarrow 16x^2y$
- $(-3ab)(-5ac) \Rightarrow 15a^{1+1}bc = 15a^2bc$
- $a^2(a^5) \Rightarrow a^{2+5} = a^7$

By calculator:



- $xy^2(5x^2) \Rightarrow 5x^{1+2}y^2 = 5x^3y^2$
- $(3.72x^2a)(-4.26xa^3) \Rightarrow -15.8472x^{2+1}a^{1+3} = -15.84x^3$

By calculator:



- $(-6.72ab^4)(5.27a^2bc) \Rightarrow -35.4114a^{1+2}b^{4+1}c = -35.4a^3b^5c$
- $(3.11a^m x^2)(2.94a^3) \Rightarrow 9.1434a^{m+3}x^2 = 9.14a^{m+3}x^2$
- $(6.82b^3x^3)(5.16b^2x) \Rightarrow 35.1912b^{3+2}x^{3+1} = 35.2b^5x^4$
- $(x^m y^n)(3x^3 y^2) \Rightarrow 3x^{m+3}y^{n+2}$

By calculator:

Calculator screen showing the expansion of $(3x^3 + y^2)(x^m \cdot y^n)$. The display shows the result $3 \cdot x^m + 3 \cdot y^2 \cdot y^n$.

16. $(2w^3y^3)(7w^ay^b) \Rightarrow 14w^{a+3}y^{b+3}$

Exercise 5 • Product of a Multinomial and a Monomial

Multiply the multinomial by the monomial, and simplify. [See Examples 52, 53, 54, 55 and 56]

1. $3(-2 - x) \Rightarrow 3(-2) - 3(x) = -6 - 3x$

2. $x(b + 2) \Rightarrow x(b) + x(2) = bx + 2x$

3. $2(a + 3b) \Rightarrow 2(a) + 2(3b) = 2a + 6b$

By calculator:

Calculator screen showing the expansion of $2 \cdot (a + 3 \cdot b)$. The display shows the result $2 \cdot a + 6 \cdot b$.

4. $x(x - 5) \Rightarrow x(x) - x(5) \Rightarrow x^{1+1} - 5x = x^2 - 5x$

5. $3.83b(b^2 + 1.27) \Rightarrow 3.83b^{1+2} + 3.83b(1.27) \Rightarrow 3.83b^3 + 4.8641b = 3.83b^3 + 4.86b$

6. $2.03x(1.27x - 2.36) \Rightarrow 2.03x^{1+1}(1.27) - 2.03x(2.36) \Rightarrow 2.5781x^2 - 4.7905x = 2.58x^2 - 4.79x$

7. $3x(-7 - 10x) \Rightarrow -3x(7) - 3x^{1+1}(10) \Rightarrow -21x - 30x^2 = -30x^2 - 21x$

8. $b^4(b^2 + 8) \Rightarrow b^{4+2} + b^4(8) = b^6 + 8b^4$

9. $a^2b(2a + b - ab) \Rightarrow a^{2+1}b(2) + a^2b^{1+1} - a^{2+1}b^{1+1} = 2a^3b + a^2b^2 - a^3b^2$

By calculator:

Calculator screen showing the expansion of $a^2 \cdot b \cdot (2 \cdot a + b - a \cdot b)$. The display shows the result $2 \cdot a^3 \cdot b + a^2 \cdot b^2 - a^3 \cdot b^2$.

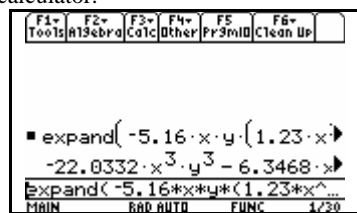
10. $-6x^3y^3(3xy + 5x^2y^3 - 2xy^2) \Rightarrow -6(3)x^{3+1}y^{3+1} + (-6)(5)x^{3+2}y^{3+3} - (-6)(2)x^{3+1}y^{3+2} = -30x^5y^6 + 12x^4y^5 - 18x^4y^4$

11. $2ab(9a^2 + 6ab - 3b^2) \Rightarrow 2(9)a^{1+2}b + 2(6)a^{1+1}b^{1+1} - 2(3)ab^{1+2} = 18a^3b + 12a^2b^2 - 6ab^3$

12. $-4.27m^2(2.83m^4 + 6.82m^2 - 3.25m^3 + 2.47)$
 $\Rightarrow -4.27(2.83)m^{2+4} + (-4.27)(6.82)m^{2+2} - (-4.27)(3.25)m^{2+3} + (-4.27)(2.47)m^2$
 $\Rightarrow -12.0841m^6 - 29.1214m^4 + 13.8775m^5 - 10.5469m^2$
 $\Rightarrow -12.1m^6 + 13.9m^5 - 29.1m^4 - 10.5m^2$

$$\begin{aligned}
 13. & -5.16xy(1.23x^2y - 5.83xy^2 + 4.27x^2y^2 - 2.94xy) \\
 & \Rightarrow -5.16(1.23)x^{1+2}y^{1+1} - (-5.16)(5.83)x^{1+1}y^{1+2} + (-5.16)(4.27)x^{1+2}y^{1+2} - (-5.16)(2.94)x^{1+1}y^{1+1} \\
 & \Rightarrow -6.3468x^3y^2 + 30.0828x^2y^3 - 22.0332x^3y^3 + 15.1704x^2y^2 \\
 & = -6.35x^3y^2 + 30.1x^2y^3 - 22.0x^3y^3 + 15.2x^2y^2
 \end{aligned}$$

By calculator:



$$\begin{aligned}
 14. & 6mn^2(5m^3n + 4mn^2 - 2m^2n + mn) \Rightarrow 6(5)m^{1+3}n^{2+1} + 6(4)m^{1+1}n^{2+2} - 6(2)m^{1+2}n^{2+1} + 6m^{1+1}n^{2+1} \\
 & = 30m^4n^3 - 12m^3n^3 + 24m^2n^4 + 6m^2n^3
 \end{aligned}$$

Exercise 6 • Product of Two Binomials

Multiply the following binomials, and simplify. [See Examples 57, 58, 59 and 60]

- $(x + y)(x + z) \Rightarrow x(x) + x(z) + y(x) + y(z) \Rightarrow x^{1+1} + xz + xy + yz = x^2 + xz + xy + yz$
- $(4a - 3)(a + 2) \Rightarrow 4a(a) + 4a(2) - 3(a) - 3(2) \Rightarrow 4a^{1+1} + 8a - 3a - 6 = 4a^2 + 5a - 6$
- $(4m + n)(2m^2 - n) \Rightarrow 4m(2m^2) + 4m(-n) + n(2m^2) + n(-n) \Rightarrow 8m^3 - 4mn + 2m^2n - n^2 = 8m^3 + 2m^2n - 4mn - n^2$
- $(y + 2)(y - 2) \Rightarrow y(y) - 2y + 2y + 2(-2) = y^2 - 4$ (you will factor this later as the difference of squares)
- $(2x - y)(x + y) \Rightarrow 2x(x) + 2x(y) - y(x) - y(y) = 2x^2 + xy - y^2$
By the distributive property:

$$\begin{aligned}
 (2x - y)(x + y) &= 2x(x + y) - y(x + y) \\
 &= 2x(x) + 2x(y) - y(x) - y(y) \\
 &= 2x^2 + 2xy - xy - y^2 \\
 &= 2x^2 + xy - y^2
 \end{aligned}$$
- $(a^2 - 3b)(a^2 + 5b) \Rightarrow a^2(a^2) + a^2(5b) - 3b(a^2) - 3b(5b) = a^4 + 2a^2b - 15b^2$
- $(4xy^2 - 3a^3b)(3xy^2 + 4a^3b) \Rightarrow 4xy^2(3xy^2) + 4xy^2(4a^3b) - 3a^3b(3xy^2) - 3a^3b(4a^3b)$

$$\Rightarrow 12x^2y^4 + 16xy^2a^3b - 9xy^2a^3b - 12a^6b^2 = 12x^2y^4 + 7a^3bxy^2 - 12a^6b^2$$
- $(2m^2 - 2n^2)(2m^2 + 2n^2) \Rightarrow 2m^2(2m^2) + 2m^2(2n^2) - 2n^2(2m^2) - 2n^2(2n^2)$

$$\Rightarrow 4m^4 + 2m^2n^2 - 2m^2n^2 - 4n^4 \Rightarrow 4m^4 - 4n^4$$

You will see later that a shortcut to multiplying the sum and difference of the same terms is to square the first term, square the second term, and then put a minus (subtraction) sign between the first and second terms.

$$9. (a - 7x)(2a + 3x) \Rightarrow a(2a) + a(3x) - 7x(2a) - 7x(3x) \Rightarrow 2a^2 + 3ax - 14ax - 21x^2 = 2a^2 - 11ax - 21x^2$$

By calculator:

F1+	F2+	F3+	F4+	F5	F6+
Tools	Algebra	Calc	Other	Pr3mID	Clean Up

$$\blacksquare \text{expand}((a - 7 \cdot x) \cdot (2 \cdot a + 3 \cdot x))$$

$$\frac{-21 \cdot x^2 - 11 \cdot a \cdot x + 2 \cdot a^2}{\text{expand}((a - 7 * x) * (2 * a + 3 * x))}$$

MAIN	BAC AUTO	FUNC	1/230
------	----------	------	-------

$$10. (3x - z^2)(4x - 3z^2) \Rightarrow 3x(4x) + 3x(-3z^2) - z^2(4x) - z^2(-3z^2) \Rightarrow 12x^2 - 9xz^2 - 4xz^2 + 3z^4 = 12x^2 - 13xz^2 + 3z^4$$

$$11. (ax - 5b)(ax + 5b) \Rightarrow ax(ax) + ax(5b) - 5b(ax) - 5b(5b) = a^2x^2 - 25b^2$$

By the distributive property:

$$\begin{aligned} (ax - 5b)(ax + 5b) &= ax(ax + 5b) - 5b(ax + 5b) \\ &= a^2x^2 + 5abx - 5abx - 25b^2 \\ &= a^2x^2 - 25b^2 \end{aligned}$$

$$12. (5y^2 + 3z)(5y^2 - 3z) \Rightarrow 5y^2(5y^2) + 5y^2(-3z) + 3z(5y^2) + 3z(-3z) \Rightarrow 25y^4 - 9z^2$$

$$13. (2.93x - 1.11y)(x + y) \Rightarrow 2.93x(x) + 2.93x(y) - 1.11y(x) - 1.11y(y) \Rightarrow 2.93x^2 + 2.93xy - 1.11xy - 1.11y^2$$

$$\Rightarrow 2.93x^2 + 1.82xy - 1.11y^2$$

$$14. (2.84a^2 - 3.82b)(a^2 + 5.11b) \Rightarrow 2.84a^2(a^2) + 2.84a^2(5.11b) - 3.82b(a^2) - 3.82b(5.11b)$$

$$\Rightarrow 2.84a^4 + 14.5124a^2b - 3.82a^2b - 19.5202b^2 \Rightarrow 2.84a^4 + 10.6924a^2b - 19.5202b^2 = 2.84a^4 + 10.7a^2b - 19.5b^2$$

$$15. (4.03y^2 - 3.92a^3b)(3.26y^2 + 4.73a^3b) \Rightarrow 4.03y^2(3.26y^2) + 4.03y^2(4.73a^3b) - 3.92a^3b(3.26y^2) - 3.92a^3b(4.73a^3b)$$

$$\Rightarrow 13.1378y^4 + 19.0619a^3by^2 - 12.7792a^3by^2 - 18.5416a^6b^2 \Rightarrow 13.1378y^4 + 6.2827a^3by^2 - 18.5416a^6b^2$$

$$= 13.1y^4 + 6.28a^3by^2 - 18.5a^6b^2$$

$$16. (2.83m^2 - 2.12n^2)(2.83m^2 + 2.12n^2) \Rightarrow 2.83m^2(2.83m^2) + 2.83m^2(2.12n^2) - 2.12n^2(2.83m^2) - 2.12n^2(2.12n^2)$$

$$\Rightarrow 8.0089m^4 + 5.999m^2n^2 - 5.999m^2n^2 - 4.4944n^4 \Rightarrow 8.0089m^4 - 4.4944n^4 = 8.01m^4 - 4.49n^4$$

Applications

$$17. \text{Area of rectangle} = \text{length} \times \text{width} \Rightarrow (L + 2)(W - 3) \Rightarrow L(W) + L(-3) + 2(W) + 2(-3) = LW - 3L + 2W - 6$$

$$18. RT(\text{the distance}) = (R - 8.5\text{mi/h})(T + 2.4\text{ h}) \Rightarrow R(T) + R(2.4) - 8.5(T) - 8.5(2.4) \Rightarrow RT + 2.4R - 8.5T - 20.4$$

By calculator:

F1+	F2+	F3+	F4+	F5	F6+
Tools	Algebra	Calc	Other	Pr3mID	Clean Up

$$\blacksquare \text{expand}((r - 8.5) \cdot (t + 2.4))$$

$$\frac{r \cdot t + 2.4 \cdot r - 8.5 \cdot t - 20.4}{\text{expand}((R - 8.5) * (T + 2.4))}$$

MAIN	BAC AUTO	FUNC	1/230
------	----------	------	-------

Exercise 7 • Product of Two Multinomials

Multiply the following binomials and trinomials, and simplify. [See Examples 61 and 62]

$$1. (x - 3)(x + 4 - y) \Rightarrow x(x) + x(4) + x(-y) - 3(x) - 3(4) - 3(-y) \Rightarrow x^2 + 4x - xy - 3x - 12 + 3y$$

$$= x^2 - xy + x + 3y - 12$$

2. $(a-d)(a-2d+5) \Rightarrow a(a) + a(-2d) + a(5) - d(a) - d(-2d) - d(5) \Rightarrow a^2 - 2ad + 5a - ad + 2d^2 - 5d$
 $= a^2 - 3ad + 5a + 2d^2 - 5d$
3. $(w^2 + w - 5)(4w - 2) \Rightarrow w^2(4w) + w^2(-2) + w(4w) + w(-2) - 5(4w) - 5(-2)$
 $\Rightarrow 4w^3 - 2w^2 + 4w^2 - 2w - 20w + 10 = 4w^3 - 2w^2 - 22w + 10$
4. $(a^2 - 5)(3a^2 - 7a - 4) \Rightarrow a^2(3a^2) + a^2(-7a) + a^2(-4) - 5(3a^2) - 5(-7a) - 5(-4)$
 $\Rightarrow 3a^4 - 7a^3 - 4a^2 - 15a^2 + 35a + 20 = 3a^4 - 7a^3 - 19a^2 + 35a + 20$
5. $(b^7 - 2.82b^5 + 4.27b^3)(b + 2.93) \Rightarrow b^7(b) + b^7(2.93) - 2.82b^5(b) - 2.82b^5(2.93) + 4.27b^3(b) + 4.27b^3(2.93)$
 $\Rightarrow b^8 + 2.93b^7 - 2.82b^6 - 8.2626b^5 + 4.27b^4 + 12.5111b^3 = b^8 + 2.93b^7 - 2.82b^6 - 8.26b^5 + 4.27b^4 + 12.5b^3$
6. $(x + 3.88)(x^3 - 2.15x - 6.03) \Rightarrow x(x^3) + x(-2.15x) + x(-6.03) + 3.88(x^3) + 3.88(-2.15x) + 3.88(-6.03)$
 $\Rightarrow x^4 - 2.15x^2 - 6.03x + 3.88x^3 - 8.342x - 23.3964 \Rightarrow x^4 + 3.88x^3 - 2.15x^2 - 14.372x - 23.3964$
 $= x^4 + 3.88x^3 - 2.15x^2 - 14.4x - 23.4$
7. $(1 + c^2)(4c^2 + 7c - 3) \Rightarrow 1(4c^2) + 1(7c) - 1(3) + c^2(4c^2) + c^2(7c) - c^2(3) \Rightarrow 4c^2 + 7c - 3 + 4c^4 + 7c^3 - 3c^2$
 $= 4c^4 + 7c^3 + c^2 + 7c - 3$
8. $(2x^2 - 6xy + 3y^2)(3x + 3y) \Rightarrow 2x^2(3x) + 2x^2(3y) - 6xy(3x) - 6xy(3y) + 3y^2(3x) + 3y^2(3y)$
 $\Rightarrow 6x^3 + 6x^2y - 18xy^2 - 18xy^2 + 9xy^2 + 9y^3 = 6x^3 - 12x^2y - 9xy^2 + 9y^3$
9. $(b^2 - bx + x^2)(b + x) \Rightarrow b^2(b) + b^2(x) - bx(b) - bx(x) + x^2(b) + x^2(x)$
 $\Rightarrow b^3 + b^2x - b^2x - bx^2 + x^2b + x^3 = x^3 + b^3$
10. $(a^2 + 2a - 2)(a + 1) \Rightarrow a^2(a) + a^2(1) + 2a(a) + 2a(1) - 2(a) - 2(1) \Rightarrow a^3 + a^2 + 2a^2 + 2a - 2a - 2$
 $= a^3 + 3a^2 - 2$
11. $(b^2 + bx + x^2)(b - x) \Rightarrow b^2(b) + b^2(-x) + bx(b) + bx(-x) + x^2(b) + x^2(-x)$
 $\Rightarrow b^3 - b^2x + b^2x - bx^2 + x^2b - x^3 = b^3 - x^3$
12. $(b^2 + bx - x^2)(b - x) \Rightarrow b^2(b) + b^2(-x) + bx(b) + bx(-x) - x^2(b) - x^2(-x)$
 $\Rightarrow b^3 - b^2x + b^2x - bx^2 - x^2b + x^3 = x^3 - 2bx^2 + b^3$

Multiply the following binomials and polynomials and simplify [See Example 63]

13. $(a^2 + 5a - xy)(a + z) \Rightarrow a^2(a) + a^2(z) + 5a(a) + 5a(z) - xy(a) - xy(z) \Rightarrow a^3 + a^2z + 5a^2 + 5az - axy - xyz$
 $= a^3 - xyz - axy + a^2z + 5az + 5a^2$
14. $(c^2 - cm + cn + mn)(c - m) \Rightarrow c^2(c) + c^2(-m) - cm(c) - cm(-m) + cn(c) + cn(-m) + mn(c) + mn(-m)$
 $\Rightarrow c^3 - c^2m - c^2m + cm^2 + c^2n - cmn + cmn - m^2n = c^3 - 2c^2m + c^2n + cm^2 - m^2n$
15. $(y^2 - x^2)(y^3 + ay^2 - abxy + bx^2 - x^3)$
 $\Rightarrow y^2(y^3) + y^2(ay^2) + y^2(-abxy) + y^2(bx^2) + y^2(-x^3) - x^2(y^3) - x^2(ay^2) - x^2(-abxy) - x^2(bx^2) - x^2(-x^3)$
 $\Rightarrow y^5 + ay^4 - abxy^3 + y^2bx^2 - x^3y^3 - ax^2y^2 + abx^3y - bx^4 + x^5$

$$= x^5 - bx^4 - x^3y^2 + abx^3y - x^2y^3 - ax^2y^2 + bx^2y^2 - abxy^3 + y^5 + ay^4$$

Multiply the following monomials and binomials and simplify [See Example 64]

$$16. \quad 4(5b + 3c)(5b + 3c) \Rightarrow 4[5b(5b) + 5b(3c) + 3c(5b) + 3c(3c)] \Rightarrow 4[25b^2 + 15bc + 15bc + 9c^2] \\ = 100b^2 + 120bc + 36c^2$$

$$17. \quad (4w - 5z)(4w - 5z)d \Rightarrow [4w(4w) + 4w(-5z) - 5z(4w) - 5z(-5z)]d \Rightarrow [16w^2 - 20wz - 20wz + 25z^2]d \\ = 16dw^2 - 40dwz + 25dz^2$$

$$18. \quad z(22.1a - 3.03b)(2.26a + 38.2b) \Rightarrow z[22.1a(2.26a) + 22.1a(38.2b) - 3.03b(2.26a) - 3.03b(38.2b)] \\ \Rightarrow z[49.946a^2 + 844.22ab - 6.8478ab - 115.746b^2] \Rightarrow z[49.946a^2 + 837.37ab - 115.746b^2] \\ = 49.946a^2z + 837.37abz - 115.746b^2z$$

$$19. \quad (d - 4.11)(d - 4.93)(d + 2.26) \Rightarrow [d(d) + d(-4.93) - 4.11(d) - 4.11(-4.93)][(d + 2.26)] \\ \Rightarrow d(d)(d) + d(-4.93)(d) - 4.11(d)(d) - 4.11(-4.93)(d) + d(d)(2.26) + d(-4.93)(2.26) \\ - 4.11(d)(2.26) - 4.11(-4.93)(2.26) \\ \Rightarrow d^3 - 4.93d^2 - 4.11d^2 + 20.2623d + 2.26d^2 - 11.1418d - 9.2886d + 45.79279 \\ \Rightarrow d^3 - 6.78d^2 - 0.1681d + 45.79279 = d^3 - 6.78d^2 - 0.168d + 45.8$$

$$20. \quad (m + 3)(m + 3)(m - 5) \Rightarrow (m^2 + 6m + 9)(m - 5) \Rightarrow m^2(m) + m^2(-5) + 6m(m) + 6m(-5) + 9(m) + 9(-5) \\ \Rightarrow m^3 - 5m^2 + 6m^2 - 30m + 9m - 45 = m^3 + m^2 - 21m - 45$$

$$21. \quad (c + m)(c + n)(c - p) \Rightarrow (c^2 + cn + cm + mn)(c - p) \\ \Rightarrow c^2(c) + c^2(-p) + cn(c) + cn(-p) + cm(c) + cm(-p) + mn(c) + mn(-p) \\ \Rightarrow c^3 - c^2p + c^2n + c^2m - cmp + cmn - cnp - mnp$$

$$22. \quad (z + 5)(z - 2)(z - 5)(z + 2) \Rightarrow (z + 5)(z - 5)(z - 2)(z + 2) \Rightarrow (z^2 - 25)(z^2 - 4) \\ (z^2 - 25)(z^2 - 4) \Rightarrow z^2(z^2) + z^2(-4) - 25(z^2) - 25(-4) \Rightarrow z^4 - 4z^2 - 25z^2 + 100 = z^4 - 29z^2 + 100$$

$$23. \quad (2 + y)(2 - y)(2 - y)(4 + y^2) \Rightarrow (4 - y^2)(4 + y^2)(2 - y) \Rightarrow (16 - y^4)(2 - y) \\ \Rightarrow 16(2) + 16(-y) - y^4(2) - y^4(-y) \Rightarrow 32 - 16y - 2y^4 + y^5 = y^5 - 2y^4 - 16y + 32$$

Multiply the following trinomials and simplify. [See Example 65]

$$24. \quad (x - y - z)(x + y + z) \Rightarrow x(x) + x(y) + x(z) - y(x) - y(y) - y(z) - z(x) - z(y) - z(z) \\ \Rightarrow x^2 + xy + xz - xy - y^2 - yz - xz - yz - z^2 = x^2 - y^2 - 2yz - z^2$$

$$25. \quad (5x^3 + 2xy^2 - 2x)(5x^2 - 2x) \Rightarrow 5x^3(5x^2) + 5x^3(-2x) + 2xy^2(5x^2) + 2xy^2(-2x) - 2x(5x^2) - 2x(-2x) \\ = 25x^5 - 10x^4 + 10x^3y^2 - 4x^2y^2 - 10x^3 + 4x^2$$

$$26. \quad (5x - y + 2x)(4x - y + 6) \Rightarrow 5x(4x) + 5x(-y) + 5x(6) - y(4x) - y(-y) - y(6) + 2x(4x) + 2x(-y) + 2x(6) \\ \Rightarrow 20x^2 - 5xy + 30x - 4xy + y^2 - 6y + 8x^2 - 2xy + 12x = 28x^2 - 11xy + 42x + y^2 - 6y$$

$$27. \quad (x + y - z)(x - y - z) \Rightarrow x(x) + x(-y) + x(-z) + y(x) + y(-y) + y(-z) - z(x) - z(-y) - z(-z)$$

$$\Rightarrow x^2 - xy - xz + xy - y^2 - yz - xz + yz + z^2 = x^2 - 2xz - y^2 + z^2$$

$$28. (a^2 - 5.93a + 31.4)(a^2 - 5.37a + 4.03) \Rightarrow a^2(a^2) + a^2(-5.37a) + a^2(4.03) - 5.93a(a^2) \\ - 5.93a(-5.37a) - 5.93a(4.03) + 31.4(a^2) + 31.4(-5.37a) + 31.4(4.03)$$

$$\Rightarrow a^4 - 5.37a^3 + 4.03a^2 - 5.93a^3 + 31.8441a^2 - 23.8979a + 31.4a^2 - 168.618a + 126.542$$

$$\Rightarrow a^4 - 11.3a^3 + 67.2741a^2 - 192.5159a + 126.542 = a^4 - 11.3a^3 + 67.3a^2 - 193a + 127$$

$$29. (m^3 - 4.83 + 32.4m)(m^2 - 3.37m + 2.26) \Rightarrow m^3(m^2) + m^3(-3.37m) + m^3(2.26) - 4.83(m^2) - 4.83(-3.37m) \\ - 4.83(2.26) + 32.4m(m^2) + 32.4m(-3.37m) + 32.4m(2.26)$$

$$\Rightarrow m^5 - 3.37m^4 + 2.26m^3 - 4.83m^2 + 16.2771m - 10.9158 + 32.4m^3 - 109.188m^2 + 73.224m$$

$$\Rightarrow m^5 - 3.37m^4 + 34.66m^3 - 114.018m^2 + 89.5501m - 10.9158 = m^5 - 3.37m^4 + 34.7m^3 - 114m^2 + 89.6m - 10.9$$

$$30. (am - ym + yx)(am + ym - yx) \Rightarrow am(am) + am(my) + am(-yx) - my(am) - my(my) - my(-yx) \\ + xy(am) + xy(my) + xy(-yx)$$

$$a^2m^2 + am^2y - amxy - am^2y - m^2y^2 + mxy^2 + amxy + mxy^2 - x^2y^2 = a^2m^2 - m^2y^2 + 2mxy^2 - x^2y^2$$

Exercise 8• Powers of Multinomials

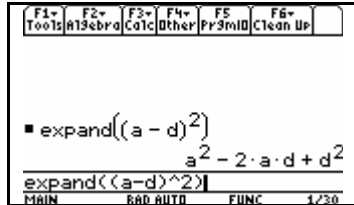
Square each binomial. [See Example 66]

$$1. (x + y)^2 \Rightarrow (x + y)(x + y) \Rightarrow x^2 + xy + xy + y^2 = x^2 + 2xy + y^2$$

$$2. (m + n)^2 \Rightarrow m^2 + mn + mn + n^2 = m^2 + 2mn + n^2$$

$$3. (a - d)^2 \Rightarrow a^2 - ad - ad + d^2 = a^2 - 2ad + d^2$$

By calculator:



$$4. (z - w)^2 \Rightarrow z^2 - wz - wz + w^2 = z^2 - 2wz + w^2$$

$$5. (B + D)^2 \Rightarrow B^2 + BD + BD + D^2 = B^2 + 2BD + D^2$$

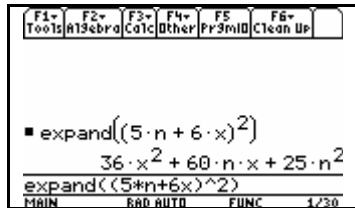
$$6. (C - D)^2 \Rightarrow C^2 - CD - CD + D^2 = C^2 - 2CD + D^2$$

$$7. (4.92y + 3.12z)^2 \Rightarrow (4.92y)^2 + 4.92y(3.12z) + 3.12z(4.92y) + (3.12z)^2 \\ \Rightarrow 24.2064y^2 + 15.3504yz + 15.3504yz + 9.7344z^2 = 24.2y^2 + 30.7yz + 9.73z^2$$

$$8. (2.45d - 1.93x)^2 \Rightarrow (2.45d)^2 - 2.45d(1.93x) - 2.45d(1.93x) + (1.93x)^2 \\ \Rightarrow 6.0025d^2 - 4.7285dx - 4.7285dx + 3.7249x^2 = 6.00d^2 - 9.46dx + 3.72x^2$$

$$9. (5n + 6x)^2 \Rightarrow 25n^2 + 30nx + 30nx + 36x^2 = 25n^2 + 60nx + 36x^2$$

By calculator:



$$10. (d^3 + d^2)^2 \Rightarrow (d^3)^2 + d^3d^2 + d^3d^2 + (d^2)^2 = d^6 + 2d^5 + d^4$$

$$11. (1 - w)^2 \Rightarrow 1 - 2[(1)(w)] + w^2 \Rightarrow 1 - 2w + w^2 = w^2 - 2w + 1$$

$$12. (cy^2 - c^3y)^2 \Rightarrow (cy^2)^2 - 2(cy^2c^3y) + (c^3y)^2 = c^2y^4 - 2c^4y^3 + c^6y^2$$

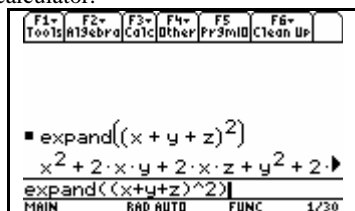
$$13. (b^3 - 13)^2 \Rightarrow (b^3)^2 - 2[b^3(13)] + (13)^2 = b^6 - 26b^3 + 169$$

$$14. (a^n + b^3)^2 \Rightarrow (a^n)^2 + 2[a^n(b^3)] + (b^3)^2 = a^{2n} + 2a^nb^3 + b^6$$

Square each trinomial. [See Example 67]

$$15. (x + y + z)^2 \Rightarrow (x + y + z)(x + y + z) \Rightarrow x^2 + xy + xz + xy + y^2 + yz + xz + yz + z^2 = x^2 + 2xy + 2xz + y^2 + 2yz + z^2$$

By calculator:



$$16. (x - y - z)^2 \Rightarrow x^2 - xy - xz - xy + y^2 + yz - xz + yz + z^2 = x^2 - 2xy - 2xz + y^2 + 2yz + z^2$$

$$17. (a + b - 1)^2 \Rightarrow a^2 + ab - a + ab + b^2 - b - a - b + 1 = a^2 + 2ab - 2a + b^2 - 2b + 1$$

$$18. (5a^3 - 3a + 16)^2 \Rightarrow 5a^3(5a^3) + 5a^3(-3a) + 5a^3(16) - 3a(5a^3) - 3a(-3a) - 3a(16) + 16(5a^3) + 16(-3a) + 16(16) \\ 25a^6 - 15a^4 + 80a^3 - 15a^4 + 9a^2 - 48a + 80a^3 - 48a + 256 = 25a^6 - 30a^4 + 160a^3 + 9a^2 - 96a + 256$$

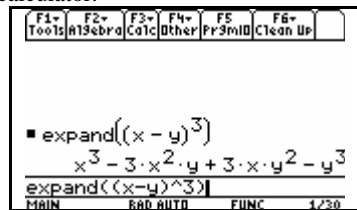
$$19. (c^2 - cd + d^2)^2 = c^4 - 2c^3d + 3c^2d^2 - 2cd^3 + d^4$$

$$20. (w^2 - 5w + 2)^2 \Rightarrow w^4 - 10w^3 + 29w^2 - 20w + 4$$

Cube each binomial. [See Example 68]

$$21. (x - y)^3 \Rightarrow (x^2 - 2xy + y^2)(x - y) \Rightarrow x^2(x) - x^2(y) - 2xy(x) - 2xy(-y) + y^2(x) + y^2(-y) \\ = x^3 - 3x^2y + 3xy^2 - y^3$$

By calculator:



22. $(1.93 - 3.24a)^3 \Rightarrow (3.7249 - 12.5064a + 10.4976a^2)(1.93 - 3.24a) \Rightarrow 3.7249(1.93)$
 $+ 3.7249(-3.24a) - 12.5064a(1.93) - 12.5064a(-3.24a) + 10.4976a^2(1.93) + 10.4976a^2(-3.24a)$
 $= -34.0a^3 + 60.8a^2 - 36.2a + 7.19$
23. $(3.02m + 2.16n)^3 \Rightarrow (9.1204m^2 + 13.0464mn + 4.6656n^2)(3.02m + 2.16n)$
 $\Rightarrow 9.1204m^2(3.02m) + 9.1204m^2(2.16n)$
 $\Rightarrow 13.0464mn(3.02m) + 13.0464mn(2.16n) + 4.6656n(3.02m) + 4.6656n(2.16n)$
 $= 27.5m^3 + 59.1m^2n + 42.3mn^2 + 10.1n^3$
24. $(a^2 + 1)^3 \Rightarrow (a^4 + 2a^2 + 1)(a^2 + 1) \Rightarrow a^4(a^2) + a^4(1) + 2a^2(a^2) + 2a^2(1) + 1(a^2) + 1(1) = a^6 + 3a^4 + 3a^2 + 1$
25. $(c + d)^3 \Rightarrow (c^2 + 2cd + d^2)(c + d) \Rightarrow c^2(c) + c^2(d) + 2cd(c) + 2cd(d) + d^2(c) + d^2(d) = c^3 + 3c^2d + 3cd^2 + d^3$
26. $(4p - q)^3 \Rightarrow (16p^2 - 8pq + q^2)(4p - q) \Rightarrow 16p^2(4p) + 16p^2(-q) - 8pq(4p) - 8pq(-q) + q^2(4p) + q^2(-q)$
 $= 64p^3 - 48p^2q + 12pq^2 - q^3$

Applications

27. $(x + 2)^2 \Rightarrow (x + 2)(x + 2) = x^2 + 4x + 4$
28. $I^2R = (I + 2.5)^2 R \Rightarrow [(I + 2.5)(I + 2.5)]R \Rightarrow [I^2 + 5.0I + 6.25]R = I^2R + 5.0IR + 6.25R$
29. $\frac{4}{3}\pi(r - 2)^3 \Rightarrow \frac{4}{3}\pi[(r^2 - 4r + 4)(r - 2)] \Rightarrow \frac{4}{3}\pi(r^3 - 2r^2 - 4r^2 + 8r + 4r - 8) \Rightarrow \frac{4}{3}\pi(r^3 - 6r^2 + 12r - 8)$
 $\Rightarrow \left(\frac{4}{3}\pi\right)r^3 - \left(\frac{4}{3}\pi\right)6r^2 + \left(\frac{4}{3}\pi\right)12r - \left(\frac{4}{3}\pi\right)8 = 4.19r^3 - 25.1r^2 + 50.3r - 33.5$

Exercise 9• Removing Symbols of Grouping

Remove symbols of grouping, and simplify. [See Examples 70 and 71]

- $a + (b + a) \Rightarrow a + b + a = 2a + b$
- $x + (2 - x) \Rightarrow x + 2 - x = 2$
- $x + (x - y) \Rightarrow x + x - y = 2x - y$
- $-(a + 3.92) - (a - 4.14) \Rightarrow -a - 3.92 - a + 4.14 = 0.22 - 2a$
- $(4z + 2) + (z - 5) \Rightarrow 4z + 2 + z - 5 = 5z - 3$

6. $(2x + 5) + (x - 2) \Rightarrow 2x + 5 + x - 2 = 3x + 3$

7. $(c - 5) - (6 + 3c) \Rightarrow c - 5 - 6 - 3c = -2c - 11$

8. $(2x + 6a) - (4x - a) \Rightarrow 2x + 6a - 4x + a = 7a - 2x$

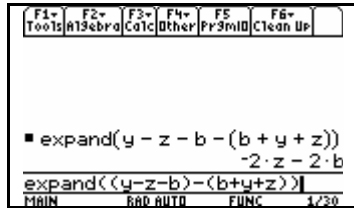
9. $(0.826c - 5.37) - (2.76 + 0.273c) \Rightarrow 0.826c - 0.273c - 5.37 - 2.76 = 0.533c - 8.13$

10. $(7262x + 1.26a) - (2844x - 8.23a) \Rightarrow 7262x + 1.26a - 2844x + 8.23a = 4418x + 9.49a$

11. $(5 - 2a^3 + 3a^4) + (4a^4 - 6a^3 - 7) \Rightarrow 5 - 2a^3 + 3a^4 + 4a^4 - 6a^3 - 7 = 7a^4 - 8a^3 - 2$

12. $(y - z - b) - (b + y + z) \Rightarrow y - z - b - b - y - z = -2b - 2z$

By calculator:



13. $(5bc + 6c - a) - (8c - 2a + 3bc) \Rightarrow 5bc + 6c - a - 8c + 2a - 3bc = 2bc - 2c + a$

14. $(2z + 5c - 3a) - (6a + 2c - 4z) \Rightarrow 2z + 5c - 3a - 6a - 2c + 4z = 6z + 3c - 9a$

15. $x - 2.66[y - 3.02(x + 6.22y) + 4.98y] \Rightarrow x - 2.66[y - 3.02x - 18.7844y + 4.98y] \Rightarrow x - 2.66[-3.02x - 13.8044y]$
 $\Rightarrow x + 8.0332x + 36.7197y = 9.03x + 36.7y$

16. $[(5x - 4y) - (2x - 5y)][(-3x - 6y) - (2x + 4y)] \Rightarrow [5x - 4y - 2x + 5y][-3x - 6y - 2x - 4y]$
 $\Rightarrow [3x + y][-5x - 10y] \Rightarrow -15x^2 - 30xy - 5xy - 10y^2 = -15x^2 - 35xy - 10y^2$

17. $\{[3 - (x + 7)] - 3x\} - (x + 7) \Rightarrow (3 - x - 7 - 3x) - x - 7 \Rightarrow 3 - x - 7 - 3x - x - 7 = -5x - 11$

18. $[a(a + b) - a^2](a^2 + b^2)(a^3 - 4ab + b^3) \Rightarrow [a^2 + ab - a^2](a^2 + b^2)(a^3 - 4ab + b^3) \Rightarrow (ab)(a^2 + b^2)(a^3 - 4ab + b^3)$
 $\Rightarrow (ab)(a^5 - 4a^3b + a^2b^3 + a^3b^2 - 4ab^3 + b^5) \Rightarrow a^6b - 4a^4b^2 + a^3b^4 + a^4b^3 - 4a^2b^4 + ab^6$

19. $6p - \{3p + [2q - ((5p + 4q) + p) - (3p + 2)] - 2p\} \Rightarrow 6p - \{3p + 2q - 5p - 4q - p - 3p - 2 - 2p\}$
 $\Rightarrow 6p - (-8p - 2q - 2) \Rightarrow 6p + 8p + 2q + 2 = 14p + 2q + 2$

20. $[(5a - 2)(2a + 3) - 6a^2][(3a - 4)(2a - 4) + 5a^2] \Rightarrow [10a^2 + 11a - 6 - 6a^2][6a^2 - 20a + 16 + 5a^2]$
 $\Rightarrow [4a^2 + 11a - 6][11a^2 - 20a + 16] \Rightarrow 44a^4 + 41a^3 - 222a^2 + 296a - 96$

21. $(2.25c - 9.28b + 3.82a) - (-3.92a - 1.72b - 8.33c) \Rightarrow 2.25c - 9.28b + 3.82a + 3.92a + 1.72b + 8.33c$
 $= 7.74a - 7.56b + 10.58c$

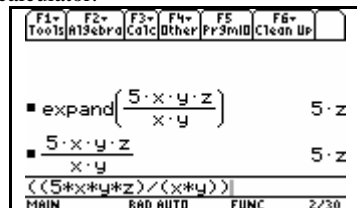
22. $(23y^2 + 4y^3 - 12) - (11y^3 - 8y^2 + y) \Rightarrow 23y^2 + 4y^3 - 12 - 11y^3 + 8y^2 - y = -7y^3 + 31y^2 - y - 12$
23. $24ab - (16ab - 3x^2 + 7z - 2y^2) \Rightarrow 24ab - 16ab + 3x^2 - 7z + 2y^2 = 8ab + 3x^2 - 7z + 2y^2$
24. $(18y^2 - 12xy) - (6y^2 + xy - a) \Rightarrow 18y^2 - 12xy - 6y^2 - xy + a = 12y^2 - 13xy + a$
25. $(4y + 2y^2) - (3y - 6b + 4y^2 + 5) \Rightarrow 4y + 2y^2 - 3y + 6b - 4y^2 - 5 = y - 2y^2 + 6b - 5$
26. $(-6x - z) - \{3y + [7x - (3z + 8y + x)]\} \Rightarrow -6x - z - \{3y + 7x - 3z - 8y - x\} \Rightarrow -6x - z - 3y - 7x + 3z + 8y + x = -12x + 5y + 2z$

Exercise 10• Quotient of Two Monomials

Divide the following monomials. [See Examples 74, 75, 76, 77, 78, 79 and 80]

1. $\frac{x^7}{x^4} \Rightarrow x^{7-4} = x^3$
2. $\frac{21a^4}{3a^2} \Rightarrow \frac{21}{3}a^{4-2} = 7a^2$
3. $\frac{5xyz}{xy} \Rightarrow 5\left(\frac{xyz}{xy}\right) \Rightarrow 5x^{1-1}y^{1-1}z = 5z$

By calculator:



4. $\frac{m^3n}{mn} \Rightarrow m^{3-1}n^{1-1} = m^2$
5. $\frac{4a^2d}{-2ad} \Rightarrow \left(\frac{4}{-2}\right)a^{2-1}d^{1-1} = -2a$
6. $\frac{-360x^4y^2}{-30x^2y} \Rightarrow \left(\frac{-360}{-30}\right)x^{4-2}y^{2-1} = 12x^2y$
7. $\frac{31ab^9}{ab^8} \Rightarrow 31a^{1-1}b^{9-8} = 31b$
8. $\frac{54x^2z}{-9x} \Rightarrow \left(\frac{54}{-9}\right)x^{2-1}z = -6xz$
9. $\frac{-99ad}{3a} \Rightarrow \left(\frac{-99}{3}\right)a^{1-1}d = -33d$

$$10. \frac{49xyz}{-7y} \Rightarrow \left(\frac{-49}{7}\right)xy^{1-1}z = -7xz$$

$$11. 42p^5q^4r^2 \div 7p^3qr \Rightarrow 6p^{5-3}q^{4-1}r^{2-1} = 6p^2q^3r \text{ (divide the coefficients, subtract the exponents of the same base)}$$

$$12. 50x^3y^5z^3 \div (-10xy^3z^2) = -5x^2y^2z$$

$$13. -32m^2nx \div 4mx = -8mn$$

$$14. 48cd^2z^3 \div (-24cd) = -2dz^3$$

$$15. -36a^4b^2c \div 9ab = -4a^3bc$$

$$16. \frac{45m^2q}{-5mq} = -9m$$

$$17. \frac{-24m^3n^3z}{4m^3z} = -6n^3 \text{ (divide the coefficients, subtract the exponents of the same base)}$$

$$18. \frac{-27a^5p^2}{-9a^3q^2} = \frac{3a^2p^2}{q^2}$$

$$19. \frac{25a^4bcxyz}{5a^2bcxz} = 5a^2y$$

$$20. \frac{18d^3f^2}{3d^2f} = 6df$$

$$21. 32a^2bc \div (-8ab) = -4ac \text{ (divide the coefficients, subtract the exponents of the same base)}$$

$$22. -12m^2n^3 \div 4mn^2 = -3mn$$

$$23. -36a^2by^2 \div 12a^2y = -3by$$

$$24. 44a^2b^3c^4 \div 11a^2bc = 4b^2c^3$$

$$25. \frac{64x^2y^2}{8xy} = 8xy$$

$$26. \frac{24pq^2r^3s}{8r} = 3pq^2r^2s \text{ (divide the coefficients, subtract the exponents of the same base)}$$

$$27. \frac{x^2y^6z^2}{x^2z^2} = y^6$$

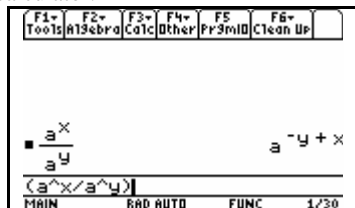
$$28. \frac{(a-x)^5}{(a-x)^2} \Rightarrow (a-x)^{5-2} = (a-x)^3$$

Remember, $(a-x)^5$ is equivalent to $(a-x)(a-x)(a-x)(a-x)(a-x)$

$$\begin{aligned} \text{Thus, } \frac{(a-x)^5}{(a-x)^2} &\Rightarrow \frac{(a-x)(a-x)(a-x)(a-x)(a-x)}{(a-x)(a-x)} \Rightarrow \frac{\cancel{(a-x)}\cancel{(a-x)}(a-x)(a-x)(a-x)}{\cancel{(a-x)}\cancel{(a-x)}} \\ &\Rightarrow (a-x)(a-x)(a-x) = (a-x)^3 \end{aligned}$$

29. $\frac{a^x}{a^y} = a^{x-y}$ (the laws of exponents remain the same)

By calculator:



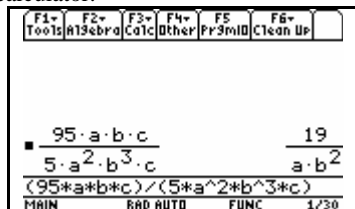
30. $66c^2dy^3 \div (-22cy) \Rightarrow \left(\frac{66}{-22}\right)c^{2-1}dy^{3-1} = -3cdy^2$

31. $-35a^3b^2z \div 7ab^2 = -5a^2z$

32. $19e^2m^2n^2 \div (-em^2n) \Rightarrow \left(\frac{19}{-1}\right)e^{2-1}m^{2-2}n^{2-1} = -19en$

33. $95abc \div 5a^2b^3c \Rightarrow \left(\frac{95}{5}\right)a^{1-2}b^{1-3}c^{1-1} \Rightarrow 19a^{-1}b^{-2}c^0 \Rightarrow \frac{19}{1}\left(\frac{1}{a}\right)\left(\frac{1}{b^2}\right) = \frac{19}{ab^2}$

By calculator:



34. $45a^2b^2d^2 \div 15abd = 3abd$

Exercise 11 • Dividing a Polynomial by a Monomial

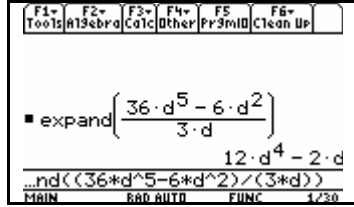
Divide each polynomial by the given monomial. [See Examples 81 and 82]

1. $\frac{15x^3 + 3x^2}{x} \Rightarrow \frac{15x^3}{x} + \frac{3x^2}{x} \Rightarrow 15x^{3-1} + 3x^{2-1} = 15x^2 + 3x$

2. $\frac{42m^6 - 2m^3}{2m} \Rightarrow \frac{42m^6}{2m} - \frac{2m^3}{2m} \Rightarrow \left(\frac{42}{2}\right)m^{6-1} - \left(\frac{2}{2}\right)m^{3-1} = 21m^5 - m^2$

3. $\frac{36d^5 - 6d^2}{3d} \Rightarrow \frac{36d^5}{3d} - \frac{6d^2}{3d} = 12d^4 - 2d$

By calculator:



4. $\frac{22x^2 + 11y^5}{11} \Rightarrow \frac{22x^2}{11} + \frac{11y^5}{11} = 2x^2 + y^5$
5. $\frac{48c^4 + 36c^5}{12c^2} \Rightarrow \frac{48c^4}{12c^2} + \frac{36c^5}{12c^2} = 4c^2 + 3c^3$
6. $\frac{27n^3 - 9n^2}{-3n} \Rightarrow \frac{27n^3}{-3n} - \frac{9n^2}{-3n} \Rightarrow -9n^2 - (-3n) = -9n^2 + 3n$
7. $\frac{39p^2 + 52p^3}{-13p^2} \Rightarrow \frac{39p^2}{-13p^2} + \frac{52p^3}{-13p^2} = -3 - 4p$
8. $\frac{40a^5 - 20a^2}{10a} \Rightarrow \frac{40a^5}{10a} - \frac{20a^2}{10a} = 4a^4 - 2a$
9. $\frac{-25x^3 - 15x^2}{5x} \Rightarrow \frac{-25x^3}{5x} - \frac{15x^2}{5x} = -5x^2 - 3x$
10. $\frac{-55a^3d + 22a^2}{-a^2} \Rightarrow \frac{-55a^3d}{-a^2} + \frac{22a^2}{-a^2} = 55ad - 22$
11. $\frac{-bm^3n^2 - 3bm^2n}{-m^2n} \Rightarrow \frac{-bm^3n^2}{-m^2n} - \frac{3bm^2n}{-m^2n} \Rightarrow bmn - (-3b) = bmn + 3b$
12. $\frac{-5a^5b + 5ab^2}{-ab} \Rightarrow \frac{-5a^5b}{-ab} + \frac{5ab^2}{-ab} \Rightarrow 5a^4 + (-5b) = 5a^4 - 5b$
13. $\frac{8a^3b^2 + 4a^2b^3}{4ab^2} \Rightarrow \frac{8a^3b^2}{4ab^2} + \frac{4a^2b^3}{4ab^2} = 2a^2 + ab$
14. $\frac{10x^3y - 5xy^4}{-5xy} \Rightarrow \frac{10x^3y}{-5xy} - \frac{5xy^4}{-5xy} = -2x^2 + y^3$
15. $\frac{x^2y^3z - xy^4z^2}{-xy^2z} \Rightarrow \frac{x^2y^3z}{-xy^2z} - \frac{xy^4z^2}{-xy^2z} = -xy + y^2z$
16. $\frac{16a^3bc^2 + 12a^2b^5c}{-4abc} \Rightarrow \frac{16a^3bc^2}{-4abc} + \frac{12a^2b^5c}{-4abc} = -4a^2c - 3ab^4$
17. $\frac{m^2n^2 + m^3n^2 - m^2n^3}{-mn} \Rightarrow \frac{m^2n^2}{-mn} + \frac{m^3n^2}{-mn} - \frac{m^2n^3}{-mn} \Rightarrow -mn - m^2n + mn^2 = mn^2 - mn - m^2n$

$$18. \frac{p^5q^2 - p^2q^5 - p^3q^3}{p^2q^2} \Rightarrow \frac{p^5q^2}{p^2q^2} - \frac{p^2q^5}{p^2q^2} - \frac{p^3q^3}{p^2q^2} = p^3 - q^3 - pq$$

$$19. \frac{x^3y^3 - x^4y + xy^4}{-xy} \Rightarrow \frac{x^3y^3}{-xy} - \frac{x^4y}{-xy} + \frac{xy^4}{-xy} = -x^2y^2 + x^3 - y^3$$

$$20. \frac{-a^3b^3 - a^2b^2 - ab}{-ab} \Rightarrow \frac{-a^3b^3}{-ab} - \frac{a^2b^2}{-ab} - \frac{ab}{-ab} = a^2b^2 + ab + 1$$

$$21. \frac{c^3 - 4c^2d^2 + d^3}{cd^2} \Rightarrow \frac{c^3}{cd^2} - \frac{4c^2d^2}{cd^2} + \frac{d^3}{cd^2} \Rightarrow \frac{c^2}{d^2} - 4c + \frac{d}{c}$$

$$22. \frac{r^4s^3 - r^2s^2 + r^4s^2}{-r^2s} \Rightarrow \frac{r^4s^3}{-r^2s} - \frac{r^2s^2}{-r^2s} + \frac{r^4s^2}{-r^2s} = -r^2s^2 + s - r^2s$$

$$23. \frac{a^4 + 2a^2b^2 - b^4}{a^2b^2} \Rightarrow \frac{a^4}{a^2b^2} + \frac{2a^2b^2}{a^2b^2} - \frac{b^4}{a^2b^2} = \frac{a^2}{b^2} + 2 - \frac{b^2}{a^2}$$

By calculator:

The image shows a calculator screen with the following text:

 ■ expand((a^4 + 2 * a^2 * b^2 - b^4) / (a^2 * b^2))

 a^2 / b^2 - b^2 / a^2 + 2

 ...*a^2*b^2-b^4)/(a^2*b^2)

 The calculator interface includes function keys (F1-F6), a numeric keypad, and a status bar at the bottom showing 'MAIN', 'RAD AUTO', 'FUNC', and '1/20'.

$$24. \frac{m^5n^2 + m^2n^2 - m^2n^4}{-m^2n} \Rightarrow \frac{m^5n^2}{-m^2n} + \frac{m^2n^2}{-m^2n} - \frac{m^2n^4}{-m^2n} = -m^3n - n + n^3$$

$$25. \frac{4x^3z + 2xz^2 - 3z^4}{-xz} \Rightarrow \frac{4x^3z}{-xz} + \frac{2xz^2}{-xz} - \frac{3z^4}{-xz} = -4x^2 - 2z + \frac{3z^3}{x}$$

$$26. \frac{ab^3 + a^3c^2 - b^2c^4}{-abc} \Rightarrow \frac{ab^3}{-abc} + \frac{a^3c^2}{-abc} - \frac{b^2c^4}{-abc} = -\frac{b^2}{c} - \frac{a^2c}{b} + \frac{bc^3}{a}$$

$$27. \frac{p^3q^3 + pq^2r^3 - p^2r^4}{-p^2r} \Rightarrow \frac{p^3q^3}{-p^2r} + \frac{pq^2r^3}{-p^2r} - \frac{p^2r^4}{-p^2r} = -\frac{pq^3}{r} - \frac{q^2r^2}{p} + r^3$$

$$28. \frac{7m^4n^2 + 7m^3n^3 - 7m^2n^4}{-7m^2n^3} \Rightarrow \frac{7m^4n^2}{-7m^2n^3} + \frac{7m^3n^3}{-7m^2n^3} - \frac{7m^2n^4}{-7m^2n^3} = -\frac{m^2}{n} - m + n$$

$$29. \frac{4c^4d + 3c^2d^3 - cd^5}{-cd^2} \Rightarrow \frac{4c^4d}{-cd^2} + \frac{3c^2d^3}{-cd^2} - \frac{cd^5}{-cd^2} = -\frac{4c^3}{d} - 3cd + d^3$$

$$30. \frac{3a^2x + 5a^3x^3 - 2ax^2}{-a^2x^2} \Rightarrow \frac{3a^2x}{-a^2x^2} + \frac{5a^3x^3}{-a^2x^2} - \frac{2ax^2}{-a^2x^2} = -\frac{3}{x} - 5ax + \frac{2}{a}$$

$$31. \frac{8b^2c^4 + 4b^2c - 12b^3c^3}{-b^3c^2} \Rightarrow \frac{8b^2c^4}{-b^3c^2} + \frac{4b^2c}{-b^3c^2} - \frac{12b^3c^3}{-b^3c^2} = -\frac{8c^2}{b} - \frac{4}{bc} + 12c$$

Exercise 12 • Quotient of Two Polynomials

Divide each polynomial by the given binomial. [See Examples 84 and 85]

1. $a^2 + 15a + 56 \div a + 7$

$$\begin{array}{r}
 a+8 \\
 a+7 \overline{) a^2 + 15a + 56} \\
 \underline{-(a^2 + 7a)} \downarrow \\
 8a + 56 \\
 \underline{-(8a + 56)} \\
 0
 \end{array}$$

Thus, $a^2 + 15a + 56 \div a + 7 = a + 8$

Some problems may also be done by factoring (which you will see in Chapter 10)

$$\frac{a^2 + 15a + 56}{a + 7} \Rightarrow \frac{(a + 8)(a + 7)}{a + 7} \Rightarrow \frac{(a + 8)\cancel{(a + 7)}}{\cancel{a + 7}} = a + 8$$

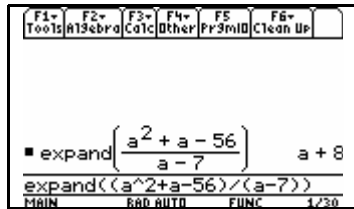
2. $a^4 + 3a^2 + 2 \div a^2 + 1$

$$\begin{array}{r}
 a^2 + 2 \\
 a^2 + 1 \overline{) a^4 + 3a^2 + 2} \\
 \underline{-(a^4 + a^2)} \downarrow \\
 2a^2 + 2 \\
 \underline{-(2a^2 + 2)} \\
 0
 \end{array}$$

3. $a^2 + a - 56 \div a - 7$

$$\begin{array}{r}
 a+8 \\
 a-7 \overline{) a^2 + a - 56} \\
 \underline{-(a^2 - 7a)} \downarrow \\
 8a - 56 \\
 \underline{-(8a - 56)} \\
 0
 \end{array}$$

By calculator:



The image shows a calculator screen with the following text:

 ■ expand $\left(\frac{a^2 + a - 56}{a - 7} \right)$ a + 8

 expand($\langle a^2 + a - 56 \rangle / \langle a - 7 \rangle$)

 The calculator interface includes function keys (F1-F6), a numeric keypad, and a status bar at the bottom showing 'MAIN', 'RAD AUTO', 'FUNC', and '1/30'.

4. $4x^2 + 23x + 15 \div 4x + 3$

$$\begin{array}{r}
 \overline{x+5} \\
 4x+3 \overline{)4x^2+23x+15} \\
 \underline{-(4x^2+3x)} \downarrow \\
 20x+15 \\
 \underline{-(20x+15)} \\
 0
 \end{array}$$

$$\begin{array}{r}
 5. \quad 2x^2+11x+5 \div 2x+1 \\
 \overline{x+5} \\
 2x+1 \overline{)2x^2+11x+5} \\
 \underline{-(2x^2+x)} \downarrow \\
 10x+5 \\
 \underline{-(10x+5)} \\
 0
 \end{array}$$

$$\begin{array}{r}
 6. \quad 6x^2-7x-3 \div 2x-3 \\
 \overline{3x+1} \\
 2x-3 \overline{)6x^2-7x-3} \\
 \underline{-(6x^2-9x)} \downarrow \\
 2x-3 \\
 \underline{-(2x-3)} \\
 0
 \end{array}$$

$$\begin{array}{r}
 7. \quad a^2-15a+56 \div a-7 \\
 \overline{a-8} \\
 a-7 \overline{)a^2-15a+56} \\
 \underline{-(a^2-7a)} \downarrow \\
 -8a+56 \\
 \underline{-(-8a+56)} \\
 0
 \end{array}$$

$$\begin{array}{r}
 8. \quad a^2-a-56 \div a+7 \\
 \overline{a-8} \\
 a+7 \overline{)a^2-a-56} \\
 \underline{-(a^2+7a)} \downarrow \\
 -8a-56 \\
 \underline{-(-8a-56)} \\
 0
 \end{array}$$

$$9. \quad 3x^2-4x-4 \div 2-x$$

$$\begin{array}{r}
 \overline{-3x-2} \\
 2-x \overline{)3x^2-4x-4} \\
 \underline{-(3x^2-6x)} \downarrow \\
 2x-4 \\
 \underline{-(2x-4)} \\
 0
 \end{array}$$

10. $27x^3 - 8y^3 \div 3x - 2y$

$$\begin{array}{r}
 \overline{9x^2 + 6xy + 4y^2} \\
 3x-2y \overline{)27x^3 + 0x^2y + 0xy^2 - 8y^3} \\
 \underline{-(27x^3 - 18x^2y)} \downarrow \downarrow \\
 18x^2y + 0xy^2 \downarrow \\
 \underline{-(18x^2y - 12xy^2)} \downarrow \\
 12xy^2 - 8y^3 \\
 \underline{-(12xy^2 - 8y^3)} \\
 0
 \end{array}$$

11. $a^8 - 3a^4 + 2 \div a^4 - 1$

$$\begin{array}{r}
 \overline{a^4-2} \\
 a^4-1 \overline{)a^8-3a^4+2} \\
 \underline{-(a^8 - a^4)} \downarrow \\
 -2a^4+2 \\
 \underline{-(-2a^4+2)} \\
 0
 \end{array}$$

By calculator:

Calculator screen showing the expansion of $\frac{a^8 - 3a^4 + 2}{a^4 - 1}$. The display shows the expression $\text{expand}\left(\frac{a^8 - 3 \cdot a^4 + 2}{a^4 - 1}\right)$ and the result $a^4 - 2$.

12. $a^3 - 8a - 3 \div a - 3$

$$\begin{array}{r}
 \overline{a^2+3a+1} \\
 a-3 \overline{)a^3+0a^2-8a-3} \\
 \underline{-(a^3-3a^2)} \downarrow \downarrow \\
 3a^2-8a \downarrow \\
 \underline{-(3a^2-9a)} \downarrow \\
 a-3 \\
 \underline{-(a-3)} \\
 0
 \end{array}$$

13. $x^2 - 4x + 3 \div x + 2$

$$\begin{array}{r}
 x-6 \\
 x+2 \overline{)x^2 - 4x + 3} \\
 \underline{-(x^2 + 2x)} \downarrow \\
 -6x + 3 \\
 \underline{-(-6x - 12)} \\
 15
 \end{array}$$

Thus $x^2 - 4x + 3 \div x + 2 = x - 6 + \frac{15}{x+2}$

14. $2 + 4x - x^2 \div 4 - x$

$$\begin{array}{r}
 x \\
 -x+4 \overline{) -x^2 + 4x + 2} \quad (\text{write the terms in descending order of the powers}) \\
 \underline{-(-x^2 + 4x)} \downarrow \\
 0x + 2
 \end{array}$$

Thus $2 + 4x - x^2 \div 4 - x = x + \frac{2}{4-x}$

15. $4 + 2x - 5x^2 \div 3 - x$

$$\begin{array}{r}
 5x+13 \\
 -x+3 \overline{) -5x^2 + 2x + 4} \quad (\text{write the terms in descending order of the powers}) \\
 \underline{-(-5x^2 + 15x)} \downarrow \\
 -13x + 4 \\
 \underline{-(-13x + 39)} \\
 -35
 \end{array}$$

Thus $4 + 2x - 5x^2 \div 3 - x = 5x + 13 - \frac{35}{3-x}$

By calculator:

Calculator screen showing the expansion of $\frac{4 + 2x - 5x^2}{3 - x}$. The screen displays the expression and its simplified form: $\frac{35}{x - 3} + 5x + 13$. Below the expression, it shows the remainder calculation: $\dots\text{and}((4+2*x-5*x^2)/(3-x))|$.

16. $2x^2 - 5x + 4 \div x + 1$

$$\begin{array}{r}
 \overline{2x-7} \\
 x+1 \overline{)2x^2-5x+4} \\
 \underline{-(2x^2+2x)} \quad \downarrow \\
 -7x+4 \\
 \underline{-(-7x-7)} \\
 11
 \end{array}$$

$$\text{Thus } 2x^2 - 5x + 4 \div x + 1 = 2x - 7 + \frac{11}{x+1}$$

CHAPTER 2 REVIEW PROBLEMS

- $$(b^4 + b^2x^3 + x^4)(b^2 - x^2) \Rightarrow b^4(b^2) + b^4(-x^2) + b^2x^3(b^2) + b^2x^3(-x^2) + x^4(b^2) + x^4(-x^2)$$

$$= b^6 - b^4x^2 + b^4x^3 - b^2x^5 + b^2x^4 - x^6$$
- $$(x + y - 2)^2 \Rightarrow (x + y - 2)(x + y - 2) \Rightarrow x(x) + x(y) + x(-2) + y(x) + y(y) + y(-2) - 2(x) - 2(y) - 2(-2)$$

$$\Rightarrow x^2 + xy - 2x + xy + y^2 - 2y - 2x - 2y + 4 = x^2 + 2xy - 4x + y^2 - 4y + 4$$
- $$(7.28 \times 10^4)^3 \Rightarrow (7.28)^3 \times (10^4)^3 \Rightarrow 385.82835 \times 10^{12} = 3.86 \times 10^{14}$$
- $$(xy + 5)^2 \Rightarrow (xy + 5)(xy + 5) \Rightarrow xy(xy) + xy(5) + 5(xy) + 5(5) \Rightarrow x^2y^2 + 10xy + 25$$
- $$(3x - m)(x^2 + m^2)(3x - m) \Rightarrow (3x - m)(3x - m)(x^2 + m^2) \Rightarrow (9x^2 - 6mx + m^2)(x^2 + m^2)$$

$$\Rightarrow 9x^2(x^2) + 9x^2(m^2) - 6mx(x^2) - 6mx(m^2) + m^2(x^2) + m^2(m^2) \Rightarrow 9x^4 + 10x^2m^2 - 6mx^3 - 6m^3x + m^4$$
- $$\frac{-x^6 - 2x^5 - x^4}{-x^4} \Rightarrow \frac{-x^6}{-x^4} - \frac{2x^5}{-x^4} - \frac{x^4}{-x^4} \Rightarrow x^{6-4} + 2x^{5-4} + x^{4-4} = x^2 + 2x + 1$$
- $$(2x + 1)^3 \Rightarrow (2x + 1)(2x + 1)(2x + 1) \Rightarrow (4x^2 + 4x + 1)(2x + 1) \Rightarrow 4x^2(2x) + 4x^2(1) + 4x(2x) + 4x(1) + 1(2x) + 1(1)$$

$$\Rightarrow 8x^3 + 4x^2 + 8x^2 + 4x + 2x + 1 = 8x^3 + 12x^2 + 6x + 1$$
- $$7x - \{-6x - [-5x - (-4x - 3x) - 2]\} \Rightarrow 7x - \{-6x - [-5x - (-7x) - 2]\} \Rightarrow 7x - \{-6x - [2x - 2]\}$$

$$\Rightarrow 7x - \{-6x - 2x + 2\} \Rightarrow 7x - \{-8x + 2\} \Rightarrow 7x + 8x - 2 = 15x - 2$$
- $$(3ax^2)(2ax^3) \Rightarrow (3)(2)(a^{1+1})(x^{2+3}) = 6a^2x^5$$
- $$\left(\frac{3a^2}{2b^3}\right)^3 \Rightarrow \frac{3^{1 \cdot 3} a^{2 \cdot 3}}{2^{1 \cdot 3} b^{3 \cdot 3}} \Rightarrow \frac{3^3 a^6}{2^3 b^9} = \frac{27a^6}{8b^9}$$
- $$\frac{a^2x - abx - acx}{ax} \Rightarrow \frac{a^2x}{ax} - \frac{abx}{ax} - \frac{acx}{ax} \Rightarrow a^{2-1}x^{1-1} - a^{1-1}bx^{1-1} - a^{1-1}cx^{1-1} = a - b - c$$

12. $\frac{3x^5y^3 - 3x^4y^3 - 3x^2y^4}{3x^3y^2} \Rightarrow \frac{3x^5y^3}{3x^3y^2} - \frac{3x^4y^3}{3x^3y^2} - \frac{3x^2y^4}{3x^3y^2} = x^2y - xy - \frac{y^2}{x}$
13. $(4a - 3b)^2 \Rightarrow (4a - 3b)(4a - 3b) \Rightarrow 16a^2 - 12ab - 12ab + 9b^2 = 16a^2 - 24ab + 9b^2$
14. $(x^3 - xy + y^2)(x + y) \Rightarrow x^3(x) + x^3(y) - xy(x) - xy(y) + y^2(x) + y^2(y) \Rightarrow x^4 + x^3y - x^2y - xy^2 + xy^2 + y^3$
 $= x^4 + x^3y - x^2y + y^3$
15. $(xy - 2)(xy - 4) \Rightarrow xy(xy) + xy(-4) - 2(xy) - 2(-4) \Rightarrow x^2y^2 - 4xy - 2xy + 8 = x^2y^2 - 6xy + 8$
16. $\frac{x^{m+1} + x^{m+2} + x^{m+3} + x^{m+4}}{x^4} \Rightarrow \frac{x^{m+1}}{x^4} + \frac{x^{m+2}}{x^4} + \frac{x^{m+3}}{x^4} + \frac{x^{m+4}}{x^4} \Rightarrow x^{m+1-4} + x^{m+2-4} + x^{m+3-4} + x^{m+4-4}$
 $= x^{m-3} + x^{m-2} + x^{m-1} + x^m$
17. $(2a - 3b)^2 \Rightarrow (2a - 3b)(2a - 3b) \Rightarrow 4a^2 - 6ab - 6ab + 9b^2 = 4a^2 - 12ab + 9b^2$
18. $(a^2 - 3a + 8)(a + 3) \Rightarrow a^2(a) + a^2(3) - 3a(a) - 3a(3) + 8(a) + 8(3) \Rightarrow a^3 + 3a^2 - 3a^2 - 9a + 8a + 24 = a^3 - a + 24$
19. $\frac{a^3b^2 - a^2b^5 - a^4b^2}{a^2b} \Rightarrow \frac{a^3b^2}{a^2b} - \frac{a^2b^5}{a^2b} - \frac{a^4b^2}{a^2b} = ab - b^4 - a^2b$
20. $(2x - 5)(x + 2) \Rightarrow 2x(x) + 2x(2) - 5(x) - 5(2) \Rightarrow 2x^2 + 4x - 5x - 10 = 2x^2 - x - 10$
21. $(2m - c)(2m + c)(4m^2 + c^2) \Rightarrow (4m^2 - c^2)(4m^2 + c^2) = 16m^4 - c^4$
22. $\left(\frac{8x^5y^{-2}}{4x^3y^{-3}}\right)^3 \Rightarrow \frac{8^{1 \cdot 3} x^{5 \cdot 3} y^{-2 \cdot 3}}{4^{1 \cdot 3} x^{3 \cdot 3} y^{-3 \cdot 3}} \Rightarrow \frac{8^3 x^{15} y^{-6}}{4^3 x^9 y^{-9}} \Rightarrow \left(\frac{512}{64}\right) x^{15-9} y^{-6-(-9)} = 8x^6 y^3$
23. $\frac{2a^6}{a^4} \Rightarrow 2a^{6-4} = 2a^2$
24. $(a^2 + a^2y + ay^2 + y^3)(a - y) \Rightarrow a^2(a) + a^2(-y) + a^2y(a) + a^2y(-y) + ay^2(a) + ay^2(-y) + y^3(a) + y^3(-y)$
 $= a^3 - a^2y + a^3y - y^4$
25. $y - 3[y - 2(4 - y)] \Rightarrow y - 3[y - 8 + 2y] \Rightarrow y - 3[3y - 8] \Rightarrow y - 9y + 24 = -8y + 24$
26. $\frac{-a^7}{a^5} \Rightarrow -a^{7-5} = -a^2$
27. $(2x^2 + xy - 2y^2)(3x + 3y) \Rightarrow 2x^2(3x) + 2x^2(3y) + xy(3x) + xy(3y) - 2y^2(3x) - 2y^2(3y)$
 $= 6x^3 + 9x^2y - 3xy^2 - 6y^3$
28. $\frac{x^4 - \frac{1}{2}x^3 - \frac{1}{3}x^2 - 2x - 1}{2x} \Rightarrow \frac{x^4}{2x} - \frac{\frac{1}{2}x^3}{2x} - \frac{\frac{1}{3}x^2}{2x} - \frac{2x}{2x} - \frac{1}{2x} = \frac{x^3}{2} - \frac{x^2}{4} - \frac{x}{6} - 1 - \frac{1}{2x}$

$$29. -2[w - 3(2w - 1)] + 3w \Rightarrow -2[w - 6w + 3] + 3w \Rightarrow -2[-5w + 3] + 3w \Rightarrow 10w - 6 + 3w = 13w - 6$$

$$30. (a^2 + b)(a + b^2) \Rightarrow a^2(a) + a^2(b^2) + b(a) + b(b^2) = a^3 + a^2b^2 + ab + b^3$$

$$31. (a^4 - 2a^3c + 4a^2c^2 - 8ac^3 + 16c^4)(a + 2c) \Rightarrow a^4(a) + a^4(2c) - 2a^3c(a) - 2a^3c(2c) + 4a^2c^2(a) + 4a^2c^2(2c) - 8ac^3(a) - 8ac^3(2c) + 16c^4(a) + 16c^4(2c) = a^5 + 32c^5$$

$$32. \frac{16x^3}{4x} \Rightarrow \left(\frac{16}{4}\right)x^{3-1} = 4x^2$$

$$33. \frac{(a-c)^m}{(a-c)^2} = (a-c)^{m-2}$$

$$34. 7 - 8c^2 + 5c^3 + 8c \div 5c - 3$$

$$\begin{array}{r} c^2 - c + 1 \\ 5c - 3 \overline{) 5c^3 - 8c^2 + 8c + 7} \\ \underline{-(5c^3 - 3c^2)} \\ -5c^2 + 8c \\ \underline{-(-5c^2 + 3c)} \\ 5c + 7 \\ \underline{-(5c - 3)} \\ 10 \end{array}$$

$$\text{Thus } 7 - 8c^2 + 5c^3 + 8c \div 5c - 3 = c^2 - c + 1 + \frac{10}{5c - 3}$$

$$35. \frac{-x^2y - xy^2}{-xy} \Rightarrow \frac{-x^2y}{-xy} - \frac{xy^2}{-xy} = x + y$$

$$36. (-2x^2 - x + 6) - (7x^2 - 2x + 4) + (x^2 - 3) \Rightarrow -2x^2 - x + 6 - 7x^2 + 2x - 4 + x^2 - 3 = -8x^2 + x - 1$$

$$37. (b - 3)^3 \Rightarrow (b - 3)(b - 3)(b - 3) \Rightarrow (b^2 - 6b + 9)(b - 3) \Rightarrow b^2(b) + b^2(-3) - 6b(b) - 6b(-3) + 9(b) + 9(-3) \Rightarrow b^3 - 3b^2 - 6b^2 + 18b + 9b - 27 = b^3 - 9b^2 + 27b - 27$$

$$38. (2x^2y^3z^{-1})^3 \Rightarrow 2^{1 \cdot 3} x^{2 \cdot 3} y^{3 \cdot 3} z^{-1 \cdot 3} \Rightarrow 2^3 x^6 y^9 z^{-3} \Rightarrow 8x^6 y^9 \left(\frac{1}{z^3}\right) = \frac{8x^6 y^9}{z^3}$$

$$39. \frac{2x^{-2}y^3}{4x^{-4}y^6} \Rightarrow \frac{2\left(\frac{1}{x^2}\right)y^3}{4\left(\frac{1}{x^4}\right)y^6} \Rightarrow \frac{\frac{2y^3}{x^2}}{\frac{4y^6}{x^4}} \Rightarrow \frac{2y^3}{x^2} \div \frac{4y^6}{x^4} \Rightarrow \left(\frac{2y^3}{x^2}\right)\left(\frac{x^4}{4y^6}\right) \Rightarrow \frac{2x^4y^3}{4x^2y^6} = \frac{x^2}{2y^3}$$

$$40. -\{-[-(a - 3) - a]\} + 2a \Rightarrow -\{-[-a + 3 - a]\} + 2a \Rightarrow -\{-a - 3 + a\} + 2a \Rightarrow -a + 3 - a + 2a \Rightarrow -2a + 2a + 3 = 3$$

$$41. (x - 2)(x + 4) \Rightarrow x^2 + 4x - 2x - 8 = x^2 + 2x - 8$$

$$42. (x^2 + 2)^2 \Rightarrow (x^2 + 2)(x^2 + 2) \Rightarrow x^4 + 2x^2 + 2x^2 + 4 = x^4 + 4x^2 + 4$$

$$43. \frac{a^2b^2 - 2ab - 3ab^3}{ab} \Rightarrow \frac{a^2b^2}{ab} - \frac{2ab}{ab} - \frac{3ab^3}{ab} = ab - 2 - 3b^2$$

$$44. \frac{3a^3c^3 + 3a^2c - 3ac^2}{3ac} \Rightarrow \frac{3a^3c^3}{3ac} + \frac{3a^2c}{3ac} - \frac{3ac^2}{3ac} = a^2c^2 + a - c$$

$$45. \frac{6a^3x^2 - 15a^4x^2 + 30a^3x^3}{-3a^3x^2} \Rightarrow \frac{6a^3x^2}{-3a^3x^2} - \frac{15a^4x^2}{-3a^3x^2} + \frac{30a^3x^3}{-3a^3x^2} = -2 + 5a - 10x$$

$$46. \frac{20x^2y^4 - 14xy^3 + 8x^2y^2}{2x^2y^2} \Rightarrow \frac{20x^2y^4}{2x^2y^2} - \frac{14xy^3}{2x^2y^2} + \frac{8x^2y^2}{2x^2y^2} = 10y^2 - \frac{7y}{x} + 4$$

$$47. (2.83 \times 10^3)^2 \Rightarrow (2.83)^2 \times 10^{3 \cdot 2} \Rightarrow 8.0089 \times 10^6 = 8.01 \times 10^6$$

$$48. (x - 3) - [x - (2x + 3) + 4] \Rightarrow (x - 3) - [x - 2x - 3 + 4] \Rightarrow (x - 3) - [-x + 1] \Rightarrow x - 3 + x - 1 = 2x - 4$$

$$49. (2xy^3)(5x^2y) \Rightarrow (2)(5)x^{1+2}y^{3+1} = 10x^3y^4$$

$$50. \frac{27xy^5z^2}{3x^2yz^2} \Rightarrow \left(\frac{27}{3}\right)x^{1-2}y^{5-1}z^{2-2} \Rightarrow 9x^{-1}y^4 \Rightarrow 9\left(\frac{1}{x}\right)y^4 = \frac{9y^4}{x}$$

$$51. (x - 1)(x^2 + 4x) \Rightarrow x^3 + 4x^2 - x^2 - 4x = x^3 + 3x^2 - 4x$$

$$52. (z^2 - 3)^2 \Rightarrow (z^2 - 3)(z^2 - 3) \Rightarrow z^4 - 3z^2 - 3z^2 + 9 = z^4 - 6z^2 + 9$$

$$53. (1.33 \times 10^4)^2 \Rightarrow (1.33)^2 \times 10^{4 \cdot 2} \Rightarrow 1.7689 \times 10^8 = 1.77 \times 10^8$$

$$54. (y + 1) - [y(y + 1) + (3y - 1) + 5] \Rightarrow (y + 1) - [y^2 + y + 3y - 1 + 5] \Rightarrow (y + 1) - [y^2 + 4y + 4] \Rightarrow y + 1 - y^2 - 4y - 4 = -y^2 - 3y - 3$$

$$55. (2ab^2)(3a^2b) \Rightarrow (2)(3)a^{1+2}b^{2+1} = 6a^3b^3$$

$$56. \frac{64ab^4c^3}{8a^2bc^3} \Rightarrow \left(\frac{64}{8}\right)a^{1-2}b^{4-1}c^{3-3} \Rightarrow 8a^{-1}b^3 \Rightarrow 8\left(\frac{1}{a}\right)b^3 = \frac{8b^3}{a}$$

$$57. x^8 + x^4 + 1 \div x^4 - x$$

$$\begin{array}{r}
 \frac{x^4 + x + 1}{x^4 - x} \overline{)x^8 + 0x^5 + x^4 + 1} \\
 \underline{-(x^8 - x^5)} \\
 x^5 + x^4 \\
 \underline{-(x^5 - x^2)} \\
 x^4 + x^2 \\
 \underline{-(x^4 - x)} \\
 x^2 + x + 1
 \end{array}$$

$$\text{Thus } x^8 + x^4 + 1 \div x^4 - x = x^4 + x + 1 + \frac{x^2 + x + 1}{x^4 - x}$$

$$58. 1 - a^3b^3 \div 1 - ab$$

$$\begin{array}{r}
 \frac{a^2b^2 + ab + 1}{-ab + 1} \overline{)-a^3b^3 + 0a^2b^2 + 0ab + 1} \\
 \underline{-(-a^3b^3 + a^2b^2)} \\
 -a^2b^2 + 0ab \\
 \underline{-(-a^2b^2 + ab)} \\
 -ab + 1 \\
 \underline{-(-ab + 1)} \\
 0
 \end{array}$$

$$59. 0.85x + 0.72(750 - x) \Rightarrow 0.85 + 540 - 0.72x = 0.13x + 540$$

$$60. \pi(3r)^2 \Rightarrow \pi(3r)(3r) = 9r^2\pi$$

$$61. 16.1\left(\frac{t}{2}\right)^2 \Rightarrow 16.1\left(\frac{t}{2}\right)\left(\frac{t}{2}\right) \Rightarrow 16.1\left(\frac{t^2}{4}\right) \Rightarrow \frac{16.1t^2}{4} \Rightarrow 4.025t^2 = 4.02t^2 \text{ ft}$$

$$62. \frac{(2V)^2}{R} \Rightarrow \frac{(2V)(2V)}{R} = \frac{4V^2}{R}$$