## 2.1 Exercises

1. complementary; cotangent

2. $\dfrac{24}{7}$; reciprocals

3. opposite; hypotenuse

4. adjacent; hypotenuse

5. Answers will vary.

6. Answers will vary.
The numerator of these ratios will always be smaller than the denominator (hypotenuse).

7. a. $\sec 30° = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{2}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}$

b. $\csc 30° = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{2}{1} = 2$

c. $\cot 30° = \dfrac{\text{adj}}{\text{opp}} = \dfrac{\sqrt{3}}{1} = \sqrt{3}$

8. a. $\sec 60° = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{2}{1} = 2$

b. $\csc 60° = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{2}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}$

c. $\cot 60° = \dfrac{\text{adj}}{\text{opp}} = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

9. a. $\cos 45° = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

b. $\sin 45° = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

c. $\tan 45° = \dfrac{\text{opp}}{\text{adj}} = \dfrac{1}{1} = 1$

10. a. $\sec 45° = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{\sqrt{2}}{1} = \sqrt{2}$

b. $\csc 45° = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{\sqrt{2}}{1} = \sqrt{2}$

c. $\cot 45° = \dfrac{\text{adj}}{\text{opp}} = \dfrac{1}{1} = 1$

11. a. $\cos A = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{4}{5}$

b. $\sin A = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{3}{5}$

c. $\tan A = \dfrac{\text{opp}}{\text{adj}} = \dfrac{3}{4}$

12. a. $\cos B = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{3}{5}$

b. $\sin B = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{4}{5}$

c. $\tan B = \dfrac{\text{opp}}{\text{adj}} = \dfrac{4}{3}$

13. a. $\sec B = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{5}{3}$

b. $\csc B = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{5}{4}$

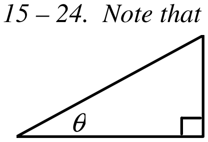
c. $\cot B = \dfrac{\text{adj}}{\text{opp}} = \dfrac{3}{4}$

14. a. $\sec A = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{5}{4}$

b. $\csc A = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{5}{3}$

c. $\cot A = \dfrac{\text{adj}}{\text{opp}} = \dfrac{4}{3}$

*Use the following diagram in 15 – 24. Note that it is not drawn to scale.*




15. $\cos\theta = \dfrac{5}{13} = \dfrac{\text{adj}}{\text{hyp}}$

$\text{opp} = \sqrt{13^2 - 5^2} = \sqrt{144} = 12$

$\sin\theta = \dfrac{12}{13}$; $\csc\theta = \dfrac{13}{12}$; $\sec\theta = \dfrac{13}{5}$;

$\tan\theta = \dfrac{12}{5}$; $\cot\theta = \dfrac{5}{12}$

16. $\sin\theta = \dfrac{20}{29} = \dfrac{\text{opp}}{\text{hyp}}$

$\text{adj} = \sqrt{29^2 - 20^2} = \sqrt{441} = 21$

$\cos\theta = \dfrac{21}{29}$; $\sec\theta = \dfrac{29}{21}$; $\csc\theta = \dfrac{29}{20}$;

$\tan\theta = \dfrac{20}{21}$; $\cot\theta = \dfrac{21}{20}$

17. $\tan\theta = \frac{84}{13} = \frac{\text{opp}}{\text{adj}}$

$\text{hyp} = \sqrt{13^2 + 84^2} = \sqrt{7225} = 85$

$\cos\theta = \frac{13}{85};\ \sec\theta = \frac{85}{13};\ \cot\theta = \frac{13}{84};$

$\sin\theta = \frac{84}{85};\ \csc\theta = \frac{85}{84}$

18. $\sec\theta = \frac{53}{45} = \frac{\text{hyp}}{\text{adj}}$

$\text{opp} = \sqrt{53^2 - 45^2} = \sqrt{784} = 28$

$\cos\theta = \frac{45}{53};\ \tan\theta = \frac{28}{45};\ \cot\theta = \frac{45}{28}$

$\sin\theta = \frac{28}{53};\ \csc\theta = \frac{53}{28}$

19. $\cot\theta = \frac{2}{11} = \frac{\text{adj}}{\text{opp}}$

$\text{hyp} = \sqrt{2^2 + 11^2} = \sqrt{125} = 5\sqrt{5}$

$\sin\theta = \frac{11}{5\sqrt{5}};\ \tan\theta = \frac{11}{2};\ \csc\theta = \frac{5\sqrt{5}}{11}$

$\cos\theta = \frac{2}{5\sqrt{5}};\quad \sec\theta = \frac{5\sqrt{5}}{2}$

20. $\cos\theta = \frac{2}{3} = \frac{\text{adj}}{\text{hyp}}$

$\text{opp} = \sqrt{3^2 - 2^2} = \sqrt{5}$

$\sin\theta = \frac{\sqrt{5}}{3};\ \csc\theta = \frac{3}{\sqrt{5}};\ \cot\theta = \frac{2}{\sqrt{5}}$

$\tan\theta = \frac{\sqrt{5}}{2};\ \sec\theta = \frac{3}{2}$

21. $\tan\theta = 2 = \frac{2}{1} = \frac{\text{opp}}{\text{adj}}$

$\text{hyp} = \sqrt{1^2 + 2^2} = \sqrt{5}$

$\cos\theta = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5};\ \sin\theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

$\sec\theta = \sqrt{5};\ \csc\theta = \frac{\sqrt{5}}{2};\ \cot\theta = \frac{1}{2}$

22. $\csc\theta = 3 = \frac{3}{1} = \frac{\text{hyp}}{\text{opp}}$

$\text{adj} = \sqrt{3^2 - 1^2} = \sqrt{8} = 2\sqrt{2}$

$\cos\theta = \frac{2\sqrt{2}}{3};\ \sin\theta = \frac{1}{3};\ \tan\theta = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$

$\sec\theta = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4};\ \cot\theta = \frac{2\sqrt{2}}{1} = 2\sqrt{2}$

23. $\cot\theta = t = \frac{t}{1} = \frac{\text{adj}}{\text{opp}}$

$\text{hyp} = \sqrt{t^2 + 1^2} = \sqrt{t^2 + 1}$

$\cos\theta = \frac{t}{\sqrt{t^2+1}} = \frac{t\sqrt{t^2+1}}{t^2+1};$

$\sin\theta = \frac{1}{\sqrt{t^2+1}} = \frac{\sqrt{t^2+1}}{t^2+1};$

$\tan\theta = \frac{1}{t};\ \sec\theta = \frac{\sqrt{t^2+1}}{t};\ \csc\theta = \sqrt{t^2+1}$

24. $\sin\theta = t = \frac{t}{1} = \frac{\text{opp}}{\text{hyp}}$

$\text{adj} = \sqrt{1^2 - t^2} = \sqrt{1 - t^2}$

$\cos\theta = \sqrt{1-t^2};\ \tan\theta = \frac{t}{\sqrt{1-t^2}} = \frac{t\sqrt{1-t^2}}{1-t^2};$

$\sec\theta = \frac{1}{\sqrt{1-t^2}} = \frac{\sqrt{1-t^2}}{1-t^2};\ \cos\theta = \frac{1}{t};$

$\cot\theta = \frac{\sqrt{1-t^2}}{t}$

25. For $\angle A$, $(\text{adj, opp}) = (21, 20)$.

26. For $\angle B$, $(\text{adj, opp}) = (20, 21)$.

27. For $\angle A$, $(\text{adj, opp}) = (\sqrt{5}, 2)$.

28. For $\angle B$, $(\text{adj, opp}) = (2, \sqrt{5})$.

29. For $\angle B$, $(\text{adj, opp}) = (6.5, 7.2)$.

30. For $\angle A$, $(\text{adj, opp}) = (7.2, 6.5)$.

31. For $\angle B$, $(\text{adj, opp}) = (a, b)$.

32. For $\angle A$, $(\text{adj, opp}) = (b, a)$.

33. For $\angle A$, $\text{adj} = \sqrt{73^2 - 48^2} = \sqrt{3025} = 55$
$(\text{adj}, \text{opp}) = (55, 48)$

34. For $\angle B$, $\text{opp} = \sqrt{73^2 - 48^2} = \sqrt{3025} = 55$
$(\text{adj}, \text{opp}) = (48, 55)$

35. $\sin 47° = \cos(90° - 47°) = \cos 43°$

36. $\sin 12° = \cos(90° - 12°) = \cos 78°$

37. $\cot 69° = \tan(90° - 69°) = \tan 21°$

38. $\csc 17° = \sec(90° - 17°) = \sec 73°$

39. $\sin(4x)° = \cos(5x)°$
$\sin(4x)° = \cos(90 - 4x)°$
$5x = 90 - 4x$
$9x = 90$
$x = 10$

40. $\cos(2x)° = \sin(3x)°$
$\cos(2x)° = \sin(90 - 2x)°$
$3x = 90 - 2x$
$5x = 90$
$x = 18$

41. $\cot(6x-1)° = \tan(5x+3)°$
$\cot(6x-1)° = \tan[90 - (6x-1)]°$
$5x + 3 = 90 - (6x - 1)$
$5x + 3 = 90 - 6x + 1$
$5x + 3 = 91 - 6x$
$11x = 88$
$x = 8$

42. $\csc(6x-3)° = \sec(2x+5)°$
$\csc(6x-3)° = \sec[90 - (6x-3)]°$
$2x + 5 = 90 - (6x - 3)$
$2x + 5 = 90 - 6x + 3$
$2x + 5 = 93 - 6x$
$8x = 88$
$x = 11$

43.

| $\theta$ | 30° |
|---|---|
| $\sin\theta$ | $\frac{1}{2}$ |
| $\cos\theta$ | $\frac{\sqrt{3}}{2}$ |
| $\tan\theta$ | $\frac{\sqrt{3}}{3}$ |
| $\sin(90-\theta)$ | $\frac{\sqrt{3}}{2}$ |
| $\cos(90-\theta)$ | $\frac{1}{2}$ |

| $\theta$ | 30° |
|---|---|
| $\tan(90-\theta)$ | $\sqrt{3}$ |
| $\csc\theta$ | 2 |
| $\sec\theta$ | $\frac{2\sqrt{3}}{3}$ |
| $\cot\theta$ | $\sqrt{3}$ |

44.

| $\theta$ | 45° |
|---|---|
| $\sin\theta$ | $\frac{\sqrt{2}}{2}$ |
| $\cos\theta$ | $\frac{\sqrt{2}}{2}$ |
| $\tan\theta$ | 1 |
| $\sin(90-\theta)$ | $\frac{\sqrt{2}}{2}$ |
| $\cos(90-\theta)$ | $\frac{\sqrt{2}}{2}$ |

| $\theta$ | 45° |
|---|---|
| $\tan(90-\theta)$ | 1 |
| $\csc\theta$ | $\sqrt{2}$ |
| $\sec\theta$ | $\sqrt{2}$ |
| $\cot\theta$ | 1 |

45. $\sqrt{6}\csc 15° = \sqrt{6}\sec 75° = \sqrt{6}\left(\sqrt{6} + \sqrt{2}\right)$
$= 6 + \sqrt{12} = 6 + 2\sqrt{3}$

46. $\csc^2 15° = \sec^2 75° = \left(\sqrt{6} + \sqrt{2}\right)^2$
$= 6 + 2\sqrt{12} + 2 = 8 + 2\sqrt{12} = 8 + 4\sqrt{3}$

47. $\cot^2 15° = \tan^2 75° = \left(2 + \sqrt{3}\right)^2$
$= 4 + 4\sqrt{3} + 3 = 7 + 4\sqrt{3}$

48. $\sqrt{3}\cot 15° = \sqrt{3}\tan 75° = \sqrt{3}\left(2 + \sqrt{3}\right)$
$= 2\sqrt{3} + 3 = 3 + 2\sqrt{3}$

49. $\dfrac{\sin\alpha}{\sin\beta} = \dfrac{\sin\alpha}{\cos\alpha} = \tan\alpha$

50. $\dfrac{\cos\beta}{\cos\alpha} = \dfrac{\sin\alpha}{\cos\alpha} = \tan\alpha$

51. $\sec^2\alpha = \tan^2\alpha + 1 = \cot^2\beta + 1$

52. $\cot^2\alpha = \csc^2\alpha - 1 = \sec^2\beta - 1$

53. $\tan^2\alpha\sec^2\beta - \cot^2\beta$
$= \tan^2\alpha\csc^2\alpha - \tan^2\alpha$
$= \tan^2\alpha\left(\csc^2\alpha - 1\right)$
$= \tan^2\alpha\cot^2\alpha$
$= 1$

54. $\sin^2\alpha\tan^2\beta+\cos^2\beta$
$=\sin^2\alpha\cot^2\alpha+\sin^2\alpha$
$=\sin^2\alpha\left(\cot^2\alpha+1\right)$
$=\sin^2\alpha\csc^2\alpha$
$=1$

55. $v=\dfrac{32kt}{\cos\theta}$
$=\dfrac{32(0.2)(0.8)}{\cos 45^\circ}$
$=\dfrac{5.12}{\frac{\sqrt{2}}{2}}$
$\approx 7.2$ ft/sec

56. $\sigma=\dfrac{9.8m\sin\theta}{0.01\pi r\sec\theta}$
$700\geq\dfrac{9.8m\sin 30^\circ}{0.01\pi\cdot 1\cdot\sec 30^\circ}$
$m\leq\dfrac{700\cdot 0.01\pi\cdot 1\cdot\sec 30^\circ}{9.8\sin 30^\circ}$
$m\leq 5.2$ kg

57. $\tan\theta=\dfrac{\text{opp}}{\text{adj}}=\dfrac{30}{8}=3.75$

58. $\cot\theta=\dfrac{\text{adj}}{\text{opp}}=\dfrac{36}{6}=6$

59. $\sec\theta=\dfrac{\text{hyp}}{\text{adj}}=\dfrac{12}{8}=1.5$

60. $\csc\theta=\dfrac{\text{hyp}}{\text{opp}}=\dfrac{8}{6}=\dfrac{4}{3}$

61. $\text{hyp}=\sqrt{12^2+9^2}=\sqrt{225}=15$
$\cos\theta=\dfrac{\text{adj}}{\text{hyp}}=\dfrac{12}{15}=0.8$

62. $\text{hyp}=\sqrt{7.5^2+10^2}=\sqrt{156.25}=12.5$
$\sin\theta=\dfrac{\text{opp}}{\text{hyp}}=\dfrac{10}{12.5}=0.8$

63. a. Let $x$ be the length of the diagonal of one side of the box.
$x=\sqrt{10^2+10^2}=\sqrt{200}=10\sqrt{2}$ cm

Now, let $d$ be the length of the diagonal that passes through the center of the box.
$x=\sqrt{10^2+\left(10\sqrt{2}\right)^2}=\sqrt{300}=10\sqrt{3}$ cm

b. $\sin\theta=\dfrac{\text{opp}}{\text{hyp}}=\dfrac{10}{10\sqrt{3}}=\dfrac{\sqrt{3}}{3}$

64. a. Let $x$ be the length of the diagonal of bottom of the box.
$x=\sqrt{25^2+45^2}=\sqrt{2650}=5\sqrt{106}$ cm

Now, let $d$ be the length of the diagonal that passes through the center of the box.
$x=\sqrt{10^2+\left(5\sqrt{106}\right)^2}$
$=\sqrt{2750}=5\sqrt{110}$ cm

b. $\cos\theta=\dfrac{\text{adj}}{\text{hyp}}=\dfrac{5\sqrt{106}}{5\sqrt{110}}=\sqrt{\dfrac{53}{55}}$

65. $\cot u=\dfrac{x}{h}\rightarrow x=h\cot u$
$\cot v=\dfrac{x-d}{h}\rightarrow\cot v=\dfrac{h\cot u-d}{h}$
$h\cot v=h\cot u-d$
$d=h\cot u-h\cot v=h(\cot u-\cot v)$
$h=\dfrac{d}{\cot u-\cot v}$

66. $\sin\alpha=\dfrac{a}{c}\rightarrow c=\dfrac{a}{\sin\alpha}$
$\tan\alpha=\dfrac{d}{c}\rightarrow c=\dfrac{d}{\tan\alpha}$
$\dfrac{a}{\sin\alpha}=\dfrac{d}{\tan\alpha}\rightarrow\dfrac{a}{d}=\dfrac{\sin\alpha}{\tan\alpha}=\cos\alpha$
$\cos\alpha=\sin\beta=\dfrac{a}{d}$
$\cos\beta=\sqrt{1-\sin^2\beta}=\sqrt{1-\dfrac{a^2}{d^2}}=\dfrac{\sqrt{d^2-a^2}}{d}$

67. $\dfrac{\sin\theta}{\tan^2\theta}=\cot\theta\cos\theta$
$\sin\theta\cdot\cot^2\theta=\cot\theta\cos\theta$
$\sin\theta\cdot\dfrac{\cos^2\theta}{\sin^2\theta}=\cot\theta\cos\theta$
$\dfrac{\cos^2\theta}{\sin\theta}=\cot\theta\cos\theta$
$\cot\theta\cos\theta=\cot\theta\cos\theta$

68. $\cos\alpha + \tan\alpha \sin\alpha = \sec\alpha$

$$\cos\alpha + \frac{\sin\alpha}{\cos\alpha}\cdot\sin\alpha = \sec\alpha$$
$$\cos\alpha + \frac{\sin^2\alpha}{\cos\alpha} = \sec\alpha$$
$$\frac{\cos^2\alpha}{\cos\alpha} + \frac{\sin^2\alpha}{\cos\alpha} = \sec\alpha$$
$$\frac{1}{\cos\alpha} = \sec\alpha$$
$$\sec\alpha = \sec\alpha$$

69. $c = \sqrt{a^2 + b^2}$

$$= \sqrt{7^2 + 9^2}$$
$$= \sqrt{130}$$

$$e = \sqrt{c^2 + d^2} \qquad \cos\beta = \frac{\sqrt{d^2 - a^2}}{d} = \frac{d}{e}$$
$$= \sqrt{\left(\sqrt{130}\right)^2 + (8.75)^2}$$
$$= \sqrt{130 + 76.5625}$$
$$\approx 14.4 \text{ m}$$

70. $\sin\theta = -\frac{2}{3} \Rightarrow y = -2,\ r = 3$

Since $\cos\theta > 0$, $x$ is positive.
The angle $\theta$ is in quadrant IV.

$$x = \sqrt{3^2 - (-2)^2} = \sqrt{5}$$
$$\cos\theta = \frac{\sqrt{5}}{3};\ \tan\theta = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5};$$
$$\csc\theta = -\frac{3}{2};\ \sec\theta = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5};\ \cot\theta = -\frac{\sqrt{5}}{2}$$

71. $\text{hyp} = \sqrt{(-2)^2 + (-5)^2} = \sqrt{29}$

$$\cos^2\theta\tan^2\theta = (\cos\theta\tan\theta)^2$$
$$= \left[\left(-\frac{2}{\sqrt{29}}\right)\left(\frac{5}{2}\right)\right]^2$$
$$= \frac{25}{29}$$

72. Since $2k+1$ is odd for any integer $k$, $[90(2k+1)]°$ represents an odd multiple of $90°$. Therefore, $\cos[90(2k+1)]° = 0$.

## 2.2 Exercises

1. $\theta = \tan^{-1} x$

2. solve; three; sides

3. Pythagorean; hypotenuse; $\tan^{-1}$; angle

4. tangent; cotangent

5. To find the measure of all three angles and all three sides.

6. cosine, secant, tangent, and cotangent

7. $B = 90° - 30° = 60°$

$$\sin 30° = \frac{a}{196};\ a = 196\sin 30° = 98 \text{ cm}$$
$$\cos 30° = \frac{b}{196};\ b = 196\cos 30° = 98\sqrt{3} \text{ cm}$$

| Angle | Side |
|---|---|
| $A = 30°$ | $a = 98$ cm |
| $B = 60°$ | $b = 98\sqrt{3}$ cm |
| $C = 90°$ | $c = 196$ cm |

8. $B = 90° - 60° = 30°$

$$\sin 60° = \frac{420}{c};\ c\sin 60° = 420$$
$$c = \frac{420}{\sin 60°} = \frac{420}{\sqrt{3}/2} = \frac{840}{\sqrt{3}} = 280\sqrt{3} \text{ ft}$$
$$\cos 60° = \frac{b}{280\sqrt{3}};\quad b = 280\sqrt{3}\cos 60°$$
$$= 140\sqrt{3} \text{ ft}$$

| Angle | Side |
|---|---|
| $A = 60°$ | $a = 420$ ft |
| $B = 30°$ | $b = 140\sqrt{3}$ ft |
| $C = 90°$ | $c = 280\sqrt{3}$ ft |

9. $B = 90° - 45° = 45°$

$$\sin 45° = \frac{9.9}{c};\ c\sin 45° = 9.9$$
$$c = \frac{9.9}{\sin 45°} = \frac{9.9}{\sqrt{2}/2} = \frac{19.8}{\sqrt{2}} = 9.9\sqrt{2} \text{ mm}$$
$$\cos 45° = \frac{a}{9.9\sqrt{2}}$$
$$a = 9.9\sqrt{2}\cos 45° = 9.9\sqrt{2}\cdot\frac{\sqrt{2}}{2} = 9.9$$

9. (continued)

| Angle | Side |
|---|---|
| $A = 45°$ | $a = 9.9$ mm |
| $B = 45°$ | $b = 9.9$ mm |
| $C = 90°$ | $c = 9.9\sqrt{2}$ mm |

10. $A = 90° - 45° = 45°$

$\sin 45° = \dfrac{81.9}{c};\ c\sin 45° = 81.9$

$c = \dfrac{81.9}{\sin 45°} = \dfrac{81.9}{\sqrt{2}/2} = \dfrac{163.8}{\sqrt{2}} = 81.9\sqrt{2}$ m

$b = 81.9$ m (Isosceles triangle)

| Angle | Side |
|---|---|
| $A = 45°$ | $a = 81.9$ m |
| $B = 45°$ | $b = 81.9$ m |
| $C = 90°$ | $c = 81.9\sqrt{2}$ m |

11. $B = 90° - 22° = 68°$

$\sin 22° = \dfrac{14}{c};\ c\sin 22° = 14$

$c = \dfrac{14}{\sin 22°} \approx 37.37$ m

$\tan 22° = \dfrac{14}{b};\ b\tan 22° = 14$

$b = \dfrac{14}{\tan 22°} \approx 34.65$ m

| Angle | Side |
|---|---|
| $A = 22°$ | $a = 14$ m |
| $B = 68°$ | $b \approx 34.65$ m |
| $C = 90°$ | $c \approx 37.37$ m |

12. $B = 90° - 49° = 41°$

$\sin 49° = \dfrac{89}{c};\ c\sin 49° = 89$

$c = \dfrac{89}{\sin 49°} \approx 117.93$ in.

$\tan 49° = \dfrac{89}{b};\ b\tan 49° = 89$

$b = \dfrac{89}{\tan 49°} \approx 77.37$ in.

| Angle | Side |
|---|---|
| $A = 49°$ | $a = 89$ in. |
| $B = 41°$ | $b \approx 77.37$ in. |
| $C = 90°$ | $c \approx 117.93$ in. |

13. $A = 90° - 58° = 32°$

$\cos 58° = \dfrac{5.6}{c};\ c\cos 58° = 5.6$

$c = \dfrac{5.6}{\cos 58°} \approx 10.57$ mi

$\tan 58° = \dfrac{b}{5.6}$

$b = 5.6\tan 58° \approx 8.96$ mi

| Angle | Side |
|---|---|
| $A = 32°$ | $a = 5.6$ mi |
| $B = 58°$ | $b \approx 8.96$ mi |
| $C = 90°$ | $c \approx 10.57$ mi |

14. $B = 90° - 51° = 39°$

$\sin 51° = \dfrac{a}{238};\ a = 238\sin 51° \approx 184.96$ ft

$\cos 51° = \dfrac{b}{238};\ b = 238\cos 51° \approx 149.78$ ft

| Angle | Side |
|---|---|
| $A = 51°$ | $a \approx 184.96$ ft |
| $B = 39°$ | $b \approx 149.78$ ft |
| $C = 90°$ | $c = 238$ ft |

15. $B = 90° - 65° = 25°$

$\sin 65° = \dfrac{625}{c};\ c\sin 65° = 625;$

$c = \dfrac{625}{\sin 65°} \approx 689.61$ mm

$\tan 65° = \dfrac{625}{b};\ b\tan 65° = 625;$

$b = \dfrac{625}{\tan 65°} \approx 291.44$ mm

| Angle | Side |
|---|---|
| $A = 65°$ | $a = 625$ mm |
| $B = 25°$ | $b \approx 291.44$ mm |
| $C = 90°$ | $c \approx 689.61$ mm |

16. $A = 90° - 28° = 62°$

$\sin 28° = \dfrac{45.8}{c};\ c\sin 28° = 45.8;$

$c = \dfrac{45.8}{\sin 28°} \approx 97.56$ m

$\tan 28° = \dfrac{45.8}{a};\ a\tan 28° = 45.8;$

$a = \dfrac{45.8}{\tan 28°} \approx 86.14$ m

| Angle | Side |
|---|---|
| $A = 62°$ | $a \approx 86.14$ m |
| $B = 28°$ | $b \approx 45.8$ m |
| $C = 90°$ | $c \approx 97.56$ m |

17. $\sin 27° = 0.4540$

18. $\cos 72° = 0.3090$

19. $\tan 40° = 0.8391$

20. $\cot 57.3° = 0.6420$

21. $\sec 40.9° = 1.3230$

22. $\csc 39° = 1.5890$

23. $\sin 65° = 0.9063$

24. $\tan 84.1° = 9.6768$

25. $A = \sin^{-1}(0.4540) \approx 27°$

26. $B = \cos^{-1}(0.3090) \approx 72°$

27. $\theta = \tan^{-1}(0.8390) \approx 40°$

28. $A = \tan^{-1}\left(\frac{1}{0.6420}\right) \approx 57.3°$

29. $B = \cos^{-1}\left(\frac{1}{1.3230}\right) \approx 40.9°$

30. $\beta = \sin^{-1}\left(\frac{1}{1.5890}\right) \approx 39°$

31. $A = \sin^{-1}(0.9063) \approx 65°$

32. $B = \tan^{-1}(9.6768) \approx 84.1°$

33. $\alpha = \tan^{-1}(0.9896) \approx 44.7°$

34. $\alpha = \cos^{-1}(0.7408) \approx 42.2°$

35. $\alpha = \sin^{-1}(0.3453) \approx 20.2°$

36. $\alpha = \tan^{-1}(3.1336) \approx 72.3°$

37. $\tan\theta = \frac{6}{18};\ \theta = \tan^{-1}\left(\frac{1}{3}\right) \approx 18.4°$

38. $\sin\beta = \frac{14}{15};\ \beta = \sin^{-1}\left(\frac{14}{15}\right) \approx 69.0°$

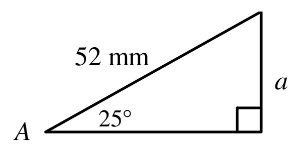
39. $\tan\gamma = \frac{19.5}{18.7};\ \gamma = \tan^{-1}\left(\frac{19.5}{18.7}\right) \approx 46.2°$

40. $\sin A = \frac{5}{6.2};\ A = \sin^{-1}\left(\frac{5}{6.2}\right) \approx 53.8°$

41. $\cos B = \frac{20}{42};\ B = \cos^{-1}\left(\frac{20}{42}\right) \approx 61.6°$

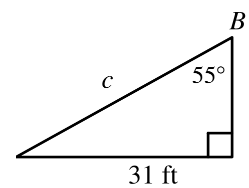
42. $\tan\alpha = \frac{221}{207};\ \alpha = \tan^{-1}\left(\frac{221}{207}\right) \approx 46.9°$

43.




$\sin 25° = \frac{a}{52};\ a = 52\sin 25° \approx 21.98 \text{ mm}$

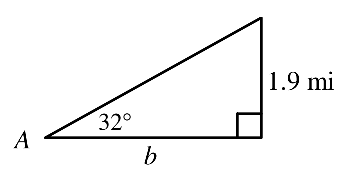
44.




$\sin 55° = \frac{31}{c};\ c\sin 55° = 31$

$c = \frac{31}{\sin 55°} \approx 37.84 \text{ ft}$

45.




$\tan 32° = \frac{1.9}{b};\ b\tan 32° = 1.9$

$b = \frac{1.9}{\tan 32°} \approx 3.04 \text{ mi}$

46. $\cos 29.6° = \frac{a}{9.5}$

$a = 9.5\cos 29.6° \approx 8.26 \text{ yd}$

47. $\cos 62.3° = \frac{82.5}{c};\ c\cos 62.3° = 82.5$

$c = \frac{82.5}{\cos 62.3°} \approx 177.48 \text{ furlongs}$

48. $\tan 12.5^\circ = \dfrac{b}{32.8}$

$b = 32.8\tan 12.5^\circ \approx 7.27$ km

49. $\sin\theta = \dfrac{2A}{ab}$

$\sin\theta = \dfrac{2(38.9)}{(17)(24)}$

$\theta = \sin^{-1}\left(\dfrac{2(38.9)}{(17)(24)}\right) \approx 11.0^\circ$

Repeat for $\beta$ using 24 and 8 for $a$ and $b$.

$\sin\beta = \dfrac{2(38.9)}{8(24)}$; $\beta = \sin^{-1}\dfrac{2(38.9)}{8(24)} \approx 23.9^\circ$

$\gamma = 180^\circ - (11.0^\circ + 23.9^\circ) = 145.1^\circ$

50. $E = \dfrac{I\cos\theta}{d^2}$

$E = 18$; $I = 90$; $\theta = 25^\circ$

$18 = \dfrac{90\cos 25^\circ}{d^2}$; $18d^2 = 90\cos 25^\circ$

$d^2 = \dfrac{90\cos 25^\circ}{18}$; $d \approx 2.1$ ft

51. $\cos 34^\circ = \dfrac{320}{Z}$; $Z\cos 34^\circ = 320$

$Z = \dfrac{320}{\cos 34^\circ} \approx 386.0\,\Omega$

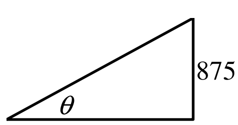
52. $\cos\theta = \dfrac{290}{420}$; $\theta = \cos^{-1}\left(\dfrac{290}{420}\right) \approx 46.3^\circ$

53. a. Five contour lines, so change in elevation is $5(175) = 875$ m.

b. $\dfrac{2.4\text{ cm}}{x\text{ m}} = \dfrac{1\text{ cm}}{500\text{ m}}$;

$x = 2.4(500) = 1200$ m

c.




$d^2 = 1200^2 + 875^2$;

$d = \sqrt{2{,}205{,}625} \approx 1485$ m

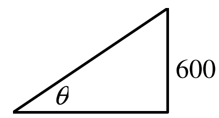
$\tan\theta = \dfrac{875}{1200}$; $\theta = \tan^{-1}\dfrac{875}{1200} \approx 36.1^\circ$

54. a. Four contour lines, so change in elevation is $4(150) = 600$ m.

b. $\dfrac{1\text{ cm}}{250\text{ m}} = \dfrac{4.5\text{ cm}}{x\text{ m}}$

$x = 4.5(250) = 1125$ m

c.




$d^2 = 600^2 + 1125^2$;

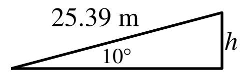
$d = \sqrt{1{,}625{,}625}d = 1275$ m

$\tan\theta = \dfrac{60}{1125}$

$\theta = \tan^{-1}\left(\dfrac{60}{1125}\right) \approx 28.1^\circ$

55. $\tan 42^\circ = \dfrac{h}{500}$; $h = 500\tan 42^\circ \approx 450$ ft

56.




a. $\sin 10^\circ = \dfrac{h}{25.39}$

$h = 25.39\sin 10^\circ \approx 4.4$ m

b. $\cos 10^\circ = \dfrac{d}{25.39}$

$d = 25.39\cos 10^\circ \approx 25.0$ m

57. a. The triangle at the base of the box is isosceles with sides $x$, so the dotted diagonal on the bottom has length $x\sqrt{2}$ (45-45-90 triangle). The triangle formed by that diagonal, the diagonal across the box, and one edge of the box is right, and we can apply the Pythagorean Theorem.

$\left(x\sqrt{2}\right)^2 + x^2 = 35^2$

$2x^2 + x^2 = 1225$

$3x^2 = 1225$

$x^2 \approx 408.3 \rightarrow x \approx 20.2$ cm

b. Now we can find the angle using cosine.

$\cos\theta = \dfrac{20.2\sqrt{2}}{35}$

$\theta = \cos^{-1}\dfrac{20.2\sqrt{2}}{35} \approx 35.3^\circ$

58. a. Let $x$ be length of the dotted diagonal of the bottom of the box.

$x^2 = 50^2 + 70^2; \quad x = \sqrt{7400}$ cm

Now use the triangle formed by the two diagonals (90 cm and $\sqrt{7400}$ cm) and the height of the box ($h$).

$$\sqrt{7400}^2 + h^2 = 90^2$$
$$h^2 = 90^2 - 7400$$
$$h = \sqrt{700} \approx 26.5 \text{ cm}$$

b. $\cos\theta = \dfrac{\sqrt{7400}}{90}$

$$\theta = \cos^{-1}\left(\frac{\sqrt{7400}}{90}\right) \approx 17.1°$$

59. $\tan v = \dfrac{h}{d}; \quad d = \dfrac{h}{\tan v}$

$\tan u = \dfrac{h-x}{d}; \quad \tan u = \dfrac{h-x}{\dfrac{h}{\tan v}}$

$$\tan u = \frac{\tan v(h-x)}{h}$$
$$h\tan u = h\tan v - x\tan v$$
$$h\tan u - h\tan v = -x\tan v$$
$$h\tan v - h\tan u = x\tan v$$
$$h(\tan v - \tan u) = x\tan v$$
$$h = \frac{x\tan v}{\tan v - \tan u};$$
$$h = \frac{75\tan 50°}{\tan 50° - \tan 40°} \approx 253.45 \text{ m}$$

60. a. $\tan v = \dfrac{h}{d}; \; h = d\tan v$

$\tan u = \dfrac{h-x}{d}; \; \tan u = \dfrac{d\tan v - x}{d}$

$$d\tan u = d\tan v - x$$
$$x = d\tan v - d\tan u = d(\tan v - \tan u)$$
$$d = \frac{x}{\tan v - \tan u}$$

b. $d = \dfrac{x}{\tan v - \tan u}$

$$= \frac{75}{\tan 50° - \tan 40°} \approx 212.67 \text{ m}$$

61. $132°42'54'' = \left[132 + 42\left(\dfrac{1}{60}\right) + 54\left(\dfrac{1}{3600}\right)\right]^{\circ}$

$$= 132.715°$$

62. $36.4525° = 36° + 0.4525(60)' = 36° + 27.15'$

$$= 36°27' + 0.15(60)'' = 36°27'9''$$

63. If $\tan 22.5° = \sqrt{2} - 1$, then

$\cot 22.5° = \dfrac{1}{\sqrt{2}-1} = \sqrt{2} + 1.$

$$\sqrt{8}\cot 22.5° = \sqrt{8}\left(\sqrt{2}+1\right)$$
$$= \sqrt{16} + \sqrt{8}$$
$$= 4 + 2\sqrt{2}$$

64. $r = \sqrt{(-2)^2 + 5^2} = \sqrt{29}$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{29}}{5}$$

65. For angle $\beta$, the opposite side is proportional to 3 and the proportional is similar to 4.

$$\sin\beta = \frac{\text{opp}}{\text{hyp}} = \frac{3}{4} = 0.75$$

66. $\sec\theta = \tan\theta\sin\theta + \cos\theta$

$$\sec\theta = \frac{\sin^2\theta}{\cos\theta} + \cos\theta$$
$$\sec\theta = \frac{\sin^2\theta}{\cos\theta} + \frac{\cos^2\theta}{\cos\theta}$$
$$\sec\theta = \frac{1}{\cos\theta}$$
$$\sec\theta = \sec\theta$$

## Mid-Chapter Check

1. $\cos\theta = \frac{12}{13} = \frac{\text{adj}}{\text{hyp}}$

   $\text{opp} = \sqrt{13^2 - 12^2} = \sqrt{25} = 5$

   $\sin\theta = \frac{5}{13};\ \csc\theta = \frac{13}{5};\ \sec\theta = \frac{13}{12};$

   $\tan\theta = \frac{5}{12};\ \cot\theta = \frac{12}{5}$

2. a. $\tan\alpha = \frac{\text{opp}}{\text{adj}} = \frac{4}{3}$

   b. $\sec\beta = \frac{\text{hyp}}{\text{adj}} = \frac{5}{4}$

   c. $\csc\alpha = \frac{\text{hyp}}{\text{opp}} = \frac{5}{4}$

   d. $\cot\beta = \frac{\text{adj}}{\text{opp}} = \frac{4}{3}$

3. $\frac{\cos\beta}{\cos\alpha} = \frac{\sin(90^\circ - \beta)}{\cos\alpha} = \frac{\sin\alpha}{\cos\alpha} = \tan\alpha$

4. a. $\tan 53^\circ = 1.3270$

   b. $\sin 28.4^\circ = 0.4756$

   c. $\cos 61.6^\circ = 0.4756$

   d. $\tan 45^\circ = 1$

5. $A = 90^\circ - 55^\circ = 35^\circ$

   $\sin 55^\circ = \frac{24}{c};\ c\sin 55^\circ = 24;$

   $c = \frac{24}{\sin 55^\circ} \approx 29.30 \text{ cm}$

   $\tan 55^\circ = \frac{24}{a};\ a\tan 55^\circ = 24;$

   $a = \frac{24}{\tan 55^\circ} \approx 16.80 \text{ cm}$

| Angle | Side |
|---|---|
| $A = 35^\circ$ | $a \approx 16.80 \text{ cm}$ |
| $B = 55^\circ$ | $b \approx 24 \text{ cm}$ |
| $C = 90^\circ$ | $c \approx 29.30 \text{ cm}$ |

6. $\tan 75^\circ = 2 + \sqrt{3},$

   $\cot 15^\circ = \tan(90 - 15)^\circ = \tan 75^\circ = 2 + \sqrt{3}$

7. $c = \sqrt{7^2 + 14^2} = \sqrt{245} \approx 15.7 \text{ ft}$

   $\tan A = \frac{7}{14} = \frac{1}{2};\ \ A = \tan^{-1}\frac{1}{2} = 26.6^\circ$

   $\tan B = \frac{14}{7} = 2;\ \ B = \tan^{-1} 2 = 63.4^\circ$

| Angle | Side |
|---|---|
| $A = 26.6^\circ$ | $a \approx 7 \text{ ft}$ |
| $B = 63.4^\circ$ | $b \approx 14 \text{ ft}$ |
| $C = 90^\circ$ | $c \approx 15.7 \text{ ft}$ |

8.

| | $\theta = 30^\circ$ | $\theta = 45^\circ$ | $\theta = 60^\circ$ |
|---|---|---|---|
| $\sin\theta^\circ$ | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ |
| $\cos\theta^\circ$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ |
| $\tan\theta^\circ$ | $\frac{\sqrt{3}}{3}$ | 1 | $\sqrt{3}$ |
| $\csc\theta^\circ$ | 2 | $\sqrt{2}$ | $\frac{2\sqrt{3}}{3}$ |
| $\sec\theta^\circ$ | $\frac{2\sqrt{3}}{3}$ | $\sqrt{2}$ | 2 |
| $\cot\theta^\circ$ | $\sqrt{3}$ | 1 | $\frac{\sqrt{3}}{3}$ |

9. $\sin\theta = \frac{16}{20} = \frac{4}{5};\ \theta = \sin^{-1}\frac{4}{5} = 53^\circ$

   The acute angle the anchor lines makes with the street is $53^\circ$.

10. $\tan 4^\circ = \frac{24}{x}$

    $x = \frac{24}{\tan 4^\circ} = 343.2$ in. or 28 ft, 7.2 in.

    The ramp must begin 343.2 inches, or 28 feet, 7.2 inches from the door landing.

## Reinforcing Basic Concepts

1. $A = 28°, b = 20\,\text{cm}, c = 15\,\text{cm}$

$$A = \frac{1}{2}bc\sin$$
$$= \frac{1}{2}(20)(15)\sin 28°$$
$$\approx 70.4\,\text{cm}^2$$

2. $B = 128°, a = 35\,\text{mm}, c = 32\,\text{mm}$

$$A = \frac{1}{2}ac\sin B$$
$$= \frac{1}{2}(35)(32)\sin 128°$$
$$\approx 441.3\,\text{mm}^2$$

3. $A = 26.3°, B = 79.6°, a = 45\,\text{in.}$

$$C = 180° - (26.3° + 79.6°) = 74.1°$$
$$A = \frac{a^2 \sin B \sin C}{2\sin A}$$
$$= \frac{(45)^2 \sin 79.6° \sin 74.1°}{2\sin 26.3°}$$
$$\approx 2161.7\,\text{in.}^2$$

4. $B = 117°, C = 18°, b = 45\,\text{ft}$

$$A = 180° - (117° + 18°) = 45°$$
$$A = \frac{b^2 \sin A \sin C}{2\sin B}$$
$$= \frac{(45)^2 \sin 45° \sin 18°}{2\sin 117°}$$
$$\approx 248.3\,\text{ft}^2$$

## 2.3 Exercises

1. orientation; parallel
2. sight; depression
3. calculators; sine; cosine; tangent
4. E; 35°; south
5. Answers will vary.
6. Answers will vary.
7. $h = 40\tan 15.3763° \approx 11$ m
8. $h = 40\tan 57.1715° \approx 62$ m
9. $h = 40\tan 19.2900° \approx 14$ m
10. $h = 40\tan 58.3925° \approx 65$ m
11. $h = 40\tan 23.0255° \approx 17$ m
12. $h = 40\tan 59.5346° \approx 68$ m
13. $d = 35\sin 45.5847° \approx 25$ ft
14. $d = 35\sin 8.2132° \approx 5$ ft
15. $d = 35\sin 55.9523° \approx 29$ ft
16. $d = 35\sin 14.9006° \approx 9$ ft
17. $d = 35\sin 70.5370° \approx 33$ ft
18. $d = 35\sin 21.8037° \approx 13$ ft
19. a. $63°$
    b. $90° - 63° = 27°$
20. a. $81°$
    b. $90° - 81° = 9°$
21. a. $49°$
    b. $90° - 49° = 41°$
22. a. $39°$
    b. $90° - 39° = 51°$

23. a. $23°31'$

b. $90° - 23°31'$
$= 89°60' - 23°31'$
$= 66°29'$

24. a. $42°24'$

b. $90° - 42°24'$
$= 89°60' - 42°24'$
$= 47°36'$

25. a. $15°32'49''$

b. $90° - 15°32'49''$
$= 89°59'60'' - 15°32'49''$
$= 74°27'11''$

26. a. $77°18'06''$

b. $90° - 77°18'06''$
$= 89°59'60'' - 77°18'06''$
$= 12°41'54''$

27. a. $d = \frac{80}{\cos 10°} \approx 81.2$ m

b. $81.2 \div 3 = 27.1$ sec

28. a. $d = \frac{80}{\cos 65°} \approx 189.3$ m

b. $189.3 \div 3 = 63.1$ sec

29. a. $d = \frac{80}{\cos 15°} \approx 82.8$ m

b. $82.8 \div 3 = 27.6$ sec

30. a. $d = \frac{80}{\cos 70°} \approx 233.9$ m

b. $233.9 \div 3 = 78.0$ sec

31. a. $d = \frac{80}{\cos 20°} \approx 85.1$ m

b. $85.1 \div 3 = 28.4$ sec

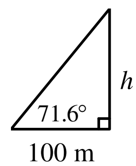
32. a. $d = \frac{80}{\cos 75°} \approx 309.1$ m

b. $309.1 \div 3 = 103.0$ sec

33. $h = \frac{d}{\cot\alpha - \cot\beta}$
$= \frac{50}{\cot 45° - \cot 60°}$
$\approx 118.3$ ft
The height of the building shown is approximately 118.3 feet.

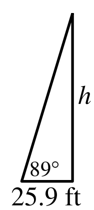
34. $d = \sqrt{r_1^2 t_1^2 + r_2^2 t_2^2 + 2 r_1 t_1 r_2 t_2 \cos\theta}$
$= \sqrt{450^2 \cdot 2.5^2 + 500^2 \cdot 3^2 + 2 \cdot 450 \cdot 2.5 \cdot 500 \cdot 3\cos 15°}$
$= \sqrt{1,265,625 + 2,250,000 + 3,375,000\cos 15°}$
$\approx 2603$ mi
The distance between the two planes is 2603 miles.

35.



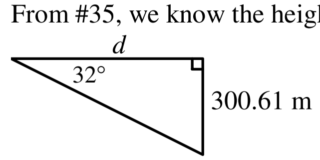

$\tan 71.6° = \frac{h}{100}$
$h = 100\tan 71.6° \approx 300.6$ m
The Eiffel Tower is approximately 300.6 meters tall.

36.



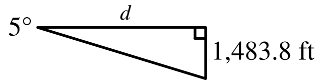

$\tan 89° = \frac{h}{25.9}$
$h = 25.9\tan 89° \approx 1483.8$ ft
The Petronas tower is approximately 1483.8 feet tall.

37. From #35, we know the height is 300.61 m.




$\tan 32° = \frac{300.61}{d}$; $d\tan 32° = 300.61$
$d = \frac{300.61}{\tan 32°} \approx 481.1$ m
The accident is approximately 481.1 meters from the base of the tower.

38. From #36, the height is 1483.8 ft.




$\tan 5° = \dfrac{1483.8}{d};\ d\tan 5° = 1483.8$

$d = \dfrac{1483.8}{\tan 5°} \approx 16{,}960 \text{ ft}$

$16{,}960 \text{ ft} \cdot \dfrac{1 \text{ mi}}{5280 \text{ ft}} \approx 3.2 \text{ mi}$

The residence is approximately 16,960 feet or 3.2 miles from the base of the tower.

39. Let $x$ be the length of the bridge.

$x = 110\tan 38°35'15''$

$= 110\tan\left[38 + 35\left(\dfrac{1}{60}\right) + 15\left(\dfrac{1}{3600}\right)\right]°$

$= 110\tan 38.5875°$

$\approx 87.77 \text{ ft} \approx 87 \text{ ft } 9 \text{ in.}$

The bridge will be approximately 87 feet 9 inches long.

40. $90° - 67°11'42'' = 89°59'60'' - 67°11'42''$
$= 22°48'18''$

Let $w$ be the width of the sign.

$w = 35\tan 22°48'18''$

$= 35\tan\left[22 + 48\left(\dfrac{1}{60}\right) + 18\left(\dfrac{1}{3600}\right)\right]°$

$\approx 14.72 \text{ m} \approx 14 \text{ m } 72 \text{ cm}$

The sign is approximately 14 meters 72 centimeters wide.

41. $\tan 83° = \dfrac{d}{50};\ d = 50\tan 83° \approx 407.22 \text{ ft}$

$\dfrac{407.22 \text{ ft}}{2.35 \text{ sec}} \cdot \dfrac{1 \text{ mi}}{5280 \text{ ft}} \cdot \dfrac{3600 \text{ sec}}{1 \text{ hr}} = 118.1 \text{ mph}$

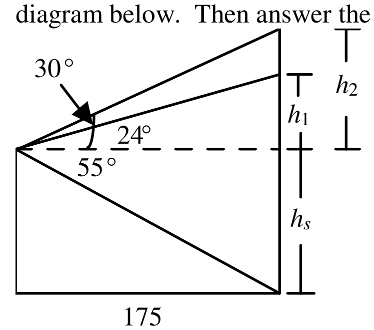
The plane is flying at approximately 118.1 miles per hour.

42. $\tan 80° = \dfrac{d}{60};\ d = 60\tan 80° \approx 340.28 \text{ ft}$

$\dfrac{340.28 \text{ ft}}{3 \text{ sec}} \cdot \dfrac{1 \text{ mi}}{5280 \text{ ft}} \cdot \dfrac{3600 \text{ sec}}{1 \text{ hr}} \approx 77.3 \text{ mph}$

The train is moving at approximately 77.3 miles per hour.

43. First, find the distances $h_s, h_1$, and $h_2$ in the diagram below. Then answer the questions.




$\tan 55° = \dfrac{h_s}{175};\ h_s = 175\tan 55° \approx 250.0 \text{ yd}$

$\tan 24° = \dfrac{h_1}{175};\ h_1 = 175\tan 24° \approx 77.9 \text{ yd}$

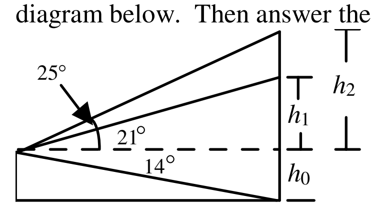
$\tan 30° = \dfrac{h_2}{175};\ h_2 = 175\tan 30° \approx 101.0 \text{ yd}$

a. The southern rim of the canyon is approximately 250.0 yards high.

b. $h_s + h_2 \approx 251.0 + 101.0 = 351.0 \text{ yd}$
The northern rim of the canyon is approximately 351.0 yards high.

c. $h_2 - h_1 \approx 101.0 - 77.9 = 23.1 \text{ yd}$
The climber has 23.1 yards to go before reaching the top.

44. First, find the distances $h_0, h_1$, and $h_2$ in the diagram below. Then answer the questions.




$\tan 14° = \dfrac{h_0}{300};\ h_0 = 300\tan 14° \approx 74.8 \text{ ft}$

$\tan 21° = \dfrac{h_1}{300};\ h_1 = 300\tan 21° \approx 115.2 \text{ ft}$

$\tan 25° = \dfrac{h_2}{300};\ h_2 = 300\tan 25° \approx 139.9 \text{ ft}$

a. The height of the observation post is approximately 74.8 feet.

b. $h_0 + h_2 \approx 74.8 + 139.9 = 214.7 \text{ ft}$
The height of the tree is approximately 214.7 feet.

c. $h_0 + h_1 \approx 74.8 + 115.2 = 190.0$
The baboons are approximately 190.0 feet above the ground.

45. Let $h_t$ be the height of the tower and $h_r$ be the height of the restaurant.

$\tan 74.6° = \frac{h_t}{500}$

$h_t = 500\tan 74.6° \approx 1815.2$ ft

The CNN Tower is approximately 1815.2 feet tall.

$\tan 66.5° = \frac{h_r}{500}$

$h_r = 500\tan 66.5° \approx 1149.9$ ft

$1815.2 - 1149.9 = 665.3$ ft

The restaurant is located approximately 665.3 feet below the pinnacle of the tower.

46. Let $h_s$ be the height to the top of the spire and $h_r$ be the height to the roof.

$\tan 79° = \frac{h_s}{159}$

$h_s = 159\tan 79° \approx 818$ m

Burj Dubai is approximately 818 meters tall.

$\tan 79° = \frac{h_r}{134}$

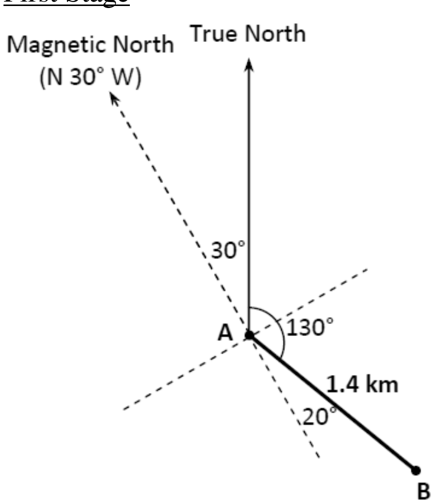
$h_r = 134\tan 79° \approx 689.4$ m

$818 - 689.4 = 128.6$ m

The spire itself is approximately 128.6 meters tall.

47. The key to solving this exercise are several careful sketches, keeping track of angles and distances, while always maintaining true North.

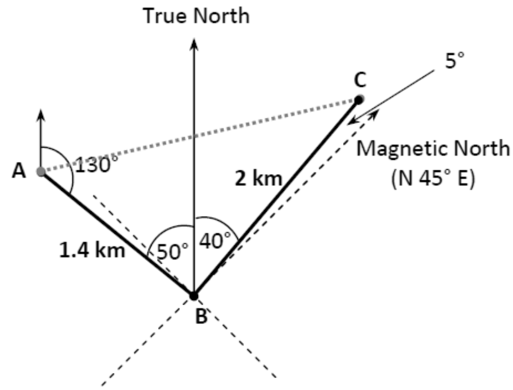
First Stage




47. (continued)

Second Stage

Here our diagram shows us angle $B$ is a right angle.



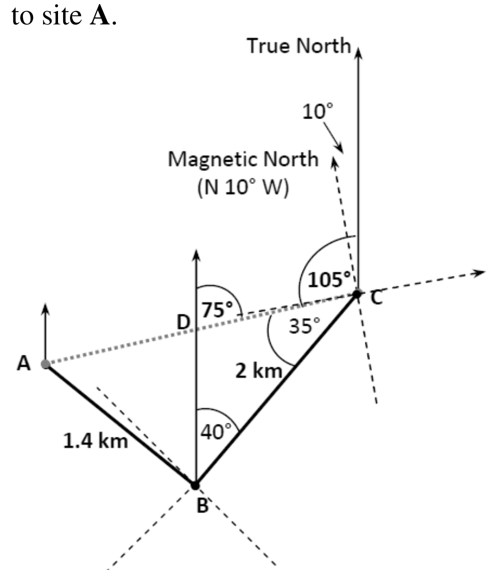

We can now use our knowledge of right triangles to determine angle $C$.

$\tan C = \frac{1.4}{2}$

$C = \tan^{-1}\left(\frac{1.4}{2}\right) = \tan^{-1}(0.7) \approx 35°$

Third Stage

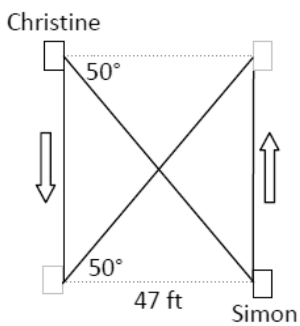
We use one more careful sketch to determine the magnetic direction to return to site **A**.




From the properties of triangles, we know $\angle BDC = 105°$. Supplementary angle rules give us the two bold angles of $75°$ and $105°$. To get back to site **A**, the rover needs to turn $105°$ counterclockwise of true north (a $95°$ counterclockwise rotation from magnetic north). This corresponds to a magnetic bearing of S 85° W.

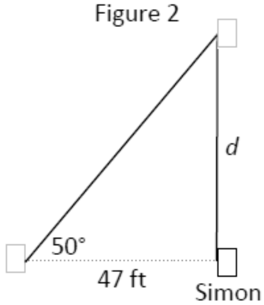
48. To begin, we draw a quick sketch, which includes both when Christine was above Simon and when she was below him (see Figure 1).

Figure 1




Because both cars have the same velocity, we can consider just Simon's. It took him 7 seconds to move from his lower position to his upper position, and the distance $d$ he traveled can easily be determined using a right triangle (See Figure 2), as follows.

Figure 2




$$\tan 50° = \frac{d}{47}$$
$$d = 47\tan 50° \approx 56\text{ ft}$$

Each car will travel 56 meters in 7 seconds.

$$\frac{56\text{ ft}}{7\text{ sec}} = 8\text{ ft/sec}$$

The cars travel at 8 feet per second.

49. $a = \frac{c}{2} = \frac{14\sqrt{3}}{2} = 7\sqrt{3}$ in.
$b = \sqrt{3}a = \sqrt{3}\cdot 7\sqrt{3} = 21$ in.

50. $\csc^2(90° - \theta) = \sec^2\theta = 5^2 = 25$

51. For $x = 1,\ y = -3(1) = -3;\ (1, -3).$
$r = \sqrt{1^2 + (-3)^2} = \sqrt{10}$

a. $\sin\theta = \frac{-3}{\sqrt{10}} = -\frac{3\sqrt{10}}{10}$

b. $\cos\theta = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$

52. 
$$\begin{aligned} C &= 180° - (48°36' + 91°44'21'') \\ &= 180° - 140°20'21'' \\ &= 179°59'60'' - 140°20'21'' \\ &= 39°39'39'' \end{aligned}$$

53. $\sin\alpha = \frac{2}{5} = \frac{\text{opp}}{\text{hyp}}$
$\text{adj} = \sqrt{5^2 - 2^2} = \sqrt{21}$
$\cos\alpha = \frac{\sqrt{21}}{5};\ \tan\alpha = \frac{2}{\sqrt{21}} = \frac{2\sqrt{21}}{21};$
$\csc\theta = \frac{5}{2};\ \sec\theta = \frac{5}{\sqrt{21}} = \frac{5\sqrt{21}}{21};$
$\cot\theta = \frac{\sqrt{21}}{2}$

54. $C = 90° - 25° = 65°$
$\sin 25° = \frac{4}{b};\ b\sin 25° = 4$
$b = \frac{4}{\sin 25°} \approx 9.5\text{ cm}$
$\tan 25° = \frac{4}{c};\ c\tan 25° = 4$
$c = \frac{4}{\tan 25°} \approx 8.6\text{ cm}$

| Angle | Side |
|---|---|
| $A = 25°$ | $a = 4\text{ cm}$ |
| $B = 90°$ | $b = 9.5\text{ cm}$ |
| $C = 65°$ | $c = 8.6\text{ cm}$ |

## 2.4 Exercises

1. coterminal; reference; $80°$

2. reference; quadrant; terminal

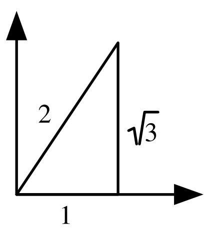
3. integers; $360°k$

4. acute; terminal

5. Answers will vary.

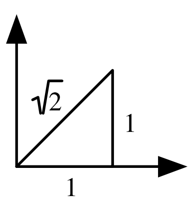
6. Answers will vary.

7.




The lengths were obtained using the special triangle relationships for 30-60-90.
The slope of the line is $\sqrt{3}$ since the rise is $\sqrt{3}$ and the run is 1.
The equation is $y=\sqrt{3}x$.
Choose the point $3, 3\sqrt{3}$ .
Then $r=\sqrt{3^2+(3\sqrt{3})^2}=\sqrt{36}=6$.
$\sin 60°=\frac{3\sqrt{3}}{6}=\frac{\sqrt{3}}{2}$; $\cos 60°=\frac{3}{6}=\frac{1}{2}$
$\tan 60°=\frac{3\sqrt{3}}{3}=\sqrt{3}$.
These match our known values.

8.




The lengths were obtained using the special triangle relationships for 45-45-90.
The slope of the line is 1 since the rise and run are both one.
The equation is $y = x$.
Choose the point (5, 5).
Then $r=\sqrt{5^2+5^2}=\sqrt{50}$.

8. (continued)
$\sin 45°=\frac{5}{\sqrt{50}}=\frac{5}{5\sqrt{2}}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$;
$\cos 45°=\frac{5}{\sqrt{50}}=\frac{5}{5\sqrt{2}}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$
$\tan 45°=\frac{5}{5}=1$.
These match our known values.

9. $50°$ is a QI angle:
$\theta_r=50°$

10. $120°$ is a QII angle:
$\theta_r=180°-120°=60°$

11. $210°$ is a QIII angle:
$\theta_r=210°-180°=30°$

12. $315°$ is a QIV angle:
$\theta_r=360°-315°=45°$

13. $-45°$ is a QIV angle:
$\theta_r=0°-(-45)°=45°$

14. $-150°$ is a QIII angle:
$\theta_r=180°+(-150)°=30°$

15. $112°$ is a QII angle:
$\theta_r=180°-112°=68°$

16. $417°$ is a QI angle:
$\theta_r=417°-360°=57°$

17. $404.4°$ is a QI angle:
$\theta_r=404.4°-360°=44.4°$

18. $222.2°$ is a QIII angle:
$\theta_r=222.2°-180°=42.2°$

19. $500°$ is a QII angle:
$\theta_r=3\cdot 180°-500°=40°$

20. $1125°$ is a QI angle:
$\theta_r=1125°-3\cdot 360°=45°$

21. $-168.4°$ is a QIII angle:
$\theta_r=180°+(-168.4)°=11.6°$

22. $-328.2°$ is a QIV angle:
$\theta_r = 360° + (-328.2)° = 31.8°$

23. $-382.1°$ is a QI angle:
$\theta_r = -(-382.1°) - 360° = 22.1°$

24. $1646.3°$ is a QIII angle:
$\theta_r = 1646.3° - 9 \cdot 180° = 26.3°$

25. $1361°30'$ is a QIV angle:
$\theta_r = 4 \cdot 360° - 1361°30' = 78°30'$

26. $-969°45'$ is a QII angle:
$\theta_r = -5 \cdot 180° - (-969°45') = 69°45'$

27. $330°$ is a QIV angle:
$\theta_r = 360° - 330° = 30°$
$\sin\theta = -\frac{1}{2};\ \cos\theta = \frac{\sqrt{3}}{2};\ \tan\theta = -\frac{1}{\sqrt{3}}$

28. $390°$ is a QI angle:
$\theta_r = 390° - 360° = 30°$
$\sin\theta = -\frac{1}{2};\ \cos\theta = \frac{\sqrt{3}}{2};\ \tan\theta = -\frac{1}{\sqrt{3}}$

29. $-45°$ is a QIV angle:
$\theta_r = 0° - (-45)° = 45°$
$\sin\theta = -\frac{\sqrt{2}}{2};\ \cos\theta = \frac{\sqrt{2}}{2};\ \tan\theta = -1$

30. $-120°$ is a QIII angle:
$\theta_r = 180° + (-120)° = 60°$
$\sin\theta = -\frac{\sqrt{3}}{2};\ \cos\theta = -\frac{1}{2};\ \tan\theta = \sqrt{3}$

31. $240°$ is a QIII angle:
$\theta_r = 240° - 180° = 60°$
$\sin\theta = -\frac{\sqrt{3}}{2};\ \cos\theta = -\frac{1}{2};\ \tan\theta = \sqrt{3}$

32. $315°$ is a QIV angle:
$\theta_r = 360° - 315° = 45°$
$\sin\theta = -\frac{\sqrt{2}}{2};\ \cos\theta = \frac{\sqrt{2}}{2};\ \tan\theta = -1$

33. $-150°$ is a QIII angle:
$\theta_r = 180° + (-150)° = 30°$
$\sin\theta = -\frac{1}{2};\ \cos\theta = -\frac{\sqrt{3}}{2};\ \tan\theta = \frac{1}{\sqrt{3}}$

34. $-210°$ is a QIII angle:
$\theta_r = -180° - (-210)° = 30°$
$\sin\theta = \frac{1}{2};\ \cos\theta = -\frac{\sqrt{3}}{2};\ \tan\theta = -\frac{1}{\sqrt{3}}$

35. $600°$ is a QIII angle:
$\theta_r = 600° - 3 \cdot 180° = 60°$
$\sin\theta = -\frac{\sqrt{3}}{2};\ \cos\theta = -\frac{1}{2};\ \tan\theta = \sqrt{3}$

36. $480°$ is a QII angle:
$\theta_r = 3 \cdot 180° - 480° = 60°$
$\sin\theta = \frac{\sqrt{3}}{2}; \cos\theta = -\frac{1}{2}; \tan\theta = -\sqrt{3}$

37. $-840°$ is a QIII angle:
$\theta_r = -840° + 5 \cdot 180° = 60°$
$\sin\theta = -\frac{\sqrt{3}}{2};\ \cos\theta = -\frac{1}{2};\ \tan\theta = \sqrt{3}$

38. $-840°$ is a QII angle:
$\theta_r = -5 \cdot 180° - (-930°) = 30°$
$\sin\theta = \frac{1}{2};\ \cos\theta = -\frac{\sqrt{3}}{2};\ \tan\theta = -\frac{1}{\sqrt{3}}$

39. $570°$ is a QIII angle:
$\theta_r = 570° - 3 \cdot 180° = 30°$
$\sin\theta = -\frac{1}{2};\ \cos\theta = -\frac{\sqrt{3}}{2};\ \tan\theta = \frac{1}{\sqrt{3}}$

40. $495°$ is a QII angle:
$\theta_r = 3 \cdot 180° - 495° = 45°$
$\sin\theta = \frac{\sqrt{2}}{2};\ \cos\theta = -\frac{\sqrt{2}}{2};\ \tan\theta = -1$

41. $-1230°$ is a QIII angle:
$\theta_r = -1230° + 7 \cdot 180° = 30°$
$\sin\theta = -\frac{1}{2};\ \cos\theta = -\frac{\sqrt{3}}{2};\ \tan\theta = \frac{1}{\sqrt{3}}$

42. $3270°$ is a QI angle:
$\theta_r = 3270° - 9 \cdot 360° = 30°$
$\sin\theta = \frac{1}{2};\ \cos\theta = \frac{\sqrt{3}}{2};\ \tan\theta = \frac{1}{\sqrt{3}}$

43. $52° + 360°k$
$52° + 360°(2) = 772°$
$52° + 360°(1) = 412°$
$52° + 360°(-1) = -308°$
$52° + 360°(-2) = -668°$

44. $12° + 360°k$
$12° + 360°(2) = 732°$
$12° + 360°(1) = 372°$
$12° + 360°(-1) = -348°$
$12° + 360°(-2) = -708°$

45. $87.5° + 360°k$
$87.5° + 360°(2) = 807.5°$
$87.5° + 360°(1) = 447.5°$
$87.5° + 360°(-1) = -272.5°$
$87.5° + 360°(-2) = -632.5°$

46. $22.8° + 360°k$
$22.8° + 360°(2) = 742.8°$
$22.8° + 360°(1) = 382.8°$
$22.8° + 360°(-1) = -337.2°$
$22.8° + 360°(-2) = -697.2°$

47. $225° + 360°k$
$225° + 360°(2) = 945°$
$225° + 360°(1) = 585°$
$225° + 360°(-1) = -135°$
$225° + 360°(-2) = -495°$

48. $175° + 360°k$
$175° + 360°(2) = 895°$
$175° + 360°(1) = 535°$
$175° + 360°(-1) = -185°$
$175° + 360°(-2) = -545°$

49. $-107° + 360°k$
$-107° + 360°(2) = 613°$
$-107° + 360°(1) = 253°$
$-107° + 360°(-1) = -467°$
$-107° + 360°(-2) = -827°$

50. $-215° + 360°k$
$-215° + 360°(2) = 505°$
$-215° + 360°(1) = 145°$
$-215° + 360°(-1) = -575°$
$-215° + 360°(-2) = -935°$

51. The three angles are coterminal in QII.
$\theta_r = 60°$
$\sin 120° = \frac{\sqrt{3}}{2}$
$\cos(-240°) = -\frac{1}{2}$
$\tan 480° = -\sqrt{3}$

52. The three angles are coterminal in QIII.
$\theta_r = 45°$
$\sin 225° = -\frac{\sqrt{2}}{2}$
$\cos 585° = -\frac{\sqrt{2}}{2}$
$\tan(-495°) = 1$

53. The three angles are coterminal in QIV.
$\theta_r = 30°$
$\sin(-30°) = -\frac{1}{2}$
$\cos(-390°) = \frac{\sqrt{3}}{2}$
$\tan 690° = -\frac{1}{\sqrt{3}}$

54. The three angles are coterminal in QIII.
$\theta_r = 30°$
$\sin 210° = -\frac{1}{2}$
$\cos 570° = -\frac{\sqrt{3}}{2}$
$\tan(-150°) = \frac{1}{\sqrt{3}}$

55. QIV; sine is negative
$\theta_r = 2 \cdot 360° - 719° = 1°$
$\sin\theta = -0.0175$
$\sin\theta_r = 0.0175$

56. QII; cosine is negative
$\theta_r = 3 \cdot 180° - 528° = 12°$
$\cos\theta = -0.9781$
$\cos\theta_r = 0.9781$

57. QIV; tangent is negative
$\theta_r = -360° - (-419°) = 59°$
$\tan\theta = -1.6643$
$\tan\theta_r = 1.6643$

58. QII; secant is negative
$\theta_r = -3 \cdot 180° - (-621°) = 81°$
$\sec\theta = -6.3925$
$\sec\theta_r = 6.3925$

59. QIV; cosecant is negative
$\theta_r = 2 \cdot 360° - 681° = 39°$
$\csc\theta = -1.5890$
$\csc\theta_r = 1.5890$

60. QIV; tangent is negative
$\theta_r = 3 \cdot 360° - 995° = 85°$
$\tan\theta = -11.4301$
$\tan\theta_r = 11.4301$

61. QI; cosine is positive
$\theta_r = 805° - 2 \cdot 360° = 85°$
$\cos\theta = 0.0872$
$\cos\theta_r = 0.0872$

62. QI; sine is positive
$\theta_r = 772° - 2 \cdot 360° = 52°$
$\sin\theta = 0.7880$
$\sin\theta_r = 0.7880$

63. a. $A = ab\sin\theta$
$= (9)(21)\sin 50°$
$\approx 144.78 \text{ units}^2$

b. Enter the function $Y_1 = 9*21\sin x$ and set up a table with TblStart = 50 and ΔTbl = 1: The first value over 150 is at 53º.
$$A = ab\sin\theta$$
$$150 < (9)(21)\sin\theta$$
$$\frac{150}{(9)(21)} < \sin\theta$$
$$\theta > \sin^{-1}\frac{150}{(9)(21)} \approx 53°$$

c. $A = ab\sin\theta$
$= ab\sin 90°$
$= ab \cdot 1$
$= ab$
The parallelogram is a rectangle whose area is $A = ab.$

d. Divide the parallelogram in half with a diagonal to form a triangle. The area given two sides and the angle between them is then $A = \frac{1}{2}ab\sin\theta = \frac{ab}{2}\sin\theta.$

64. For $y_1 = \frac{3}{4}x + 2,\ m_1 = \frac{3}{4}.$
For $y_2 = -\frac{2}{3}x + 5,\ m_1 = -\frac{2}{3}.$
$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$
$$\theta = \tan^{-1}\frac{m_2 - m_1}{1 + m_1 m_2}$$
$$\theta = \tan^{-1}\left[\frac{\left(-\frac{2}{3}\right) - \left(\frac{3}{4}\right)}{1 + \left(\frac{3}{4}\right)\left(-\frac{2}{3}\right)}\right]$$
$$\theta = \tan^{-1}\left(-\frac{17}{6}\right)$$
$$\theta \approx -70.6°$$
The angle between the lines is approximately 70.6°.

65. Cosine is positive in QI and QIV.
$\theta_r = 60°$
$\theta = 60° + 360°k;\quad \theta = 300° + 360°k$

66. Sine is positive in QI and QII.
$\theta_r = 45°$
$\theta = 45° + 360°k;\ \theta = 135° + 360°k$

67. Sine is negative in QIII and QIV.
$\theta_r = 60°$
$\theta = 240° + 360°k;\quad \theta = 300° + 360°k$

68. Tangent is negative in QII and QIV.
$\theta_r = 60°$
$\theta = 120° + 360°k;\quad \theta = 300° + 360°k$

69. Sine is positive in QI and QII.
$\theta_r = \sin^{-1} 0.8754 \approx 61.1°$
$\theta \approx 61.1° + 360°k;\ \theta \approx 118.9° + 360°k$

70. Cosine is positive in QI and QIV.
$\theta_r = \cos^{-1} 0.2378 \approx 76.2°$
$\theta \approx 76.2° + 360°k;\ \theta \approx 283.8° + 360°k$

71. Tangent is negative in QII and QIV.
$\theta_r = \tan^{-1}(-2.3512) \approx -67.0°$
$\theta \approx 113.0° + 360°k;\ \theta \approx 293.0° + 360°k$

72. Cosine is negative is QII and QIII.
$\theta_r = \cos^{-1}(-0.0562) \approx 93.2°;$
$\theta \approx 93.2° + 360°k;\ \theta \approx 266.8° + 360°k$

73. The spinner made five complete turns plus a quarter of another turn.
$5\cdot 360°+90°=1890°$
Coterminal angles: $90°+360°k$

74. The fan made three complete turns plus a third of another turn.
$3\cdot 360°+120°=1200°$
Coterminal angles: $120°+360°k$

75. Assuming he was on his feet at the start, after 2.5 revolutions, he'd go in the water head first.
$2\cdot 360°+180°=900°$

76. Charlene completes three complete rotations and a belly flop, which represents a $90°$ rotation from the horizontal.
$3\cdot 360°+90°=1170°$

77. The angle between the terminal side and the $x$-axis can be found using a right triangle with opposite 2 and adjacent 6.
$\tan\theta=\frac{2}{6};\ \theta=\tan^{-1}\left(\frac{1}{3}\right)\approx 18.4°$
$720°-18.4°=701.6°$
The spiral of Archimedes turned through an angle of $701.6°$.

78. The angle between the terminal side and the negative $x$-axis can be found using a right triangle with opposite 3.5 and adjacent 4.
$\tan\theta=\frac{3.5}{4};\ \theta=\tan^{-1}\left(\frac{3.5}{4}\right)\approx 41.2°$
$180°-41.2°=221.2°$
The involute turned through an angle of $221.2°$.

79. a. 1 o'clock represents 1/3 of the way to 3 o'clock: $1/3(90°)=30°$
2 o'clock represents 2/3 of the way to 3 o'clock: $2/3(90°)=60°$

b. From part (a), we see that the angle between any two numbers is $30°$.
At 6:30, the minute hand is on the 6 and the hour hand is halfway between the 6 and the 7. $1/2(30°)=15°$
At 7:00, the hour and minute hands are five numbers apart. $5\cdot 30°=150°$
At 7:30, the minute hand is on the 6 and the hour hand is halfway between the 7 and the 8. $3/2(30°)=45°$

80. a. First, let's find $\theta$. Note that the original ray goes through $(3,2)$, so we can form a right triangle with opposite 2 and adjacent 3.
$\tan\theta=\frac{2}{3};\ \theta=\tan^{-1}\left(\frac{2}{3}\right)=33.7°$
The final ray has terminal point $(2,5)$, so the final angle with the $x$-axis is
$\theta_2=\tan^{-1}\left(\frac{5}{2}\right)\approx 68.2°$
$68.2°-33.7°=34.5°$
The ray must be rotated through an angle of $34.5°$.

b. $d=\sqrt{2^2+5^2}=\sqrt{29}\approx 5.4$ units
The length of side $AC$ is approximately 5.4 units.

81. a. $3\text{ sec}\cdot\frac{12\text{ rev}}{\text{sec}}=36\text{ rev}$
$36\cdot 360°=12{,}960°$

b. $C=2\pi r=2\pi(20)=40\pi\approx 125.66\text{ in.}$

c. $10\text{ sec}\cdot\frac{12\text{ rev}}{\text{sec}}=120\text{ rev}$
$120\text{ rev}\cdot\frac{125.66\text{ in.}}{\text{rev}}\approx 15{,}080\text{ in.}$

d. $\frac{15{,}080\text{ in.}}{10\text{ sec}}\cdot\frac{3600\text{ sec}}{1\text{ hr}}\cdot\frac{1\text{ mi}}{12(5280)\text{ in.}}$
$\approx 85.68\text{ mph}$

82. Let $x$ be the height of the building above eye level.
$\tan 78°=\frac{x}{117}$
$x=117\tan 78°\approx 550.4$ ft
$550.4+5=555.4$ ft
The height of the monument is approximately 555.4 feet.

83. Due to the cofunction relationship,
$$\begin{aligned}\cot 50°&=\tan(90°-40°)\\&=\tan 50°\\&\approx 0.8391.\end{aligned}$$

84. $b=\sqrt{c^2-a^2}=\sqrt{52^2-25^2}=\sqrt{2079}\approx 45.6$ cm

$\sin A=\frac{25}{52};\ A=\sin^{-1}\frac{25}{52}\approx 28.7°$

$B=90°-28.7°=61.3°$

| Angle | Side |
|---|---|
| $A\approx 28.7°$ | $a=25$ cm |
| $B\approx 61.3°$ | $b\approx 45.6$ cm |
| $C=90°$ | $c=52$ cm |

85. Since $c=2a$, this is a 30-60-90 triangle.

$b=\sqrt{3}\cdot 18=18\sqrt{3}$ m

| Angle | Side |
|---|---|
| $A=30°$ | $a=18$ m |
| $B=60°$ | $b=18\sqrt{3}$ m |
| $C=90°$ | $c=36$ m |

86. Since $\sin^2\theta+\cos^2\theta=1$, then

$\sin\theta=\pm\sqrt{1-\cos^2\theta}$

87. Complement:

$$\begin{aligned}&90°-12°34'56''\\&=89°59'60''-12°34'56''\\&=(89°-12°)+(59'-34')+(60''-56'')\\&=77°+25'+04''\\&=77°25'04''\end{aligned}$$

Supplement

$$\begin{aligned}&180°-12°34'56''\\&=179°59'60''-12°34'56''\\&=(179°-12°)+(59'-34')+(60''-56'')\\&=167°+25'+04''\\&=167°25'04''\end{aligned}$$

## Summary and Concept Review 2.1

1. a. $\sin\alpha=\frac{\text{opp}}{\text{hyp}}=\frac{2\sqrt{6}}{7}$

   b. $\sin\beta=\frac{\text{opp}}{\text{hyp}}=\frac{5}{7}$

2. a. $\cos\alpha=\frac{\text{adj}}{\text{hyp}}=\frac{5}{7}$

   b. $\cos\beta=\frac{\text{adj}}{\text{hyp}}=\frac{2\sqrt{6}}{7}$

3. a. $\tan\alpha=\frac{\text{opp}}{\text{adj}}=\frac{2\sqrt{6}}{5}$

   b. $\tan\beta=\frac{\text{opp}}{\text{adj}}=\frac{5}{2\sqrt{6}}=\frac{5\sqrt{6}}{12}$

4. a. $\csc\alpha=\frac{\text{hyp}}{\text{opp}}=\frac{7}{2\sqrt{6}}=\frac{7\sqrt{6}}{12}$

   b. $\csc\beta=\frac{\text{hyp}}{\text{opp}}=\frac{7}{5}$

5. a. $\sec\alpha=\frac{\text{hyp}}{\text{adj}}=\frac{7}{5}$

   b. $\sec\beta=\frac{\text{hyp}}{\text{adj}}=\frac{7}{2\sqrt{6}}=\frac{7\sqrt{6}}{12}$

6. a. $\cot\alpha=\frac{\text{adj}}{\text{opp}}=\frac{5}{2\sqrt{6}}=\frac{5\sqrt{6}}{12}$

   b. $\cot\beta=\frac{\text{adj}}{\text{opp}}=\frac{2\sqrt{6}}{5}$

7. $\tan\theta=3=\frac{3}{1}=\frac{\text{opp}}{\text{adj}}$

   $\text{hyp}=\sqrt{1^2+3^2}=\sqrt{10}$

   $\sin\theta=\frac{3}{\sqrt{10}}=\frac{3\sqrt{10}}{10};\ \cos\theta=\frac{1}{\sqrt{10}}=\frac{\sqrt{10}}{10}$

   $\csc\theta=\frac{\sqrt{10}}{3};\ \sec\theta=\sqrt{10};\ \cot\theta=\frac{1}{3}$

8. a. $\tan 57.4°=\cot(90°-57.4°)$
   $=\cot 32.6°$

   b.
$$\begin{aligned}&\sin(19°30'15'')\\&=\cos(90°-19°30'15'')\\&=\cos[(89°59'60'')-19°30'15'']\\&=\cos[(89°-19°)+(59'-30')+(60''-15'')]\\&=\cos(70°+29'+45'')\\&=\cos 70°29'45''\end{aligned}$$

## Summary and Concept Review 2.2

9. a. $A = \cos 37° \approx 0.80$

   b. $\cos A = 0.4340$
   $A = \cos^{-1} 0.4340 \approx 64.3°$

10. $B = 90° - 49° = 41°$

$\sin 49° = \frac{89}{c};\ c \sin 49° = 89$

$c = \frac{89}{\sin 49°} \approx 117.93 \text{ in.}$

$\tan 49° = \frac{89}{b};\ b \tan 49° = 89$

$b = \frac{89}{\tan 49°} \approx 77.37 \text{ in.}$

| Angle | Side |
|---|---|
| $A = 49°$ | $a = 89$ in. |
| $B = 41°$ | $b \approx 77.37$ in. |
| $C = 90°$ | $c \approx 117.93$ in. |

11. $c = \sqrt{a^2 + b^2} = \sqrt{20^2 + 21^2} = \sqrt{841} = 29$

$\sin A = \frac{20}{29};\ A = \sin^{-1} \frac{20}{29} \approx 43.6°$

$B = 90° - 43.6° = 46.4°$

| Angle | Side |
|---|---|
| $A \approx 43.6°$ | $a = 20$ m |
| $B° \approx 46.4°$ | $b = 21$ m |
| $C = 90°$ | $c = 29$ m |

12. Let $h$ be the height of the support.

$\sin 15° = \frac{h}{20}$

$h = 20 \sin 15° \approx 5.18$ m

The support is approximately 5.18 meters.

13. $c = \sqrt{a^2 + b^2} = \sqrt{10^2 + 14^2}$

$= \sqrt{296} = 2\sqrt{74}$

$\sin A = \frac{14}{2\sqrt{74}} = \frac{7}{\sqrt{74}}$

$A = \sin^{-1} \frac{7}{\sqrt{74}} \approx 54.5°$

$B = 90° - 54.5° = 35.5°$

## Summary and Concept Review 2.3

14. Let $d$ be the distance Justin is parked from center court.

$\tan 11° = \frac{92}{d};\ d \tan 11° = 92$

$d = \frac{92}{\tan 11°} \approx 473 \text{ ft}$

Justin is parked approximately 473 feet from center court.

15. The four bearings specified are listed.
$N 31° E$, $N 31° W$, $S 31° E$, $S 31° W$

16. Let $h$ be the present height of the helicopter.

$\tan 14° = \frac{h}{120}$

$h = 120 \tan 14° \approx 30$ ft

$\frac{30 \text{ ft}}{20 \text{ sec}} = 1.5 \text{ ft/sec}$

The helicopter is descending at a rate of approximately 1.5 feet per second.

17. a. Let $d_1$ be the distance of the nearer boat from Armando.

$\theta_1 = 90° - 63.5° = 26.5°$

$\tan 26.5° = \frac{d_1}{480}$

$d_1 = 480 \tan 26.5° \approx 239.32$ m

The nearer boat is approximately 239.32 meters out to sea.

Let $d_2$ be the distance of the further boat from Armando.

$\theta_2 = 90° - 45° = 45°$

$\tan 45° = \frac{d_2}{480}$

$d_2 = 480 \tan 45° = 480$ m

The further boat is 480 meters out to sea.

   b. $480 - 239.32 = 240.68$ m

   The two boats are approximately 240.68 meters apart.

## Summary and Concept Review 2.4

18. $-152°$ is a QIII angle:
$\theta_r = 180° + (-152)° = 28°$

$521°$ is a QII angle:
$\theta_r = 3\cdot 180° - 521° = 19°$

$210°$ is a QIII angle:
$\theta_r = 210° - 180° = 30°$

19. $-870°$ is a QIII angle:
$\theta_r = -870° + 5\cdot 180° = 30°$

$\sin\theta = -\frac{1}{2};\ \cos\theta = -\frac{\sqrt{3}}{2};\ \tan\theta = \frac{\sqrt{3}}{3}$

20. The three angles are coterminal in QII.
$\theta_r = 45°$

a. $\sin 135° = \frac{\sqrt{2}}{2}$

b. $\cos(-225°) = -\frac{\sqrt{2}}{2}$

c. $\tan 855° = -1$

21. a. Tangent is negative in QII and QIV.
$\theta_r = 45°$
$\theta = 135° + 180°k$

b. Cosine is positive in QI and QIV.
$\theta_r = 30°$
$\theta = 30° + 360°k;\ \theta = 330° + 360°k$

c. Tangent is positive in QI and QIII.
$\theta_r = \tan^{-1}(4.0108) \approx 76.0°$
$\theta \approx 76.0° + 180°k$

d. Sine is negative in QIII and QIV.
$\theta_r = \sin^{-1}(-0.4540) \approx -27.0°$
$\theta \approx -27.0° + 360°k;\ \theta \approx 207.0° + 360°k$

## Mixed Review

1. $\sin\alpha = \frac{\text{opp}}{\text{hyp}} = \frac{a}{c} = \frac{6}{\sqrt{61}} = \frac{6\sqrt{61}}{61}$

$\tan\beta = \frac{\text{opp}}{\text{adj}} = \frac{b}{a} = \frac{5}{6}$

2. Let $x$ be the distance of the police station door from the city hall door.
$\tan 36° = \frac{x}{225}$
$x = 225\tan 36° \approx 163.5\text{ ft}$
The police station door is approximately 163.5 feet from the police station door.

3. $1200°$ is a QII angle:
$\theta_r = 7\cdot 180° - 1200° = 60°$

$\sin\theta = \frac{\sqrt{3}}{2};\ \cos\theta = -\frac{1}{2};\ \tan\theta = -\sqrt{3}$

4. $c = \sqrt{a^2 + b^2} = \sqrt{47^2 + 52^2} = \sqrt{4913} \approx 70.1$
$\sin A = \frac{47}{70.1};\ A = \sin^{-1}\frac{47}{70.1} \approx 42.1°$
$B = 90° - 42.1° = 47.9°$

| Angle | Side |
|---|---|
| $A \approx 42.1°$ | $a = 47$ mm |
| $B° \approx 47.9°$ | $b = 52$ mm |
| $C = 90°$ | $c \approx 70.1$ mm |

5. Let $h$ be the height of the tornado.
$3.2\text{ km} = 3200\text{ m}$
$\tan 17.3° = \frac{h}{3200}$
$h = 3200\tan 17.3° \approx 997\text{ m}$
At that moment, the tornado is approximately 997 meters tall.

6. $\csc(2x)° = \sec(3x-5)°$
$\csc(2x)° = \csc[90-(3x-5)]°$
$2x = 90 - (3x-5)$
$2x = 90 - 3x + 5$
$2x = 95 - 3x$
$5x = 95$
$x = 19$

7. $\cos\theta = \frac{36}{85} = \frac{\text{adj}}{\text{hyp}}$

$\text{opp} = \sqrt{85^2 - 36^2} = \sqrt{5929} = 77$

$\sin\theta = \frac{77}{85};\ \csc\theta = \frac{85}{77};\ \sec\theta = \frac{85}{36};$

$\tan\theta = \frac{77}{36};\ \cot\theta = \frac{36}{77}$

8. a. $c = \sqrt{a^2 + b^2} = \sqrt{36^2 + 78^2}$
$= \sqrt{7380} \approx 85.9 > 85$

Yes. Since the hypotenuse is greater than 85 inches, the painting can be brought in the service door.

b. $\cos\theta = \frac{36}{85};\ \theta = \cos^{-1}\frac{36}{85} \approx 65°$

$\sin\theta = \frac{78}{85};\ \theta = \sin^{-1}\frac{78}{85} \approx 67°$

9. The three angles are coterminal in QIV.
$\theta_r = 30°$

a. $\sin(-30°) = -\frac{1}{2}$

b. $\cos 330° = \frac{\sqrt{3}}{2}$

c. $\tan(-750°) = -\frac{\sqrt{3}}{3}$

10. Due to the cofunction relationship,
$\cos(90° - \theta) = \sin\theta = \frac{16}{63}$

11. Let $d$ be the distance of the mower from the house.

$38°39' = \left[38 + 39\left(\frac{1}{60}\right)\right]° = 38.65°$

$90° - 38.65° = 51.35°$

$\tan 51.35° = \frac{d}{16}$

$d = 16\tan 51.35° \approx 20$ ft

The mower is approximately 20 feet from the house.

12. $735°$ is a QI angle:
$\theta_r = 735° - 2 \cdot 360° = 15°$

$-135°$ is a QIII angle:
$\theta_r = 180° + (-135)° = 45°$

13. The clock's hour hand makes $1\frac{3}{4}$ rotations in a clockwise (negative) direction.
$1.75 \cdot (-360°) = -630°$

14. $\cos\alpha\cot\beta = \cos\beta$
$\cos\alpha\tan\alpha = \cos\beta$
$\cos\alpha\frac{\sin\alpha}{\cos\alpha} = \cos\beta$
$\sin\alpha = \cos\beta$
$\cos\beta = \cos\beta$

15. $\sin\theta = \frac{100}{115.47}$

$\theta = \sin^{-1}\frac{100}{115.47} \approx 60°$

You should hit the shot at a $60°$ angle.

16. a. Five contour lines, so increase in elevation is $5(150) = 750$ ft.

b. $\frac{2.25\text{ in.}}{x\text{ in.}} = \frac{1\text{ ft}}{200\text{ ft}};$
$x = 2.25(200) = 450$ ft

The horizontal distance from $A$ to $B$ is 450 feet.

c. $d^2 = 450^2 + 750^2;$
$d = \sqrt{765{,}000} \approx 874.6$ ft

The length of conduit needed is approximately 874.6 feet.

$\tan\theta = \frac{750}{450};\ \theta = \tan^{-1}\left(\frac{750}{450}\right) \approx 59°$

The installers will experience a $59°$ angle of incline.

17. $B = 90° - 11.3° = 78.7°$

$\tan 11.3° = \frac{a}{60.5}$

$a = 60.5\tan 11.3° \approx 12.1$ m

$\sin 78.7° = \frac{60.5}{c};\ c\sin 78.7° = 60.5$

$c = \frac{60.5}{\sin 78.7°} \approx 61.7$ m

| Angle | Side |
|---|---|
| $A = 11.3°$ | $a \approx 12.1$ m |
| $B° = 78.7°$ | $b = 60.5$ m |
| $C = 90°$ | $c \approx 61.7$ m |

18. Let $d$ be the distance the ducks swam.

$\tan 65°30' = \tan 65.5° = \frac{d}{135}$

$d = 135 \tan 65.5° \approx 296.2$ yd

The ducks swam 296.2 yards.

$9 \text{ min } 52 \text{ sec} = (9 \cdot 60 + 52) \text{ sec}$
$= 592 \text{ sec}$

$\frac{296.2 \text{ yd}}{592 \text{ sec}} = 0.5 \text{ yd/sec} = 1.5 \text{ ft/sec}$

They were swimming 0.5 yards per second, or 1.5 feet per second.

19. $780°$ is a QI angle:

$\theta_r = 780° - 2 \cdot 360° = 60°$

$\tan 780° = \sqrt{3}$

20. Let $h_1$ be the height to the top of the statue.

$\tan 25.60° = \frac{h_1}{600}$

$h_1 = 600 \tan 25.60° \approx 287.5$ ft

The top of the statue is 287.5 feet high.

Let $h_2$ be the height to the base of the statue.

$\tan 24.07° = \frac{h_2}{600}$

$h_2 = 600 \tan 24.07° \approx 268.0$ ft

The base of the statue is 268.0 feet high.

$287.5 - 268.0 = 19.5$ ft

The statue Freedom is about 19.5 feet tall.

## Practice Test

1. $\cos 41° = \frac{\text{adj}}{\text{hyp}} = \frac{b}{c} = \frac{b}{25}$

$b = 25 \cos 41° \approx 18.87$ cm

2. a. $225°$ is a QIII angle:

$\theta_r = 225° - 180° = 45°$

b. $-510°$ is a QIII angle:

$\theta_r = 3 \cdot 180° + (-510)° = 30°$

3. $\csc\theta = 4 = \frac{4}{1} = \frac{\text{hyp}}{\text{opp}}$

$\text{adj} = \sqrt{4^2 - 1^2} = \sqrt{15}$

$\sin\theta = \frac{1}{4}$; $\cos\theta = \frac{\sqrt{15}}{4}$; $\tan\theta = \frac{1}{\sqrt{15}} = \frac{\sqrt{15}}{15}$

$\sec\theta = \frac{4}{\sqrt{15}} = \frac{4\sqrt{15}}{15}$; $\cot\theta = \frac{\sqrt{15}}{1} = \sqrt{15}$

4. $B = 90° - 57° = 33°$

$\sin 33° = \frac{a}{15.0}$

$a = 15.0 \sin 33° \approx 8.2$ cm

$\sin 57° = \frac{b}{15.0}$

$b = 15.0 \sin 57° \approx 12.6$ cm

| Angle | Side |
|---|---|
| $A = 33°$ | $a \approx 8.2$ cm |
| $B° = 57°$ | $b \approx 12.6$ cm |
| $C = 90°$ | $c = 15.0$ cm |

5. $c = \sqrt{a^2 + b^2} = \sqrt{138^2 + 174^2}$
$= \sqrt{49{,}320} \approx 222.1$ ft

$\sin A = \frac{138}{222.1}$; $A = \sin^{-1} \frac{138}{222.1} \approx 38.4°$

$B = 90° - 38.4° = 51.6°$

| Angle | Side |
|---|---|
| $A \approx 38.4°$ | $a = 138$ ft |
| $B \approx 51.6°$ | $b = 174$ ft |
| $C = 90°$ | $c \approx 222.1$ ft |

6. Let $h_1$ be the height from eye level to the top of the tower.

$\tan 32° = \frac{h_1}{73}$

$h_1 = 73 \tan 32° \approx 45.6$ ft

The height from eye level to the top of the tower is 45.6 feet.

Let $h_2$ be the height from eye level to the base of the tower.

$\tan 17° = \frac{h_2}{73}$

$h_2 = 73 \tan 17° \approx 22.3$ ft

The height from eye level to the base of the tower is 22.3 feet.

$45.6 + 22.3 = 67.9$ ft

The tower is approximately 67.9 feet tall.

7.

| $\theta$ | $\sin\theta$ | $\cos\theta$ | $\tan\theta$ | $\csc\theta$ | $\sec\theta$ | $\cot\theta$ |
|---|---|---|---|---|---|---|
| $30^\circ$ | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{3}}{3}$ | $2$ | $\frac{2\sqrt{3}}{3}$ | $\sqrt{3}$ |
| $45^\circ$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{2}}{2}$ | $1$ | $\sqrt{2}$ | $\sqrt{2}$ | $1$ |
| $60^\circ$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | $\sqrt{3}$ | $\frac{2\sqrt{3}}{3}$ | $2$ | $\frac{\sqrt{3}}{3}$ |
| $135^\circ$ | $\frac{\sqrt{2}}{2}$ | $-\frac{\sqrt{2}}{2}$ | $-1$ | $\sqrt{2}$ | $-\sqrt{2}$ | $-1$ |
| $240^\circ$ | $-\frac{\sqrt{3}}{2}$ | $-\frac{1}{2}$ | $\sqrt{3}$ | $-\frac{2\sqrt{3}}{3}$ | $-2$ | $\frac{\sqrt{3}}{3}$ |
| $330^\circ$ | $-\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $-\frac{\sqrt{3}}{3}$ | $-2$ | $\frac{2\sqrt{3}}{3}$ | $-\sqrt{3}$ |

8. a. $B$ is in QI, so $(180^\circ + B)$ is in QIII.
Sine is positive in QI, negative in QIII.
$\sin B = \frac{\text{opp}}{\text{hyp}} = \frac{4}{5}$
$\sin(180^\circ + B) = -\frac{4}{5}$

b. $B$ is in QI, so $(180^\circ - B)$ is in QII.
Cosine is positive in QI, negative in QII.
$\cos B = \frac{\text{adj}}{\text{hyp}} = \frac{3}{5}$
$\cos(180^\circ - B) = -\frac{3}{5}$

c. $B$ is in QI, so $(-B)$ is in QIV.
Tangent is positive in QI, negative in QIV.
$\tan B = \frac{\text{opp}}{\text{adj}} = \frac{4}{3}$
$\tan(-B) = -\frac{4}{3}$

9. $5.5 \text{ sec} \cdot 32 \text{ ft/sec} = 176 \text{ ft}$
Maya ran down the sideline 176 feet.

$\tan\theta = \frac{176}{160};\ \theta = \tan^{-1}\frac{176}{160} \approx 47.7^\circ$
Veronica threw the disc at an angle of $47.7^\circ$.

10.
$$\begin{aligned} 4\cos^2 75^\circ &= 4(\cos 75^\circ)^2 \\ &= 4(\sin(90^\circ - 75^\circ))^2 \\ &= 4(\sin 15^\circ)^2 \\ &= 4\left(\frac{1}{2}\sqrt{2-\sqrt{3}}\right)^2 \\ &= 4\left(\frac{1}{4}\left(2-\sqrt{3}\right)\right) \\ &= 2-\sqrt{3} \end{aligned}$$

11. $b = \sqrt{88^2 - 57^2} = \sqrt{4495} \approx 67 \text{ cm}$
The distance from shoulders to toes to about 67 centimeters.

$\cos\theta = \frac{57}{88};\ \theta = \cos^{-1}\frac{57}{88} \approx 49.6^\circ$
The angle formed at the hips is $49.6^\circ$.

12. Let $d$ be the distance to the lighthouse.
$\tan 25^\circ = \frac{27}{d};\ d\tan 25^\circ = 27$
$d = \frac{27}{\tan 25^\circ} \approx 57.9 \text{ m}$
You are 57.9 meters from the lighthouse.

13. $22.1° + 67.9° = 90°$
There is a right angle between the two flights.
Let $d$ be the distance between the two planes.
$\sin 57.1° = \frac{2.3}{d};\ d\sin 57.1° = 2.3$
$d = \frac{2.3}{\sin 57.1°} \approx 2.7$
The two planes are approximately 2.7 miles apart.

14. Cosine is negative in QII and QIII.
$\theta_r = 30°$
$\theta = 150° + 360°k;\ \theta = 210° + 360°k$

15. a. $\frac{b}{a} = \cot\alpha$

b. $\frac{c}{a} = \csc\alpha$

c. $\frac{b}{c} = \cos\alpha$

d. $\frac{a}{c} = \sin\alpha$

e. $\frac{c}{b} = \sec\alpha$

f. $\frac{a}{b} = \tan\alpha$

16. Let $x$ be length of the dotted diagonal of the bottom of the box.
$x^2 = 10^2 + 20^2;\ x = \sqrt{500} = 10\sqrt{5}$ cm
$\tan\theta = \frac{10\sqrt{5}}{10} = \sqrt{5}$
$\theta = \tan^{-1}\left(\sqrt{5}\right) \approx 65.9°$

17.
$$\sin\alpha(\sec\beta - \sin\alpha) = \sin^2\beta$$
$$\sin\alpha(\csc\alpha - \sin\alpha) = \sin^2\beta$$
$$\sin\alpha\left(\frac{1}{\sin\alpha} - \sin\alpha\right) = \sin^2\beta$$
$$1 - \sin^2\alpha = \sin^2\beta$$
$$\cos^2\alpha = \sin^2\beta$$
$$\sin^2\beta = \sin^2\beta$$

18. Let h be the height of the balloon.
$\tan 38° = \frac{h}{50}$
$h = 50\tan 38° \approx 39$ ft
The balloon is about 39 feet high.

19. All angles have a reference angle of $32°$.
$\sin 148° \approx 0.53$
$\sin 212° \approx -0.53$
$\sin 328° \approx -0.53$

20. Let $d$ be the distance between Alexandria and Port Said.
$\sin 47.5° = \frac{168}{d};\ d\sin 47.5° = 168$
$d = \frac{168}{\sin 47.5°} \approx 228$
Alexandria is about 228 miles from Port Said.

## Calculator Exploration and Discovery

Exercise 1:
$$\overline{ES} = \overline{EM}\sec 87° = 240{,}000 \cdot \sec 87° \approx 4{,}586{,}000 \text{ mi}$$

Exercise 2:
$$\overline{ES} = \overline{EM}\sec 87° = 240{,}000 \cdot \sec 89.85° \approx 91{,}673{,}000 \text{ mi}$$

Exercise 3:
Use the table feature of a graphing calculator to find $\angle E \approx 89.852°$.

Exercise 4:
$$\overline{ES} = \overline{EM}\sec 87° = 240{,}000 \cdot \sec 89.852° \approx 92{,}912{,}000 \text{ mi}$$

## Strengthening Core Skills

Exercise 1:

| $\theta$ | 0° | 30° | 45° | 60° | 90° | 120° | 135° | 150° | 180° | 210° | 225° |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\sin\theta$ | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | $-\frac{\sqrt{2}}{2}$ |
| $\cos\theta$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | $-\frac{\sqrt{2}}{2}$ | $-\frac{\sqrt{3}}{2}$ | −1 | $-\frac{\sqrt{3}}{2}$ | $-\frac{\sqrt{2}}{2}$ |
| $\tan\theta$ | 0 | $\frac{\sqrt{3}}{3}$ | 1 | $\sqrt{3}$ | −− | $-\sqrt{3}$ | −1 | $-\frac{\sqrt{3}}{3}$ | 0 | $\frac{\sqrt{3}}{3}$ | 1 |

Exercise 2:

a. $2\sin\theta+\sqrt{3}=0$
$$2\sin\theta=-\sqrt{3}$$
$$\sin\theta=-\frac{\sqrt{3}}{2}$$
$$\theta=\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$
$$\theta=240°,\ 300°$$

b. $-3\sqrt{2}\cos\theta+4=1$
$$-3\sqrt{2}\cos\theta=-3$$
$$\cos\theta=\frac{-3}{-3\sqrt{2}}$$
$$\cos\theta=\frac{\sqrt{2}}{2}$$
$$\theta=\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$$
$$\theta=45°,\ 315°$$

c. $-\sqrt{3}\tan\theta+2=1$
$$-\sqrt{3}\tan\theta=-1$$
$$\tan\theta=\frac{-1}{-\sqrt{3}}$$
$$\tan\theta=\frac{\sqrt{3}}{3}$$
$$\theta=\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$$
$$\theta=30°,\ 210°$$

d. $\sqrt{2}\sec\theta+1=3$
$$\sqrt{2}\sec\theta=2$$
$$\sec\theta=\frac{2}{\sqrt{2}}$$
$$\cos\theta=\frac{\sqrt{2}}{2}$$
$$\theta=\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$$
$$\theta=45°,\ 315°$$

Exercise 3:

a. $\sqrt{6}\sin\theta-2=1$
$$\sqrt{6}\sin\theta=3$$
$$\sin\theta=\frac{3}{\sqrt{6}}$$
$$\sin\theta=\frac{\sqrt{6}}{2}$$
$$\theta=\sin^{-1}\left(\frac{\sqrt{6}}{2}\right)$$

No solution.

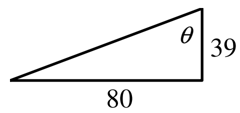
b. $-3\sqrt{2}\cos\theta+\sqrt{2}=0$
$$-3\sqrt{2}\cos\theta=-\sqrt{2}$$
$$\cos\theta=\frac{-\sqrt{2}}{-3\sqrt{2}}$$
$$\cos\theta=\frac{1}{3}$$
$$\theta=\cos^{-1}\left(\frac{1}{3}\right)$$
$$\theta\approx 70.5°,\ 289.5°$$

c. $3\tan\theta+\frac{1}{2}=-\frac{1}{4}$
$$3\tan\theta=-\frac{3}{4}$$
$$\tan\theta=-\frac{1}{4}$$
$$\theta=\tan^{-1}\left(-\frac{1}{4}\right)$$
$$\theta\approx 166.0°,\ 346.0°$$

d. $2\sec\theta=-5$
$$\sec\theta=-\frac{5}{2}$$
$$\theta=\sec^{-1}\left(-\frac{5}{2}\right)$$
$$\theta=\cos^{-1}\left(-\frac{2}{5}\right)$$
$$\theta\approx 113.6°,\ 246.4°$$

# Cumulative Review Chapters 1-2

1.




$c = \sqrt{80^2 + 39^2} = 89$

$\theta = \tan^{-1}\left(\frac{80}{39}\right) \approx 64°$

$90° - 64° = 26°$

2. Let $x$ be the length of the broken portion of the tree.

$\sin 56° = \frac{12}{x}$

$x = \frac{12}{\sin 56°} \approx 14.5$

$12 + 14.5 \approx 26.5$ m

The tree's original height was approximately 26.5 meters.

3. $\alpha = 90° - 29°24'54''$
$= (89° + 59' + 60'') - 29°24'54''$
$= (89° - 29°) + (59' - 24') + (60'' - 54'')$
$= 60° + 35' + 06''$
$= 60°35'06''$

$60°35'06'' = \left[60 + 35\left(\frac{1}{60}\right) + 6\left(\frac{1}{3600}\right)\right]°$
$= 60.585°$

4. QI; all trig functions are positive

$\theta_r = 729.5° - 2 \cdot 360° = 9.5°$

$\sin 9.5 = 0.1650$

$\cos 9.5 = 0.9863$

$\tan 9.5 = 0.1673$

5. Let $h$ be the height of the waterfall.

$\tan 60° = \frac{h}{66}$

$h = 66 \tan 60° \approx 114.3$ ft

6. a. $\tan 77° = \frac{w}{60}$; $w = 60 \tan 77° \approx 260$ ft

b. $\frac{260 \text{ ft}}{29 \text{ sec}} \cdot \frac{3600 \text{ sec}}{1 \text{ hr}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \approx 6.1$ mph

7. $x = -9$, $y = 40$, $r = \sqrt{(-9)^2 + 40^2} = 41$

The angle is in QII.

$\sin\theta = \frac{40}{41}$; $\csc\theta = \frac{41}{40}$; $\cos\theta = -\frac{9}{41}$

$\sec\theta = -\frac{41}{9}$; $\tan\theta = -\frac{40}{9}$; $\cot\theta = -\frac{9}{40}$

$\theta = \tan^{-1}\left(-\frac{40}{9}\right) \approx -77.3°$

In QII, $180° - 77.3° = 102.7°$.

8. $(\sin t)(\sec t) = 1$ False.

The reciprocal of $\sin t$ is $\csc t$.

$(\sin t)(\csc t) = 1$

9. The cofunction of sine is cosine because $\cos(90 - \theta) = \sin\theta$.

10. $\tan t = -\frac{68}{51} \Rightarrow x = -51$, $y = 68$

(Since $\sin t > 0$, $y$ is positive.)

The angle $\theta$ is in quadrant II.

$r = \sqrt{(-51)^2 + 68^2} = \sqrt{7225} = 85$

$\sin\theta = \frac{68}{85}$; $\csc\theta = \frac{85}{68}$; $\cos\theta = -\frac{51}{85}$

$\sec\theta = -\frac{85}{51}$; $\tan\theta = -\frac{68}{51}$; $\cot\theta = -\frac{51}{68}$

11. $\theta = -100°$; $\theta + 360k$

$k = 2$; $-100° + 360°(2) = 620°$

$k = 1$; $-100° + 360°(1) = 260°$

$k = -1$; $-100° + 360°(-1) = -460°$

$k = -2$; $-100° + 360°(-2) = -820°$

12. $$\frac{6x+1}{6}=\frac{7.5}{3x}$$
$$(6x+1)\cdot 3x = 6\cdot 7.5$$
$$18x^2+3x=45$$
$$18x^2+3x-45=0$$
$$3(6x^2+x-15)=0$$
$$3(3x+5)(2x-3)=0$$
$$3x+5=0 \qquad 2x-3=0$$
$$3x=-5 \qquad 2x=3$$
$$x=-\frac{5}{3} \qquad x=\frac{3}{2}$$

We must discard $x=-\frac{5}{3}$.

$$BC=6x+1=6\left(\frac{3}{2}\right)+1=9+1=10 \text{ cm}$$
$$AC=\sqrt{7.5^2+10^2}=\sqrt{156.25}=12.5 \text{ cm}$$
$$DE=3x=3\left(\frac{3}{2}\right)=4.5 \text{ cm}$$
$$DF=\sqrt{4.5^2+6^2}=\sqrt{56.25}=7.5 \text{ cm}$$

13. If $\cos(\theta-90°)=-1$, then $\theta-90°=180°$ and $\theta=270°$.

14. $$\tan\theta(\cos\theta+\csc\theta)=\sin\theta+\sec\theta$$
$$\tan\theta\cdot\cos\theta+\tan\theta\cdot\csc\theta=\sin\theta+\sec\theta$$
$$\frac{\sin\theta}{\cos\theta}\cdot\cos\theta+\frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\sin\theta}=\sin\theta+\sec\theta$$
$$\sin\theta+\frac{1}{\cos\theta}=\sin\theta+\sec\theta$$
$$\sin\theta+\sec\theta=\sin\theta+\sec\theta$$

15. If $a=b=12.1$ in., we have a 45-45-90 triangle, and $\sin\beta=\frac{\sqrt{2}}{2}$.

16. $A=90°-23°=67°$

$\tan 23°=\frac{3}{a};\ a\tan 23°=3$

$a=\frac{3}{\tan 23°}\approx 7.07$ ft

$\sin 23°=\frac{3}{c};\ c\sin 23°=3$

$c=\frac{3}{\sin 23°}\approx 7.68$ ft

| Angle | Side |
|---|---|
| $A=67°$ | $a\approx 7.07$ ft |
| $B=23°$ | $b=3$ ft |
| $C=90°$ | $c\approx 7.68$ ft |

17. $\tan A=\frac{a}{b}=\frac{11}{10};\ A=\tan^{-1}\frac{11}{10}\approx 47.7°$

$B=90°-47.7°=42.3°$

$c=\sqrt{11^2+10^2}=\sqrt{221}\approx 14.87$ m

| Angle | Side |
|---|---|
| $A\approx 47.7°$ | $a=11$ m |
| $B\approx 42.3°$ | $b=10$ m |
| $C=90°$ | $c\approx 14.87$ m |

18. $$\begin{aligned}\theta&=90°-11°18'36''\\&=(89°+59'+60'')-11°18'36''\\&=(89°-11°)+(59'-18')+(60''-36'')\\&=78°+41'+24''\\&=78°41'24''\end{aligned}$$

$$78°41'24''=\left[78+41\left(\frac{1}{60}\right)+24\left(\frac{1}{3600}\right)\right]°=78.69°$$

Let $x$ be the width of the property.

$\tan(78.69°)=\frac{150}{x};\ x\tan(78.69°)=150$

$x=\frac{150}{\tan(78.69°)}\approx 30$ ft

The property is 30 feet wide.

19. QIV; sine is negative; cosine is positive; tangent is negative

$\theta_r=-1\cdot 360°-(-390°)=30°$

$\sin(-390°)=-\frac{1}{2}$

$\cos(-390°)=\frac{\sqrt{3}}{2}$

$\tan(-390°)=-\frac{\sqrt{3}}{3}$

20. $$\begin{aligned}64.5575°&=64°+0.5575(60)'=64°+33.45'\\&=64°33'+0.45(60)''=64°33'27''\end{aligned}$$

21. The triangle is not possible. $a+c<b$

22. $\csc\theta=\frac{1}{\sin\theta}=\pm\frac{1}{\sqrt{1-\cos^2\theta}}$

23. $c = \sqrt{7^2 + \left(\sqrt{15}\right)^2} = \sqrt{49+15} = \sqrt{64} = 8$

$\sin\beta = \frac{\sqrt{15}}{8};\ \cos\beta = \frac{7}{8};\ \tan\beta = \frac{\sqrt{15}}{7}$

$\csc\beta = \frac{8}{\sqrt{15}} = \frac{8\sqrt{15}}{15};\ \sec\beta = \frac{8}{7}$

$\cot\beta = \frac{7}{\sqrt{15}} = \frac{7\sqrt{15}}{15}$

24. Cosine is negative in QII and QIII.

$\cos\theta = -0.29;\ \theta = \cos^{-1}(-0.29) \approx 106.9°$

$\theta_r = 180° - 106.9° = 73.1°$

$180° + 73.1° = 253.1°$

$\theta = 106.9° + 360°k;\ \theta = 253.1° + 360°k$

25. $a = \frac{b}{\sqrt{3}} = \frac{5}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$ in.

$c = 2a = 2 \cdot \frac{5\sqrt{3}}{3} = \frac{10\sqrt{3}}{3}$ in.